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SOME NEW PROPERTIES OF FUZZY STRONGLY g^*-CLOSED SETS AND δg^*-CLOSED SETS IN FUZZY TOPOLOGICAL SPACES

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Abstract. In this paper, a new class of fuzzy sets called fuzzy strongly g^*-closed sets is introduced and its properties are investigated. Moreover, we study some more properties of this type of closed spaces.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh in his classical paper [21]. Subsequently, many researchers have been worked in this area and other related areas which have applications in different fields based on this concept. Topology is Analysis in a general setting. As a generalization of topological spaces Chang [6] introduced the concept of fuzzy topological space in 1968. In 1986, Maki [10] continued the works of Levine [8] and Dunham [7] on generalized closed sets and closure operators by introducing the notion of a generalized Λ-set in a topological space (X, τ) and by defining an associated closure operator, i.e. the Λ-closure operator. He studied the relationships between the given topology τ and the topology τ^Λ generated by the family of generalized Λ-sets. g^*-closed sets were introduced and studied by Veerakumar [19] for general topology. Recently, Parimelazhagan and Pillai introduced strongly g^*-closed sets in topological space [14].
In the present paper, we introduce fuzzy strongly g^*-closed sets in fuzzy topological space (FTS) and investigate certain basic properties of these fuzzy sets.

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2. PRELIMINARIES

A family τ of fuzzy sets of X is called a fuzzy topology on X if 0 and 1 belong to τ and τ is closed with respect to arbitrary union and finite intersection. The members of τ are called fuzzy open sets and their complements are fuzzy closed sets.
Throughout this paper (X,τ) represent non empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), $cl\,(A)$ and $\text{int}\,(A)$ denote the closure and the interior of A respectively. (X,τ) will be replaced by X if there is no chance of confusion. We define the closure of a fuzzy set A by

$$cl\,(A) = \wedge \{\mu : \ \mu \geq A, \ 1-\mu \in \tau\} \tag{2.1}$$

and interior

$$\text{int}\,(A) = \vee \{\mu : \ \mu \leq A, \ \mu \in \tau\}\,. \tag{2.2}$$

The semi closure [17] (respectively pre-closure [3], α-closure [15] and semi pre-closure [16]) of a fuzzy set A of (X,τ) is the intersection of all fs-closed (respectively fp-closed, $f\alpha$-closed and fsp-closed) sets that contain A and is denoted by $scl\,(A)$ (respectively $pcl\,(A)$, $\alpha cl\,(A)$ and $spcl\,(A)$).

We recall some basic definitions and notations of most essential concepts needed in the following.

Definition 2.1. A fuzzy set A of (X,τ) is called,

(i) semi open (in short, fs-open) if $A \leq cl\,(\text{int}\,(A))$ and a fuzzy semi closed (fs-closed) if $\text{int}\,(cl\,(A)) \leq A$.
(ii) fuzzy pre-open (fp-open) if $A \leq \text{int}\,[cl\,(A)]$ and a fuzzy pre-closed (fp-closed) if $cl\,(\text{int}\,(A)) \leq A$.
(iii) fuzzy α-open ($f\alpha$-open) if $A \leq \text{int}\,[cl\,(\text{int}\,(A))]$ and a fuzzy α-closed ($f\alpha$-closed) if $cl\,(\text{int}\,[cl\,(A)]) \leq A$.
(iv) fuzzy semi pre-open (fsp-open) if $A \leq cl\,(\text{int}\,[cl\,(A)])$ and a fuzzy semi pre-closed (fsp-closed) if $\text{int}\,[cl\,(\text{int}\,(A))] \leq A$.
(v) fuzzy θ-open ($f\theta$-open) if $A = \text{int}_\theta\,(A)$ and a fuzzy θ-closed ($f\theta$-closed) if $A = cl_\theta\,(A)$ where $cl_\theta\,(A) = \wedge \{cl\,(\mu) : \ A \leq \mu, \ \mu \in \tau\}$.
(vi) fuzzy generalized closed (fg-closed) if $cl\,(A) \leq H$, whenever $A \leq H$ and H is fuzzy open set in X.
(vii) fuzzy generalized semi closed (gfs-closed) if $scl\,(A) \leq H$, whenever $A \leq H$ and H is fs-open set in X. This set is also called generalized fuzzy weakly semi closed set.
(viii) fuzzy generalized semi closed (fgs-closed) if $scl\,(A) \leq H$, whenever $A \leq H$ and H is fuzzy open set in X.
(ix) fuzzy generalized pre-closed (fgp-closed) if $pcl\,(A) \leq H$, whenever $A \leq H$ and H is fuzzy open set in X.

(x) fuzzy α-generalized closed ($f\alpha g$-closed) if $\alpha cl\,(A) \leq H$, whenever $A \leq H$ and H is fuzzy open set in X.
(xi) fuzzy generalized α-closed ($fg\alpha$-closed) if $\alpha cl\,(A) \leq H$, whenever H is fuzzy open set in X.
(xii) fuzzy generalized semi pre-closed (fsp-closed) if $spcl\,(A) \leq H$, whenever $A \leq H$ and H is fuzzy open set in X.
(xiii) fuzzy semi pre-generalized closed ($fspg$-closed) if $spcl\,(A) \leq H$, whenever $A \leq H$ and H is fs-open in X.
(xiv) fuzzy θ-generalized closed ($f\theta g$-closed) if $cl_\theta\,(A) \leq H$, whenever $A \leq H$ and H is fuzzy open in X.
(xv) fuzzy g^*-closed (fg^*-closed) if $cl\,(A) \leq H$, whenever $A \leq H$ and H is fg-open in X.

Definition 2.2. A fuzzy set in X is called a fuzzy point if and only if it takes the value 0 for all $y \in X$ except one, say $x \in X$. If its value at x is p $(0 < p \leq 1)$, we denote this fuzzy point by x_p, where the point x is called its support.

Definition 2.3. A fuzzy point $x_p \in A$ is said to be quasi-coincident with the fuzzy set A denoted by $x_p q A$ if and only if $p + A\,(x) > 1$. A fuzzy set A is quasi-coincident with a fuzzy set B denoted by AqB if and only if there exists $x \in X$ such that $A\,(x) + B\,(x) > 1$. If A and B are not quasi-coincident, then we write $A\overline{q}\,B$. Note that $A \leq B$ if and only if $A\overline{q}\ (1 - B)$.

3. δg^*-closed Sets

We need the following definitions in the sequel.

Definition 3.1. A fuzzy point x_a is said to be a fuzzy δ-cluster point of a fuzzy set A in a fts X if every fuzzy regular open quasi neighborhood of H of x_a is quasi coincident with A.

Definition 3.2. The union of all fuzzy δ-cluster points of A is called the fuzzy δ-closure of A, is denoted by $\delta cl\,(A)$.

Definition 3.3. A subset A of a topological space (X, τ) is called a fuzzy g-closed set if $fcl\,(A) \leq H$, whenever $A \leq H$ and H is open in (X, τ).

Definition 3.4. The complement of a fg-closed set is called a fg-open set.

Definition 3.5. A subset A of a space X is called $f\delta g$-closed if $fcl_\delta\,(A) \leq H$, whenever $A \leq H$ and H is a fuzzy open set.

Definition 3.6. A fuzzy set A in fts (X, τ) is called fuzzy δg^*-closed if and only if $fcl_\delta\,(A) \leq B$, whenever $A \leq B$ and B is fuzzy g-open in X.

In the following, we investigate the on δg^*-closed sets and study some of its fundamental properties. Several characterizations of δg^*-closed sets are given.

Theorem 3.7. *Every fuzzy δ-closed set is a fuzzy δg^*-closed set in (X,τ).*

Proof. Let A be a fuzzy δ-closed set in a fts X and B be a fuzzy g-open set in X such that $A \leq B$. Since A is a fuzzy δ-closed, $fcl_\delta(A) = A$. Therefore $fcl_\delta(A) = A \leq B$. Hence A is a fuzzy δg^*-closed set. □

Theorem 3.8. *If A is fuzzy δ-open and fuzzy δg^*-closed in (X,τ), then A is fuzzy δ-closed in (X,τ).*

Proof. Let A be fuzzy δ-open and fuzzy δg^*-closed in X. Suppose $A \leq A$, then $fcl_\delta(A) \leq A$. But $A \leq fcl_\delta(A)$, which implies that $fcl_\delta(A) = A$. Hence A is fuzzy δ-closed. □

Theorem 3.9. *Let (X,τ) be a fts and A be a fuzzy set of X. Then A is fuzzy δg^*-closed if and only if $A\bar{q}\,B$ implies $fcl_\delta(A)\,\bar{q}\,B$ for every fuzzy δg^*-closed set B of (X,τ).*

Proof. Suppose A is a fuzzy δg^*-closed set of X. Let B be a fuzzy g-closed set in X such that $A\bar{q}\,B$. Then $A \leq 1-B$ and $1-B$ is a fuzzy g-open set of X. Therefore $fcl_\delta(A) \leq 1-B$, as A is fuzzy δg^*-closed. Hence $fcl_\delta(A)\,\bar{q}\,B$. Conversely, let D be a fuzzy g-open set in X such that $A \leq D$. Then $A\bar{q}\ (1-D)$ and $1-D$ is a fuzzy g-closed set in X. By hypothesis, $fcl_\delta(A)\,\bar{q}\ (1-D)$ which implies, $cl_\delta(A) \leq D$. Hence A is fuzzy δg^*-closed. □

Theorem 3.10. *Let A be a fuzzy δg^*-closed set in (X,τ) and x_p be a fuzzy point of (X,τ) such that $x_p q fcl_\delta(A)$, then $fcl_\delta(x_p)\,qA$.*

Proof. Let A be a fuzzy δg^*-closed set in (X,τ) and x_p be a fuzzy point of (X,τ) such that $x_p q fcl_\delta(A)$. Suppose $fcl_\delta(x_p)\,\bar{q}\,A$, then $fcl_\delta(x_p) \leq 1-A$ and hence $A \leq 1-fcl_\delta(x_p)$. Now, $1-fcl_\delta(x_p)$ is fuzzy δ-open and hence fuzzy δg-open. Moreover, since A is fuzzy δg^*-closed, $fcl_\delta(A) \leq 1-fcl_\delta(x_p) \leq 1-x_p$. Hence $x_p\bar{q}\,fcl_\delta(A)$, which is a contradiction. □

Theorem 3.11. *If A is a fuzzy δg^*-closed set in (X,τ) and $A \leq B \leq fcl_\delta(A)$, then B is a fuzzy δg^*-closed set in (X,τ).*

Proof. Let A be a fuzzy δg^*-closed set in (X,τ). Given $A \leq B \leq fcl_\delta(A)$. Suppose $B \leq H$ where H is fuzzy g-open set. Since $A \leq B \leq H$ and A is a fuzzy δg^*-closed set, we get $fcl_\delta(A) \leq H$. As $B \leq fcl_\delta(A)$,

$fcl_\delta(B) \le fcl_\delta(fcl_\delta(A)) = fcl_\delta(A)$ we get $fcl_\delta(B) \le H$. Hence B is a fuzzy δg^*-closed set in (X,τ). □

Theorem 3.12. *If A is a fuzzy δg^*-open set in (X,τ) and $f\mathrm{int}_\delta(A) \le B \le A$, then B is a fuzzy δg^*-open set in (X,τ).*

Proof. Let A be fuzzy δg^*-open set and B be any fuzzy set in X such that $f\mathrm{int}_\delta(A) \le B \le A$. Then $1-A$ is a fuzzy δg^*-closed set and $1-A \le 1-B \le fcl_\delta(1-A)$, as $1-f\ \mathrm{int}_\delta(A) = fcl_\delta(1-A)$. Therefore $1-B$ is a fuzzy δg^*-closed. Hence B is fuzzy δg^*-open. □

4. Fuzzy strongly g^*-closed sets in fuzzy topological spaces

Definition 4.1. Let (X,τ) be a fuzzy topological space. A fuzzy set A of (X,τ) is called fuzzy strongly g^*-closed if $cl(\mathrm{int}(A)) \le H$, whenever $A \le H$ and H is fg-open in X.

Theorem 4.2. *Every fuzzy closed set is a fuzzy strongly g^*-closed set in the fuzzy topological space (X,τ).*

Proof. Let A be fuzzy closed set in X and H be a fg-open set in X such that $A \le H$. Since A is fuzzy closed, $cl(A) = A$. Therefore $cl(A) \le H$. Now, $cl(\mathrm{int}(A)) \le cl(A) \le H$. Hence A is fuzzy strongly g^*-closed set in X. □

Remark 4.3. The converse of the Theorem 4.2 need not be true in general. For example, let $X = \{a,b,c\}$. Fuzzy sets A and B are defined by $A(a) = 0.7$, $A(b) = 0.3$, $A(c) = 0.5$, $B(a) = 0.2$, $B(b) = 0.1$ and $B(c) = 0.3$. Let $\tau = \{0, A, 1\}$. Then B is a fuzzy strongly g^*-closed set but it is not a fuzzy closed set in (X,τ).

Remark 4.4. Every fuzzy g-closed set is a fuzzy strongly g^*-closed set in X. But the converse of this state need not be true. For example, let $X = \{a,b\}$, $\tau = \{0, B, 1\}$ and the fuzzy sets A and B be defined as follows: $A(a) = 0.3$, $A(b) = 0.3$, $B(a) = 0.5$ and $B(b) = 0.4$. Then A is a fuzzy strongly g^*-closed while it is not fg-closed.

Theorem 4.5. *Every fuzzy g^*-closed set is a fuzzy strongly g^*-closed set in (X,τ).*

Proof. Suppose that A is fg^*-closed in X. Let H be a fg-open set in X such that $A \le H$. Then $cl(A) \le H$, since A is fg^*-closed. Now, $cl(\mathrm{int}(A)) \le cl(A) \le H$, hence A is fuzzy strongly g^*-closed set in X. □

However the converse of the Theorem 4.5 need not be true as seen from the following example:

Example 4.6. Let $X = \{a, b\}$, $\tau = \{0, A, B, D, 1\}$ and fuzzy sets A, B, D and H are defined as follows: $A(a) = 0.2$, $A(b) = 0.4$, $B(a) = 0.6$, $B(b) = 0.7$, $D(a) = 0.4$, $D(b) = 0.6$, $H(a) = 0.4$ and $H(b) = 0.5$. Then H is fuzzy strongly g^*-closed set but it is not fg^*-closed in (X, τ).

Theorem 4.7. *A fuzzy set A of fuzzy topological space (X, τ) is fuzzy strongly g^*-closed if and only if $A \bar{q} B$, then $cl(\text{int}(A)) \bar{q} B$ for every fg-closed set B of (X, τ).*

Proof. Suppose that A is a fuzzy strongly g^*-closed set of X such that $A \bar{q} B$. Then $A \leq 1 - B$ and $1 - B$ is a fuzzy g-open set in X, which implies that $cl(\text{int}(A)) \leq 1 - B$, since A is fuzzy strongly g^*-closed. Hence $cl(\text{int}(A)) \bar{q} B$. Now, let E be a fuzzy g-open set in X such that $A \leq E$. Then $A \bar{q}\, 1 - E$ and $1 - E$ is fg-closed set in X. By hypothesis, $cl(\text{int}(A)) \bar{q} (1 - E)$ which implies that $cl(\text{int}(A)) \leq E$. Hence A is fuzzy strongly g^*-closed in X. □

Theorem 4.8. *Let A be a fuzzy strongly g^*-closed set in (X, τ) and x_p be a fuzzy point of (X, τ) such that $x_p q cl(int(A))$ then $cl(int(x_p)) qA$.*

Proof. Let A be a fuzzy strongly g^*-closed set in (X, τ) and x_p be a fuzzy point of (X, τ) such that $x_p q cl(\text{int}(A))$. Suppose $cl(\text{int}(x_p)) \bar{q} A$, then $cl(\text{int}(x_p)) q 1 - A$ and hence $A \leq 1 - cl(\text{int}(x_p))$. Now, $1 - cl(\text{int}(x_p))$ is fuzzy open. Moreover, since A is fuzzy strongly g^*-closed, $cl(\text{int}(A)) \leq 1 - cl(\text{int}(x_p)) \leq 1 - x_p$. Hence $x_p \bar{q}\, cl(\text{int}(A))$, which is a contradiction. □

Theorem 4.9. *If A is a fuzzy strongly g^*-closed set in (X, τ) and $A \leq B \leq cl(\text{int}(A))$, then B is fuzzy strongly g^*-closed in (X, τ).*

Proof. Let A be a fuzzy strongly g^*-closed set in (X, τ) and $B \leq H$ where H is a fuzzy g-open set in X. Then $A \leq H$. Since A is a fuzzy strongly g^*-closed set, it follows that $cl(\text{int}(A)) \leq H$. Now, $B \leq cl(\text{int}(A))$ implies $cl(\text{int}(B)) \leq cl(\text{int}(cl(\text{int}(A)))) = cl(\text{int}(A))$. We get, $cl(\text{int}(B)) \leq H$. Hence, B is a fuzzy strongly g^*-closed set in (X, τ). □

Definition 4.10. A fuzzy set A of (X, τ) is called fuzzy strongly g^*-open set in X if and only if $1 - A$ is fuzzy strongly g^*-closed in X.

In other words, A is fuzzy strongly g^*-open if and only if $H \leq cl(\text{int}(A))$, whenever $H \leq A$ of H is fg-closed in X.

Theorem 4.11. *Let (Y, τ_Y) be a subspace of a fuzzy topological space (X, τ) and A be a fuzzy set of Y. If A is fuzzy strongly g^*-closed in X, then A is a fuzzy strongly g^*-closed in Y.*

Proof. Let Y be a subspace of X and H be a fg-open set in Y such that $A \leq H$. We have to prove that $cl_y(\text{int}_y(A)) \leq H$. Since H is fg-open in Y, we have $H = G \cap Y$ where G is fg-open in X. Hence $A \leq H = G \cap Y$ implies $A \leq G$ and A is fuzzy strongly g^*-open in X. We get $cl(\text{int}(A)) \leq G$. Therefore $cl(\text{int}(A)) \cap Y \leq G \cap Y = H$. Thus $cl(\text{int}(A)) \leq H$, whenever $A \leq H$ and H is fuzzy g-open in Y. Hence A is fuzzy strongly g^*-open in Y. □

Theorem 4.12. *If a fuzzy set A of a fuzzy topological space X is both fuzzy open and fuzzy strongly g^*-closed, then it is fuzzy closed.*

Proof. Suppose that a fuzzy set A of X is both fuzzy open and fuzzy strongly g^*-closed. Now, $A \geq cl(\text{int}(A)) \geq cl(A)$. That is $A \geq cl(A)$, since $A \leq cl(A)$. So we get $A = cl(A)$. Hence A is fuzzy closed in X. □

Theorem 4.13. *If a fuzzy set A of a fuzzy topological space X is both fuzzy strongly g^*-closed and fuzzy semi open, then it is fg^*-closed.*

Proof. Suppose a fuzzy set A of X is both fuzzy strongly g^*-closed and fuzzy semi open in X. Let H be a fg-open set such that $A \leq H$. Since A is fuzzy strongly g^*-closed, therefore $cl(\text{int}(A)) \leq H$. Also since A is fs-open, $A \leq cl(\text{int}(A))$. We have $cl(A) \leq cl(\text{int}(A)) \leq H$. Hence A is fg^*-closed in X. □

Remark 4.14. Every $f\theta$-closed set is a fuzzy strongly g^*-closed set, but the converse is not true in general. For example, Let $X = \{a, b\}$, $\tau = \{0, A, 1\}$ and fuzzy sets A and B are defined as follows: $A(a) = 0.3$, $A(b) = 0.7$, $B(a) = 0.6$ and $B(b) = 0.5$. Then B is strongly g^*-closed but it is not $f\theta$-closed.

Every fp-closed, fsp-closed, gfs-closed, $fg\alpha$-closed and $fspg$-closed sets are fuzzy strongly g^*-closed, but the converse may not be true in general.
We are now ready to construct our main examples.

Example 4.15. Let $X = \{a, b\}$, $\tau = \{0, A, 1\}$ and fuzzy sets A and B are in defined by $A(a) = 0.8$, $A(b) = 0.2$, $B(a) = 0.9$ and $B(b) = 0.6$. Then B is a fuzzy strongly g^*-closed in (X, τ), while it is not a fp-closed set in (X, τ).

Example 4.16. Let $X = \{a, b\}$ and $\tau = \{0, A, 1\}$. Let fuzzy sets A and B are defined in X by $A(a) = 0.2$, $A(b) = 0.6$, $B(a) = 0.5$ and $B(b) = 0.7$. Then B is a fuzzy strongly g^*-closed set in (X, τ), while it is not a fsp-closed set in (X, τ).

Example 4.17. Let $X = \{a, b, c\}$, $\tau = \{0, A, 1\}$ and fuzzy sets A and B are defined by $A(a) = 0.8$, $A(b) = 0.3$, $A(c) = 0.1$, $B(a) = 0.8$,

$B(b) = 0.1$ and $B(c) = 0.1$. Then B is a strongly g^*-closed set in (X, τ), while it is neither fgs-closed.

Example 4.18. Let $X = \{a, b\}$ and $\tau = \{0, A, 1\}$. Define fuzzy set A, B and D in X by $A(a) = 0.3$, $A(b) = 0.6$, $B(a) = 0.6$, $B(b) = 0.6$, $D(a) = 0.9$ and $D(b) = 0.8$. Then B is fuzzy strongly g^*-closed, while it is not $fspg$-closed.

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