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DYNAMIC EQUIVALENCE RELATIONS ON THE FUZZY MEASURE ALGEBRAS

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Abstract. The main goal of the present paper is to extend classical results from the measure theory and dynamical systems to the fuzzy subset setting. In this paper, the notion of dynamic equivalence relation is introduced and then it is proved that this relation is an equivalence relation. Also, a new metric on the collection of all equivalence classes is introduced and it is proved that this metric is complete.

1. Introduction

The theory of fuzzy sets [9] has given rise to new research branches in topology and dynamical systems [3–5, 7]. In this study we assume that the fuzzy sets are members of I^X, where I is $[0,1]$, and X is a non-empty set. For any $g, h \in F$ and $x \in X$ we define,

$$(g \cup h)(x) = \inf\big\{g(x) + h(x), 1\big\},$$
$$(g \cap h)(x) = \sup\big\{g(x) + h(x) - 1, 0\big\},$$
$$g^{\perp}(x) = 1 - g(x).$$

A family $F \subseteq [0,1]^X$ of fuzzy subsets of a set X is said to be a σ -algebra, if the following axioms are satisfied:

(i) $1_X \in F$,

(ii) If $g, h \in F$ then $g.h \in F$ and $(g \cap h) \in F$ where

$$(g \cap h)(x) = \sup\big\{g(x) + h(x) - 1, 0\big\},$$

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(iii) If $\{g_i\}_{i\geq 1} \subseteq F$ then $\bigcup_{i=1}^{\infty} g_i \in F$ where

$$\bigcup_{i=1}^{\infty} g_i := \inf\Big\{\sum_{i=1}^{\infty} g_i, 1\Big\}.$$

A function $m : F \to [0, +\infty)$ is called a fuzzy set measure, if:

(i) $m(0_X) = 0$,
(ii) $m(\bigcup_{i=1}^{\infty} g_i) = \sum_{i=1}^{\infty} m(g_i)$, when ever $g_i \in F$ and $\sum_{i=1}^{\infty} g_i \leq 1$.

The triple (X, F, m) is called a fuzzy probability measure space.

A fuzzy set measure is said to be preserved by a transformation $T : X \to X$, if the following implication holds

$$g \in F \quad \Rightarrow \quad goT \in F, \qquad m(goT) = m(g).$$

Suppose that $T : X \to X$ is a continuous map on a metric space X. We also assume that $F \subseteq [0,1]^X$ is the family of all Borel measurable maps $f : X \to [0,1]$.

The set of all fuzzy set measures $m : F \to [0, +\infty)$, satisfying $m(1_X) = 1$, is denoted by $M^*(X)$. The set of all fuzzy set measures in $M^*(X)$, preserved by T, is defined as follows

$$M^*(X,T) = \big\{m \in M^*(X); m(goT) = m(g), \text{ for all } g \in F\big\}.$$

In the remaining of the paper, $T : X \to X$ is continuous and $O_T(x) = \big\{T^k(x); k = 0, 1, \cdots\big\}$ is called the orbit of x.

Definition 1.1. Let (X, F, T) be as above and $m \in M^*(X, T)$. For $x \in X$ and $g \in F$, we define

$$m_x^T(g) = \limsup_{k\to\infty} \frac{1}{k} \sum_{i=0}^{k-1} goT^i(x).$$

This definition is an extension of fuzzy measures [7, 8]. In fact, it presents a normal approach to measure as a fuzzy set, not as a function on fuzzy sets. In this paper, we will denote $m_x^T(g)$ with $m_x(g)$ when there is no confusion.

The quantity $m_x(g)$ is the average values of f over the orbit of x under the dynamic of T. We call (X, F, T) a fuzzy dynamical system. Note that, if in Definition 1.1, we consider the crisp set $g = \chi_A$ we will have

$$\begin{aligned} m_x(\chi_A) &= \limsup_{k\to\infty} \frac{1}{k} \sum_{i=0}^{k-1} \chi_A oT^i(x) \\ &= \limsup_{k\to\infty} \frac{1}{k} \text{card}\Big\{0 \leq i \leq k-1, T^i(x) \in A\Big\}. \end{aligned}$$

So, for any crisp set $g = \chi_A$, $m_x(g)$ is the average time in which $x \in X$ spends in A under the dynamic T.

2. Local approach to fuzzy measure algebras

Definition 2.1. Suppose that (X, F, T) be a fuzzy dynamical system and $x_0 \in X$. Then, a relation "$= \mod m_{x_0}$" on F is defined as follows

$$g = h(\mod m_{x_0}) \quad \text{iff} \quad m_{x_0}(g) = m_{x_0}(h) = m_{x_0}(g \cap h),$$

where $g, h \in F$.

If $x_0 \in X$ and $g = h(\mod m_{x_0})$, it is said that g and h are dynamical equivalent under T at x_0 or m_{x_0}-equivalent.

Theorem 2.2. *Let (X, F, T) be a fuzzy dynamical system and $x_0 \in X$. Then, the relation m_{x_0}- equivalence on F is an equivalence relation.*

Proof. It follows from the definition that the relation $= (\mod m_{x_0})$, is reflexive and symmetric.

For transitivity, let $g = h(\mod m_{x_0})$ and $h = k(\mod m_{x_0})$, $g, h, k \in F$ and $x_0 \in X$. Then

$$m_{x_0}(g) = m_{x_0}(h) = m_{x_0}(g \cap h),$$
$$m_{x_0}(h) = m_{x_0}(k) = m_{x_0}(h \cap k).$$

Since $\Big((g \cap h) \cup (h \cap k)\Big) \leq h$, we get $m_{x_0}\Big((g \cap h) \cup (h \cap k)\Big) \leq m_{x_0}(h)$. Thus

$$\begin{aligned} m_{x_0}(g) + m_{x_0}(h) &= m_{x_0}(g \cap h) + m_{x_0}(h \cap k) \\ &= m_{x_0}(g \cap h \cap k) + m_{x_0}\Big((g \cap h) \cup (h \cap k)\Big) \\ &\leq m_{x_0}(g \cap k) + m_{x_0}(h), \end{aligned}$$

and so

$$m_{x_0}(g) \leq m_{x_0}(g \cap k).$$

Hence

$$m_{x_0}(g) = m_{x_0}(g \cap k).$$

Since

$$\begin{aligned} m_{x_0}(h) + m_{x_0}(k) &= m_{x_0}(g) + m_{x_0}(h) \\ &\leq m_{x_0}(g \cap k) + m_{x_0}(h), \end{aligned}$$

we get

$$m_{x_0}(k) \leq m_{x_0}(g \cap k).$$

Thus $g = h(\mod m_{x_0})$. □

Theorem 2.3. *Let (X, F, T) be a fuzzy dynamical system and $x_0 \in X$. If for $g, h \in F$, $g = h(\mod m_{x_0})$, then $m_{x_0}(g \cup k) = m_{x_0}(h \cup k)$ and $m_{x_0}(g \cap k) = m_{x_0}(h \cap k)$ for all $k \in F$.*

Proof. Let $x_0 \in X$ and $g = h(\mod m_{x_0})$ and $k \in F$. We have

$$\begin{aligned} m_{x_0}(g \cup k) &\leq m_{x_0}(g \cup h \cup k) \\ &= m_{x_0}(g \cup h) + m_{x_0}(k) - m_{x_0}\big((g \cup h) \cap k\big). \end{aligned}$$

Since $(g \cup h) \cap k = (g \cap k) \cup (h \cap k)$, it follows that

$$m_{x_0}\big((g \cup h) \cap k\big) \geq m_{x_0}(h \cap k).$$

Hence,

$$\begin{aligned} m_{x_0}(g \cup k) &\leq m_{x_0}(h) + m_{x_0}(k) - m_{x_0}(h \cap k) \\ &= m_{x_0}(h \cup k). \end{aligned}$$

Similarly, $m_{x_0}(h \cup k) \leq m_{x_0}(g \cup k)$. Consequently,

$$m_{x_0}(h \cup k) = m_{x_0}(g \cup k).$$

For the second part, it is sufficient to note that $m_{x_0}(h \cup k) = m_{x_0}(g \cup k)$ if and only if $m_{x_0}(h \cap k) = m_{x_0}(g \cap k)$. ☐

Theorem 2.4. *Let (X, F, T) be a fuzzy dynamical system and $x_0 \in X$. Then*

(i) *If $g, h, k, l \in F$, $g = h(\mod m_{x_0})$, $k = l(\mod m_{x_0})$, then $g \cap k = h \cap l(\mod m_{x_0})$ and $g \cup k = h \cup l(\mod m_{x_0})$,*
(ii) *If $g_j, h_j \in F, j \in \mathbb{N}, g_j = h_j(\mod m_{x_0})$ for all $j \in \mathbb{N}$, then*

$$\bigcup_{j=1}^{\infty} g_j = \bigcup_{j=1}^{\infty} h_j(\mod m_{x_0}).$$

Proof. (i) It follows by repeated application of Theorem 2.3.
(ii) For $j \in \mathbb{N}$, we have

$$\begin{aligned} m_{x_0}(g_j) &= m_{x_0}(h_j) \\ &= m_{x_0}(g_j \cup h_j). \end{aligned}$$

Hence

$$\begin{aligned} m_{x_0}(\bigcup_{j=1}^{\infty} g_j) &= \lim_{k \to \infty} m_{x_0}(\bigcup_{j=1}^{k} g_j) \\ &= \lim_{k \to \infty} m_{x_0}\Big(\bigcup_{j=1}^{k} h_j\Big) \\ &= m_{x_0}\Big(\bigcup_{j=1}^{\infty} h_j\Big). \end{aligned}$$

Also,

$$\begin{aligned} m_{x_0}(\bigcup_{j=1}^{\infty} g_j) &= \lim_{k\to\infty} m_{x_0}(\bigcup_{j=1}^{k} g_j) \\ &= \lim_{k\to\infty} m_{x_0}\Big(\bigcup_{j=1}^{k}(g_j \cup h_j)\Big) \\ &= m_{x_0}\Big(\bigcup_{j=1}^{\infty}(g_j \cup h_j)\Big) \\ &= m_{x_0}\Big((\bigcup_{j=1}^{\infty} g_j) \cup (\bigcup_{j=1}^{\infty} h_j)\Big). \end{aligned}$$

Thus

$$\bigcup_{j=1}^{\infty} g_j = \bigcup_{j=1}^{\infty} h_j (\mod m_{x_0}).$$

□

Definition 2.5. Let (X, F, T) be a fuzzy dynamical system and $x_0 \in X$. We denote by $\bar{F}$ the collection of all equivalence classes induced by the relation m_{x_0}- equivalence on F. Also, we denote by $\bar{f}$ the class determined by $f \in F$.

In view of Theorem 2.4 (i), we may define $\bar{g} \cup \bar{h} = \overline{(g \cup h)}$ and $\bar{g} \cap \bar{h} = (g \bar{\cap} h)$. Using Theorem 2.4 (ii), for any sequence $\{h_i\}$ in $\bar{F}$, we define

$$\bigcup_{j=1}^{\infty} \bar{h}_j = \overline{(\bigcup_{j=1}^{\infty} h_j)}.$$

Since $g = h(\mod m_{x_0})$ implies $1 - g = 1 - h(\mod m_{x_0})$, we may define $(\bar{g})^{\perp} = (1 - g)^{\perp}$. Thus under this operations induced from F, $\bar{F}$ forms a fuzzy σ-algebra.

Theorem 2.6. *Suppose $\{g_j\}$ and $\{h_j\}$ are sequences in F such that $g_j = h_j(\mod m_{x_0})$ for all $j \in \mathbb{N}$. Let $g_j \uparrow g$ and $h_j \uparrow h$. Then $g = h(\mod m_{x_0})$.*

Proof. Since $g_j \uparrow g$, $h_j \uparrow h$ and $g_j = h_j(\mod m_{x_0})$ for all $j \in \mathbb{N}$, we get

$$\begin{aligned} m_{x_0}(g) &= \sup_j m_{x_0}(g_j) \\ &= \sup_j m_{x_0}(h_j) \\ &= m_{x_0}(h), \end{aligned}$$

Also $g_j \cap h_j \uparrow g \cap h$, and therefore

$$\begin{aligned} m_{x_0}(g \cap h) &= \sup_j m_{x_0}(g_j \cap h_j) \\ &= \sup_j m_{x_0}(g_j) \\ &= m_{x_0}(g). \end{aligned}$$

Hence we obtain that $g = h(\mod m_{x_0})$. □

Definition 2.7. If $g_j \uparrow g$, then we say that $\bar{g}_j \uparrow \bar{g}$ and define

$$\bar{m}_{x_0}(\bar{g}) = m_{x_0}(g).$$

We identify $\bar{g}$ with g, i.e., any member of the equivalence class $\bar{g}$, and we write $m_{x_0}(g)$ for $\bar{m}_{x_0}(\bar{g})$.

Theorem 2.8. *Let $x_0 \in X$ and $(\bar{F}, \bar{m}_{x_0})$ be as above. The function $p_{x_0} : \bar{F} \times \bar{F} \to I$ described by $p_{x_0}(g,h) = m_{x_0}(g \cup h) - m_{x_0}(g \cap h)$ for $g, h \in F$, defines a metric on $\bar{F}$.*

Proof. It follows from the definition that

$$p_{x_0}(g,h) = p_{x_0}(h,g), \qquad p_{x_0}(g,g) = 0.$$

Suppose that $p_{x_0}(g,h) = 0$. Then

$$\begin{aligned} m_{x_0}(g) &\le m_{x_0}(g \cup h) \\ &= m_{x_0}(g \cap h) \\ &\le m_{x_0}(g), \end{aligned}$$

implies $m_{x_0}(g) = m_{x_0}(g \cap h)$. Interchanging g and h, we get

$$m_{x_0}(h) = m_{x_0}(g \cap h).$$

Consequently, $g = h$.

Finally, for $g, h, k \in \bar{F}$ we have

$$\begin{aligned} m_{x_0}(g \cup h) &+ m_{x_0}(h \cup k) + m_{x_0}(g \cap k) \\ &= m_{x_0}(g \cup h \cup k) + m_{x_0}\Big((g \cup h) \cap (h \cup k)\Big) + m_{x_0}(g \cap k) \\ &= m_{x_0}(g \cup h \cup k) + m_{x_0}\bigg(h \cup \Big((g \cup h) \cap k\Big)\bigg) + m_{x_0}(g \cap k) \\ &\ge m_{x_0}(g \cup k) + m_{x_0}\bigg(h \cap \Big((g \cup h) \cap k\Big)\bigg) + m_{x_0}(g \cap h \cap k) \\ &= m_{x_0}(g \cap h) + m_{x_0}(h \cap k) + m_{x_0}(g \cup k), \end{aligned}$$

and so

$$m_{x_0}(g \cup h) - m_{x_0}(g \cap h) + m_{x_0}(h \cup k) - m_{x_0}(h \cap k) \ge m_{x_0}(g \cup k) - m_{x_0}(g \cap h),$$

i.e.,

$$p_{x_0}(g,h) + p_{x_0}(h,k) \geq p_{x_0}(g,k).$$

□

Theorem 2.9. *Suppose* $g, h, g_j, h_k (j, k \in \mathbb{N})$ *be elements of* $\bar{F}$. *The metric* p_{x_0} *in Theorem 2.8 satisfies, the following*

(i) $p_{x_0}(g,h) = p_{x_0}(g^\perp, h^\perp)$,
(ii) $p_{x_0}(g_1 \cup h_1, g_2 \cup h_2) \leq p_{x_0}(g_1, g_2) + p_{x_0}(h_1, h_2)$,
(iii) $p_{x_0}(g_1 \cap h_1, g_2 \cap h_2) \leq p_{x_0}(g_1, g_2) + p_{x_0}(h_1, h_2)$,
(iv) $p_{x_0}(g_1, g_1 \cup g_2) \leq p_{x_0}(g_1, g_2)$,
(v) $p_{x_0}(\bigcup_{j=1}^{k} g_j, \bigcup_{j=1}^{k+1} g_j) \leq p_{x_0}(g_k, g_{k+1})$, *for all* $k \in \mathbb{N}$.

Proof. (i) For, $g, h \in \bar{F}$, we have

$$\begin{aligned}
p_{x_0}(g^\perp, h^\perp) &= m_{x_0}(g^\perp \cup h^\perp) - m_{x_0}(g^\perp \cap h^\perp) \\
&= m_{x_0}\Big((g \cap h)^\perp\Big) - m_{x_0}\Big((g \cup h)^\perp\Big) \\
&= (1 - m_{x_0}(g \cap h)) - \Big(1 - m_{x_0}(g \cup h)\Big) \\
&= m_{x_0}(g \cup h) - m_{x_0}(g \cap h) \\
&= p_{x_0}(g,h).
\end{aligned}$$

(ii) For $g_1, g_2, h_1, h_2 \in \bar{F}$, we have

$$\begin{aligned}
p_{x_0}(g_1 \cup h_1, g_2 \cup h_2) &= m_{x_0}\Big((g_1 \cup h_1) \cup (g_2 \cup h_2)\Big) \\
&\quad - m_{x_0}\Big((g_1 \cup h_1) \cap (g_2 \cup h_2)\Big) \\
&\leq m_{x_0}\Big((g_1 \cup g_2) \cup (h_1 \cup h_2)\Big) \\
&\quad - m_{x_0}\Big((g_1 \cap g_2) \cup (h_1 \cap h_2)\Big) \\
&= m_{x_0}(g_1 \cup g_2) + m_{x_0}(h_1 \cup h_2) \\
&\quad - m_{x_0}\Big((g_1 \cup g_2) \cap (h_1 \cup h_2)\Big) \\
&\quad - m_{x_0}(g_1 \cap g_2) - m_{x_0}(h_1 \cap h_2) \\
&\quad + m_{x_0}\Big((g_1 \cap g_2) \cap (h_1 \cap h_2)\Big) \\
&\leq m_{x_0}(g_1 \cup g_2) + m_{x_0}(h_1 \cup h_2) \\
&\quad - m_{x_0}(g_1 \cap g_2) - m_{x_0}(h_1 \cup h_2) \\
&= p_{x_0}(g_1, g_2) + p_{x_0}(h_1, h_2).
\end{aligned}$$

(iii) For $g_1, g_2, h_1, h_2 \in \bar{F}$, we have

$$\begin{aligned} p_{x_0}(g_1 \cap h_1, g_2 \cap h_2) &= m_{x_0}\Big((g_1 \cap h_1) \cup (g_2 \cap h_2)\Big) \\ &\quad - m_{x_0}\Big((g_1 \cap h_1) \cap (g_2 \cap h_2)\Big) \\ &\leq m_{x_0}\Big((g_1 \cup g_2) \cap (h_1 \cup h_2)\Big) \\ &\quad - m_{x_0}\Big((g_1 \cap g_2) \cap (h_1 \cap h_2)\Big) \\ &= m_{x_0}(g_1 \cup g_2) + m_{x_0}(h_1 \cup h_2) \\ &\quad - m_{x_0}\Big((g_1 \cup g_2) \cup (h_1 \cup h_2)\Big) \\ &\quad - m_{x_0}(g_1 \cap g_2) - m_{x_0}(h_1 \cap h_2) \\ &\quad + m_{x_0}\Big((g_1 \cap g_2) \cup (h_1 \cap h_2)\Big) \\ &\leq p_{x_0}(g_1, g_2) + p_{x_0}(h_1, h_2). \end{aligned}$$

(iv) For $g, h \in \bar{F}$, we have

$$\begin{aligned} p_{x_0}(g, g \cup h) &= m_{x_0}(g \cup h) - m_{x_0}(g) \\ &\leq m_{x_0}(g \cup h) - m_{x_0}(g \cap h) \\ &= p_{x_0}(g, h). \end{aligned}$$

(v) By (ii) and (iv), we have

$$\begin{aligned} p_{x_0}\left(\bigcup_{j=1}^{k} g_j, \bigcup_{j=1}^{k+1} g_j\right) &= p_{x_0}\left(\Big(\bigcup_{j=1}^{k-1} g_j\Big) \cup g_k, \Big(\bigcup_{j=1}^{k-1} g_j\Big) \cup \Big(g_k \cup g_{k+1}\Big)\right) \\ &\leq p_{x_0}\Big(\bigcup_{j=1}^{k-1} g_j, \bigcup_{j=1}^{k-1} g_j\Big) + p_{x_0}\Big(g_k, g_k \cup g_{k+1}\Big) \\ &\leq p_{x_0}\Big(g_k, g_{k+1}\Big). \end{aligned}$$

□

Theorem 2.10. *Let p_{x_0} be the metric on $\bar{F}$ as defined in Theorem 2.8. Then the maps $g \to g^{\perp}, (g,h) \to g \cup h, (g,h) \to g \cap h$ are uniformly continuous.*

Proof. It follows from Theorem 2.9 (i)−(iii). □

Theorem 2.11. *For every $x_0 \in X$, the metric space $(\bar{F}, p_{x_0})$ is complete.*

Proof. Suppose that $\{g_n\}$ is a Couchy sequence of elements in $\bar{F}$. We can choose a subsequence $\{g_n\}$ such that $p_{x_0}(g_n, g_{n+1}) < 2^{-n}$ for all $n \in \mathbb{N}$. It is sufficient to prove that $\{g_n\}$ converges to g in $\bar{F}$.

For $n \in \mathbb{N}$, let $h_p = \bigcup_{i=n}^{n+p} g_i$. Then $\{h_p\}$ is a monotonic increasing sequence and

$$\lim_{p\to\infty} h_p = \bigcup_{i=n}^{\infty} g_i,$$

so,

$$m_{x_0}(\bigcup_{i=n}^{\infty} g_i) = \lim_{p\to\infty} m_{x_0}(h_p).$$

Also, by Theorem 2.9 (iv) and (v), we have

$$\begin{aligned} p_{x_0}(g_n, h_p) &\leq p_{x_0}(g_n, h_1) + p_{x_0}(h_1, h_2) + \cdots + p_{x_0}(h_{p-1}, h_p) \\ &\leq p_{x_0}(g_n, g_{n+1}) + p_{x_0}(g_{n+1}, g_{n+2}) + \cdots + p_{x_0}(g_{n+p-1}, g_{n+p}) \\ &< \frac{1}{2^n} + \frac{1}{2^{n+1}} + \cdots \\ &= \frac{1}{2^{n-1}}. \end{aligned}$$

Hence

$$\begin{aligned} p_{x_0}\Big(g_n, \bigcup_{i=n}^{\infty} g_j\Big) &= m_{x_0}(\bigcup_{i=n}^{\infty} g_j) - m_{x_0}(g_n) \\ &= \lim_{p\to\infty} \Big(m_{x_0}(h_p) - m_{x_0}(g_n)\Big) \\ &\leq 2^{-n+1}. \end{aligned}$$

Since the sequence $\{\bigcup_{j=n}^{\infty} g_j\}$ is monotonic decreasing, then

$$\begin{aligned} \lim_{n\to\infty} g_n &= \lim_{n\to\infty} \bigcup_{i=n}^{\infty} g_j \\ &= \bigcap_{n=1}^{\infty} (\bigcup_{j=n}^{\infty} g_j) \\ &= g \in \bar{F}. \end{aligned}$$

□

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