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A NEW SEQUENCE SPACE AND NORM OF CERTAIN MATRIX OPERATORS ON THIS SPACE

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ABSTRACT. In the present paper, we introduce the sequence space

$$l_p(E,\Delta)=\left\{x=(x_n)_{n=1}^{\infty}:\sum_{n=1}^{\infty}\left|\sum_{j\in E_n}x_j-\sum_{j\in E_{n+1}}x_j\right|^p<\infty\right\},$$

where $E=(E_n)$ is a partition of finite subsets of the positive integers and $p\geq 1$. We investigate its topological properties and inclusion relations. Moreover, we consider the problem of finding the norm of certain matrix operators from l_p into $l_p(E,\Delta)$, and apply our results to Copson and Hilbert matrices.

1. Introduction

Suppose that ω is the space of all real-valued sequences. Any vector subspace of ω is called a sequence space. Suppose that $E=(E_n)$ is a partition of finite subsets of the positive integers such that

$$\max E_n<\min E_{n+1}, \tag{1.1}$$

for $n=1,2,\ldots$. We introduce the sequence space $l_p(E)$ by

$$l_p(E)=\left\{x=(x_n)\in\omega:\sum_{n=1}^{\infty}\left|\sum_{j\in E_n}x_j\right|^p<\infty\right\},\quad(1\leq p<\infty),$$

with the semi-norm

$$\|x\|_{p,E}=\left(\sum_{n=1}^{\infty}\left|\sum_{j\in E_n}x_j\right|^p\right)^{1/p}.$$

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It should be noted that in the special case $E_n = \{n\}$ for $n = 1, 2, \ldots$, we have $l_p(E) = l_p$ and $\|x\|_{p,E} = \|x\|_p$. The reader can refer to [5], for more details on this sequence space $l_p(E)$.

Also, the difference sequence space $l_p(\Delta)$ is introduced by Kizmaz [9], which is defined by

$$l_p(\Delta) = \left\{ x = (x_n) : \sum_{n=1}^{\infty} |x_n - x_{n+1}|^p < \infty \right\},$$

with semi-norm

$$\|x\|_{p,\Delta} = \left(\sum_{n=1}^{\infty} |x_n - x_{n+1}|^p \right)^{\frac{1}{p}}.$$

Suppose that X, Y are two sequence spaces and $A = (a_{nk})$ is an infinite matrix of real numbers a_{nk}, where $n, k \in \mathbb{N} = \{1, 2, \ldots\}$. It is said that A defines a matrix mapping from X into Y, and is denoted by $A : X \to Y$, if for every sequence $x = (x_k) \in X$ the sequence $Ax = \{(Ax)_n\}_{n=1}^{\infty}$ exists and is in Y, where

$$(Ax)_n = \sum_{k=1}^{\infty} a_{nk} x_k,$$

for $n = 1, 2, \ldots$.

Let X be a sequence space. The matrix domain X_A of an infinite matrix A is defined by

$$X_A = \{x = (x_n) \in \omega : Ax \in X\}. \tag{1.2}$$

Note that X_A is a sequence space that can be the expansion or contraction or the overlap of the original space X. A matrix $A = (a_{nk})$ is said a triangle if $a_{nk} = 0$ for $k > n$ and $a_{nn} \neq 0$ for all $n \in \mathbb{N}$. The sequence spaces X_A and X are linearly isomorphic, i.e., $X_A \cong X$, where A is triangle.

The matrix transformations on sequence spaces that are the matrix domains of triangle matrices has been investigated for classical spaces l_p, l_∞, c and c_0, before. For example, some matrix domains of the difference operator are considered in [1, 3, 4, 9, 11]. In these studies the matrix domains are gained by triangle matrices, hence these spaces are normed sequence spaces. One can refer to Chapter 4 of [2], for more details on the domain of triangle matrices in some sequence spaces. The matrix domains which are presented in this paper are specified by the certain non-triangle matrices, so we should not expect that related spaces are normed sequence spaces.

In this paper, we want to extend the normed sequence space $l_p(\Delta)$ to semi-normed space $l_p(E, \Delta)$, investigate some topological properties of this space and derive inclusion relations concerning with its. Moreover, we investigated the inequality

$$\|Ax\|_{p,E,\Delta} \leq U\|x\|_p,$$

for all sequence $x \in l_p$. The constant U is not depending on x, and we want to find the smallest possible value of U. We use the notation $\|A\|_{p,E,\Delta}$ for the norm of A as an operator from l_p into $l_p(E,\Delta)$, and $\|A\|_{p,\Delta}$ for the norm of A as an operator from l_p into $l_p(\Delta)$. Recently, the problem of finding the upper bound of certain matrix operators are studied in [6, 8, 10] on the sequence spaces $l_p(w)$, $d(w,p)$ and $l_p(\Delta)$. In the present paper, we compute this problem for matrix operators such as Copson and Hilbert from l_p into $l_p(E,\Delta)$.

In a similar way, the Authors introduced the sequence space $l_p(\Delta, E)$ and obtained the norm of certain matrix operators on this space [12].

2. The sequence space $l_p(E,\Delta)$ of non-absolute type

Let $E = (E_n)$ be a partition of the positive integers that satisfies the condition (1.1). We define the sequence space $l_p(E,\Delta)$ by

$$l_p(E,\Delta) = \left\{ x = (x_n)_{n=1}^{\infty} : \sum_{n=1}^{\infty} \left| \sum_{j\in E_n} x_j - \sum_{j\in E_{n+1}} x_j \right|^p < \infty \right\},$$

with the semi-norm

$$\|x\|_{p,E,\Delta} = \left(\sum_{n=1}^{\infty} \left| \sum_{j\in E_n} x_j - \sum_{j\in E_{n+1}} x_j \right|^p \right)^{1/p}. \tag{2.1}$$

It should be noted that the function $\|.\|_{p,E,\Delta}$ is not a norm, since by choosing $x = (1,1,1,\ldots)$ and $E_n = \{2n-1, 2n\}$ for all n, $\|x\|_{E,\Delta} = 0$ while $x \neq 0$. It is also significant that in the special case $E_n = \{n\}$ for $n = 1, 2, \ldots$, we have

$$\|x\|_{p,E,\Delta} = \|x\|_{p,\Delta}, \qquad l_p(E,\Delta) = l_p(\Delta).$$

By the notation of (1.2), we can redefine the space $l_p(E,\Delta)$ as follows:

$$l_p(E,\Delta) = (l_p)_A,$$

where $A = (a_{nk})$ is defined by

$$a_{nk} = \begin{cases} 1 & \text{if } k \in E_n \\ -1 & \text{if } k \in E_{n+1} \\ 0 & \text{otherwise,} \end{cases}$$

Throughout this study, we assume $p \geq 1$ and $E = (E_n)$ is a partition of finite subsets of the positive integers that satisfies the condition (1.1), and also $|E_k|$ is the cardinal number of the set E_k. The main purpose of this section is to consider some properties of the sequence space $l_p(E,\Delta)$ and is to derive some inclusion relations related to these spaces. At first we bring the following theorem which is essential in the study.

Theorem 2.1. *The set $l_p(E,\Delta)$ becomes a vector space with coordinatewise addition and scalar multiplication, which is the complete seminormed space by $\|.\|_{p,E,\Delta}$ defined by (2.1).*

Proof. The proof is routine, so we omit the details. □

It must be mentioned that the absolute property does not hold on the space $l_p(E,\Delta)$, that is $\|x\|_{p,E,\Delta} \neq \||x|\|_{p,E,\Delta}$ for at least one sequence in the space $l_p(E,\Delta)$, and this says that $l_p(E,\Delta)$ is a sequence space of nonabsolute type, where $|x| = (|x_k|)$.

Theorem 2.2. *If*

$$K = \left\{ x = (x_n) : \sum_{i \in E_n} x_i = \sum_{i \in E_{n+1}} x_i, \forall\, n \right\},$$

then we have $l_p(E,\Delta)/K \simeq l_p(\Delta)$.

Proof. Consider the map $T : l_p(E,\Delta) \longrightarrow l_p(\Delta)$ defined by

$$(Tx)_n = \sum_{i \in E_n} x_i,$$

for all $x \in l_p(\Delta, E)$ and for all n. The map T is well defined and surjective also $\ker T = K$. So the proof is finished by applying the first isomorphism. □

Theorem 2.3. *We have the following statements:*

(i) $l_p(E) \subset l_p(E,\Delta)$, *furthermore the inclusion is strictly holds.*

(ii) *If*

$$E_n = \{Nn - N + 1, Nn - N + 2, \dots, Nn\}$$

for all n, then $l_p(\Delta) \subset l_p(E,\Delta)$. Moreover, this inclusion is strict when $N > 1$.

Proof. (i) By using the inequality $\|x\|_{p,E,\Delta} \leq 2\|x\|_{p,E}$, the proof is obvious. If the sequence $x = (x_k)$ is defined such that $\sum_{i \in E_k} x_i = 1$, for $k = 1, 2, \dots$. We have $x \in l_p(E,\Delta)$ while $x \notin l_p(E)$, hence the inclusion is strictly holds.

(ii) Since

$$\begin{aligned}\sum_{i\in E_n} x_i - \sum_{i\in E_{n-1}} x_i = &(x_{nN-N+1} - x_{nN-N+2})\\ &+ 2(x_{nN-N+2} - x_{nN-N+3})\\ &+ \cdots + N(x_{nN} - x_{nN+1})\\ &+ (N-1)(x_{nN+1} - x_{nN+2})\\ &+ \cdots + (x_{nN+N-1} - x_{nN+N}),\end{aligned}$$

It is clear $x \in l_p(\Delta)$ implies that $x \in l_p(E, \Delta)$, by applying Minkowski's inequality. Moreover if $N > 1$, we define the sequences $x = (x_k)$ such that

$$x_k = \begin{cases} 1 & \text{if } k = nN - N + 1 \\ -1 & \text{if } k = nN - N + 2 \\ 0 & \text{otherwise,} \end{cases}$$

obviously $x \in l_p(E, \Delta) - l_p(\Delta)$.

□

In general, neither of the spaces $l_p(E, \Delta)$ and $l_p(\Delta)$ includes the other one. Since if $E_{2n-1} = \{3n-2\}$, $E_{2n} = \{3n-1, 3n\}$ for $n = 1, 2, \ldots$, $x = (1, 1, 1, \ldots.)$ and $y = (0, 1, -1, 0, 1, -1, \ldots)$, we have $x \in l_p(\Delta) - l_p(E, \Delta)$ and $y \in l_p(E, \Delta) - l_p(\Delta)$. This statement says that there is no inclusion between these two sequence spaces.

Theorem 2.4. *If* $\sup_n |E_n| < \infty$, *then* $l_p \subset l_p(E)$. *Moreover if* $|E_n| > 1$ *for an infinite number of* n, *then the inclusion is strict.*

Proof. Let $\zeta = \sup_n |E_n|$. To prove the validity of the inclusion $l_p \subset l_p(E)$, it suffices to show

$$\|x\|_{p,E} \leq \zeta^{\frac{p-1}{p}} \|x\|_p, \tag{2.2}$$

for each $x \in l_p$. Note that $\zeta = 1$, when $p = 1$. Suppose that $x = (x_n) \in l_p$ is an arbitrary sequence. By applying Hölder's inequality, we have

$$\left| \sum_{j\in E_n} x_j \right|^p \leq |E_n|^{p-1} \sum_{j\in E_k} |x_j|^p,$$

so

$$\|x\|_{p,E}^p \leq \zeta^{p-1} \|x\|_{p,E}^p.$$

Moreover, let $|E_n| > 1$ for an infinite number of n. One can choose a sequence (n_j) such that $|E_{n_j}| > 1$ for $j = 1, 2, \ldots.$ If the sequence

$x = (x_k)$ is defined by

$$x_k = \begin{cases} 1 & \text{if } k = \min E_{n_j} \\ -1 & \text{if } k = \min E_{n_j} + 1 \\ 0 & \text{otherwise,} \end{cases} \tag{2.3}$$

for $k = 1, 2, \ldots$. It is obvious that $\sum_{i \in E_k} x_i = 0$, so $x \in l_p(E)$ while $x \notin l_p$. Hence $x \in l_p(E) - l_p$, and the inclusion $l_p \subset l_p(E)$ strictly holds. □

Corollary 2.5. *If* $\sup_n |E_n| < \infty$*, then* $l_p \subset l_p(E, \Delta)$*. Moreover if* $|E_n| > 1$ *for an infinite number of* n*, then the inclusion is strict.*

Proof. By applying Theorem 2.4 and the part (i) of Theorem 2.3, the proof is trivial. □

One may expect a similar result for the space $l_p(E, \Delta)$ as was observed for the space l_p, and ask the following natural question: Is the space $l_p(E, \Delta)$ a semi-inner product space for $p = 2$? The answer is positive and is given by the following theorem:

Theorem 2.6. *Except the case* $p = 2$*, the space* $l_p(E, \Delta)$ *is not a semi-inner product space.*

Proof. If we define

$$< x, y >= \sum_{n=1}^{\infty} \sum_{i,j \in E_n} x_i y_j,$$

then it is a semi-inner product on the space $l_2(E, \Delta)$ and

$$\begin{aligned} \|x\|^2_{2,E,\Delta} &= \sum_{k=1}^{\infty} \left| \sum_{j \in E_k} x_j - \sum_{j \in E_{k+1}} x_j \right|^2 \\ &= \|x_{E,\Delta}\|^2_2 \\ &=< x_{E,\Delta}, x_{E,\Delta} >, \end{aligned}$$

where

$$x_{E,\Delta} = \left(\sum_{i \in E_1} x_i - \sum_{i \in E_2} x_i, \sum_{i \in E_2} x_i - \sum_{i \in E_3} x_i, \ldots \right).$$

Now consider the sequences x and y such that

$$\sum_{i \in E_k} x_i = \begin{cases} 1 & k = 1, 2 \\ 0 & k \geq 3, \end{cases}$$

and

$$\sum_{i\in E_k} y_i = \begin{cases} 1 & k=1 \\ 2 & k\geq 2. \end{cases}$$

We see that

$$\|x+y\|_{p,E,\Delta}^2 + \|x-y\|_{p,E,\Delta}^2 \neq 2\left(\|x\|_{p,E,\Delta}^2 + \|y\|_{p,E,\Delta}^2\right), \quad (p\neq 2).$$

Since the equation $2 = 2^{\frac{2}{p}}$ has only one root $p=2$, the semi-norm of the space $l_p(E,\Delta)$ does not satisfy the parallelogram equality, which means that the semi-norm cannot be obtained from the semi-inner product. Hence the space $l_p(E,\Delta)$ with $p\neq 2$ is not a semi-inner product space. □

Suppose that X is a semi-normed space with a semi-norm g. A sequence (b_n) of the elements of semi-normed space X is called a Schauder basis (or briefly a basis) for X if and only if, for each $x\in X$ there exists a unique sequence of scalars (α_n) such that

$$\lim_{n\to\infty} g\left(x - \sum_{k=1}^{n} \alpha_k b_k\right) = 0.$$

The series $\sum_{k=1}^{\infty} \alpha_k b_k$ which has the sum x is then called the expansion of x with respect to (b_n), and written as $x = \sum_{k=1}^{\infty} \alpha_k b_k$. In the next, we will introduce a sequence of the points of the space $l_p(E,\Delta)$ which forms a basis for the space $l_p(E,\Delta)$.

Theorem 2.7. *If the sequence $b^{(k)} = \{b_j^{(k)}\}_{j=1}^{\infty}$ is defined such that*

$$\sum_{j\in E_n} b_j^{(k)} = \begin{cases} 0 & n<k \\ 1 & n\geq k, \end{cases}$$

and the remaining elements are zero, for $k=1,2,\ldots$. Then the sequence $\left\{b^{(k)}\right\}_{k=1}^{\infty}$ is a basis for the space $l_p(E,\Delta)$, and any $x\in l_p(E,\Delta)$ has a unique representation of the form

$$x = \sum_{k=1}^{\infty} \alpha_k b^{(k)},$$

where

$$\alpha_k = \sum_{j\in E_k} x_j, \quad k=1,2,\ldots.$$

Proof. The proof is routine, so we omit the details. □

3. Upper bound of matrix operators from l_p into $l_p(E,\Delta)$

In this section, we tend to compute the norm of certain matrix operators such as Copson and Hilbert from l_p into $l_p(E,\Delta)$ is considered, where $p \geq 1$. At first, we prove a theorem that give us the norm of operators from l_1 into $l_1(E,\Delta)$.

Theorem 3.1. *If $A = (a_{n,k})$ is a matrix operator and*

$$M = \sup_k \sum_{n=1}^{\infty} \left| \sum_{i\in E_n} a_{i,k} - \sum_{i\in E_{n+1}} a_{i,k} \right| < \infty,$$

then A is a bounded operator from l_1 into $l_1(E,\Delta)$ and $\|A\|_{1,E,\Delta} = M$. In particular if

$$\sum_{i\in E_n} a_{i,k} \geq \sum_{i\in E_{n+1}} a_{i,k},$$

for all n, k, then

$$\|A\|_{1,E,\Delta} = \sup_k \sum_{i\in E_1} a_{i,k}.$$

Proof. Suppose that x is in l_1 and

$$u_k = \sum_{n=1}^{\infty} \left| \sum_{i\in E_n} a_{i,k} - \sum_{i\in E_{n+1}} a_{i,k} \right|,$$

for all k. We have

$$\begin{aligned}\|Ax\|_{1,E,\Delta} &\leq \sum_{n=1}^{\infty}\sum_{k=1}^{\infty} \left| \sum_{i\in E_n} a_{i,k} - \sum_{i\in E_{n+1}} a_{i,k} \right| |x_k| \\ &= \sum_{k=1}^{\infty} u_k\, |x_k| \\ &\leq M\|x\|_1.\end{aligned}$$

which says that $\|A\|_{1,E,\Delta} \leqslant M$. Conversely, we take $x = e_n$ which e_n denotes the sequence having 1 in place n and 0 elsewhere, then $\|x\|_1 = 1$ and $\|Ax\|_{1,E,\Delta} = u_n$ which proves that $\|A\|_{1,E,\Delta} = M$. □

Now we are ready to compute the norms of Copson and Hilbert operators from sequence space l_1 into $l_1(E,\Delta)$. We recall that the Copson matrix operator $C = (c_{n,k})$ is defined by

$$c_{n,k} = \begin{cases} \frac{1}{k} & \text{for } n \leq k \\ 0 & \text{for } n > k. \end{cases}$$

Corollary 3.2. *If C is the Copson operator and $|E_n| \geq |E_{n+1}|$ for all n, then C is a bounded operator from l_1 into $l_1(E,\Delta)$ and $\|C\|_{1,E,\Delta} = 1$.*

Proof. Since

$$M = \sup_k \sum_{i\in E_1} c_{i,k} = c_{1,1} = 1,$$

the result will gain by Theorem 3.1. □

Corollary 3.3. *If C is the Copson operator and $E_n = \{n\}$ for all n, then C is a bounded operator from l_1 into $l_1(\Delta)$ and $\|C\|_{1,\Delta} = 1$.*

Remember that the Hilbert matrix $H = (h_{n,k})$ is defined by

$$h_{n,k} = \frac{1}{n+k}, \quad (n, k = 1, 2, \dots).$$

Corollary 3.4. *If H is the Hilbert matrix and $|E_n| \geq |E_{n+1}|$ for all n, then H is a bounded operator from l_1 into $l_1(E, \Delta)$ and*

$$\|H\|_{1,E,\Delta} = \frac{1}{2} + \cdots + \frac{1}{\max E_1 + 1}.$$

Proof. Since $M = \sup_k \sum_{i\in E_1} h_{i,k}$, we obtain the desired result from Theorem 3.1. □

Corollary 3.5. *If H is the Hilbert matrix, then H is a bounded operator from l_1 into $l_1(\Delta)$ and $\|H\|_{1,\Delta} = \frac{1}{2}$.*

Proof. Let $E_n = \{n\}$ in Corollary 3.4, so the proof is obvious. □

In the sequel, we want to find the norm of Copson and Hilbert matrix operators from l_p into $l_p(E, \Delta)$ for $p > 1$. To do this, we state the Schur's Theorem and a lemma which are needed to prove our main results.

Theorem 3.6 ([7], Theorem 275)**.** *Let $p > 1$ and $B = (b_{n,k})$ be a matrix operator with $b_{n,k} \geq 0$ for all n, k. Suppose that C, R are two strictly positive numbers such that*

$$\sum_{n=1}^{\infty} b_{n,k} \leq C \qquad \text{for all } k,$$

$$\sum_{k=1}^{\infty} b_{n,k} \leq R \qquad \text{for all } n,$$

(bounds for column and row sums respectively). Then

$$\|B\|_p \leq R^{(p-1)/p} C^{1/p}.$$

Lemma 3.7. *If $A = (a_{n,k})$ and $B = (b_{n,k})$ are two matrix operators such that*

$$b_{n,k} = \sum_{i\in E_n} a_{i,k} - \sum_{i\in E_{n+1}} a_{i,k},$$

then

$$\|A\|_{p,E,\Delta} = \|B\|_p.$$

Hence, if B is a bounded operator on l_p, then A is also a bounded operator from l_p into $l_p(E,\Delta)$.

In below, we compute the norm of the Copson matrix operator for $p > 1$.

Theorem 3.8. *Suppose that $p > 1$ and N is a positive integer and $E_n = \{nN - N + 1, nN - N + 2, \ldots, nN\}$ for all n. If C is the Copson matrix operator, then it is a bounded operator from l_p into $l_p(E,\Delta)$ and*

$$\|C\|_{p,E,\Delta} \leq \left(N + \frac{N-1}{N+1} + \frac{N-2}{N+2} + \cdots + \frac{1}{2N-1}\right)^{\frac{p-1}{p}}.$$

In particular if $E_n = \{n\}$ for all n, then we have $\|C\|_{p,E,\Delta} = 1$.

Proof. By using Lemma 3.7 $\|C\|_{p,E,\Delta} = \|B\|_p$, where

$$b_{n,k} = \sum_{i \in E_n} c_{i,k} - \sum_{i \in E_{n+1}} c_{i,k}.$$

Let C, R be defined as in Theorem 3.6. We deduce that $R_n \leq R_1$ and $C_n \leq 1$ for all n. Since

$$b_{1,k} = \begin{cases} 1 & \text{for } k \leq N \\ \frac{2N-k}{k} & \text{for } N < k \leq 2N-1 \\ 0 & \text{for } k \geq 2N, \end{cases}$$

and

$$R_1 = N + \frac{N-1}{N+1} + \frac{N-2}{N+2} + \cdots + \frac{1}{2N-1},$$

it can conclude that $\|C\|_{p,\Delta,E} \leq {R_1}^{(p-1)/p}$. In particular if $E_n = \{n\}$ for all n, then $R_1 = 1$ so $\|C\|_{p,E,\Delta} \leq 1$. Now let $x = e_1$, then $Cx = x$ and this completes the proof of the theorem. □

At last, we solve the problem of finding the norm of the Hilbert matrix operator for $p > 1$.

Theorem 3.9. *Let H be the Hilbert operator and $p > 1$. If N is a positive integer and $E_n = \{nN - N + 1, nN - N + 2, \ldots, nN\}$ for all n, then H is a bounded operator from l_p into $l_p(E,\Delta)$ and*

$$\|H\|_{p,E,\Delta} \leq \left(\frac{1}{2} + \frac{2}{3} + \cdots + \frac{N+1}{N} + \cdots + \frac{1}{2N}\right)^{\frac{p-1}{p}} \left(\frac{1}{2} + \cdots + \frac{1}{N+1}\right)^{\frac{1}{p}}.$$

Proof. By using Lemma 3.7 $\|H\|_{p,E,\Delta} = \|B\|_p$, where

$$b_{n,k} = \sum_{i\in E_n} h_{i,k} - \sum_{i\in E_{n+1}} h_{i,k}$$

Let C, R be defined as in Theorem 3.6. We deduce that $R_n \leq R_1$ and $C_n \leq C_1$ for all n. But

$$R_1 = \sum_{k=1}^{\infty} b_{1,k} = \frac{1}{2} + \frac{2}{3} + \cdots + \frac{N+1}{N} + \cdots + \frac{1}{2N},$$

and

$$C_1 = \sum_{n=1}^{\infty} b_{n,1} = \frac{1}{2} + \cdots + \frac{1}{N+1},$$

hence the result is gained. □

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