Sahand Communications in Mathematical Analysis (SCMA) Vol. 6 No. 1 (2017), 29-37
http://scma.maragheh.ac.ir

FUZZY WEAKLY e-CLOSED FUNCTIONS

VEERAPPAN CHANDRASEKAR[1] AND SOMASUNDARAM PARIMALA[2*]

Abstract. In this paper, we introduce and characterize fuzzy weakly e-closed functions in fuzzy topological spaces and the relationship between these mappings and some properties of them are investigated.

1. Introduction and preliminaries

Kubiak [14] and Šostak [19] introduced the fundamental concept of a fuzzy topological structure, as an extension of both crisp topology and fuzzy topology [1], in the sense that not only the objects are fuzzified, but also the axiomatics. In [17, 18], Šostak gave some rules and showed how such an extension can be realized. Chattopadhyay et al., [3] have redefined the same concept under the name gradation of openness. A general approach to the study of topological type structures on fuzzy power sets was developed in [5–7, 14].

In this paper, we introduce and characterize fuzzy weakly e-closed functions in fuzzy topological spaces and we investigate the relationship between these mappings and some of their properties.

Throughout this paper, let X be a non-empty set, $I = [0,1]$, $I_0 = (0,1]$. A fuzzy set λ of X is a mapping $\lambda : X \to I$, and I^X is the family of all fuzzy sets on X. The complement of a fuzzy set λ is denoted by $\overline{1} - \lambda$. For $\lambda \in I^X, \overline{\lambda}(x) = \lambda$ for all $x \in X$. For each $x \in X$ and $t \in I_0$, a fuzzy point x_t is defined by

$$x_t(y) = \begin{cases} t & \text{if } y = x, \\ 0 & \text{if } y \neq x. \end{cases}$$

2010 *Mathematics Subject Classification.* 54A05, 54A40, 54E55, 54C08.
Key words and phrases. Fuzzy topology, Fuzzy e-closed functions, Fuzzy weakly e-closed functions, Fuzzy contra e-open functions.
Received: 7 October 2016, Accepted: 16 November 2016.
* Corresponding author.

Let $Pt(X)$ be the family of all fuzzy points in X. For $\lambda, \mu \in I^X, \lambda$ is called quasi coincident with μ, denoted by $\lambda q \mu$, if there exists $x \in X$ such that $\lambda(x) + \mu(x) > 1$. Otherwise, we denote $\lambda \overline{q} \mu$. We define $x_t \in \lambda$ if $t \leq \lambda(x)$. All other notations and definitions are standard in the fuzzy set theory.

Definition 1.1 ([19])**.** A function $\tau : I^X \to I$ is called a fuzzy topology on X if it satisfies the following conditions:

(1) $\tau(\overline{0}) = \tau(\overline{1}) = 1$;
(2) $\tau\left(\bigvee_{i \in J} \mu_i\right) \geq \bigwedge_{i \in J} \tau(\mu_i)$, for any $\{\mu_i : i \in J\} \subseteq I^X$;
(3) $\tau(\mu_1 \wedge \mu_2) \geq \tau(\mu_1) \wedge \tau(\mu_2)$, for all $\mu_1, \mu_2 \in I^X$.

The pair (X, τ) is called a fuzzy topological space (for short, fts).

Definition 1.2 ([9])**.** Let (X, τ) be a fts, $\lambda, \mu \in I^X$ and $r \in I_0$. We define operators as follows:

$$C_\tau(\lambda, r) = \wedge \left\{ \mu \in I^X | \lambda \leq \mu, \tau(\overline{1} - \mu) \geq r \right\},$$

and

$$I_\tau(\lambda, r) = \vee \left\{ \mu \in I^X | \lambda \geq \mu, \tau(\mu) \geq r \right\}.$$

Definition 1.3. Let (X, τ) be a fuzzy topological space. For $\lambda, \mu \in I^X$ and $r \in I_0$, λ is called r-fuzzy regular open (for short, r-fro) [9] (resp. r-fuzzy regular closed (for short, r-frc) [9]) if $\lambda = I_\tau(C_\tau(\lambda, r), r)$ (resp. $\lambda = C_\tau(I_\tau(\lambda, r), r)$).

Definition 1.4 ([16])**.** Let (X, τ) be a fuzzy topological space. $\lambda, \mu \in I^X$ and $r \in I_0$,

(i) δ-$I_\tau(\lambda, r) = \bigvee \left\{ \mu \in I^X : \mu \leq \lambda, \mu \text{ is a } r\text{-fro set} \right\}$ is called the r-fuzzy δ-interior of λ.
(ii) δ-$C_\tau(\lambda, r) = \bigwedge \left\{ \mu \in I^X : \mu \geq \lambda, \mu \text{ is a } r\text{-fro set} \right\}$ is called the r-fuzzy δ-closure of λ.

Definition 1.5. Let (X, τ) be a fuzzy topological space. For $\lambda, \mu \in I^X$ and $r \in I_0$,

(i) λ is called a r-fuzzy δ-semiopen [16] (briefly, r-fδso) (resp. r-fuzzy δ-semiclosed [16] (briefly, r-fδsc)) set if

$$\lambda \leq C_\tau\left(\delta - I_\tau(\lambda, r), r\right), \quad (\text{resp. } I_\tau\left(\delta - C_\tau(\lambda, r), r\right) \leq \lambda).$$

(ii) λ is called a r-fuzzy δ-preopen [16] (briefly, r-fδpo) (resp. r-fuzzy δ-preclosed [16] (briefly, r-fδpc)) set if

$$\lambda \leq I_\tau\left(\delta - C_\tau(\lambda, r), r\right), \quad (\text{resp. } C_\tau\left(\delta - I_\tau(\lambda, r), r\right) \leq \lambda).$$

(iii) λ is called a r-fuzzy e-open [16] (briefly, r-feo) (resp. r-fuzzy e-closed [16] (briefly, r-fec)) set if

$$\lambda \leq C_\tau(\delta - I_\tau(\lambda, r), r) \vee I_\tau(\delta - C_\tau(\lambda, r), r),$$
$$(\text{resp. } C_\tau(\delta - I_\tau(\lambda, r), r) \wedge I_\tau(\delta - C_\tau(\lambda, r), r) \leq \lambda).$$

Definition 1.6 ([16])**.** Let (X, τ) be a fuzzy topological space. $\lambda, \mu \in I^X$ and $r \in I_0$,

(i) $eI_\tau(\lambda, r) = \bigvee \{\mu \in I^X : \mu \leq \lambda, \mu \text{ is a } r\text{-feo set}\}$ is called the r-fuzzy e-interior of λ.
(ii) $eC_\tau(\lambda, r) = \bigwedge \{\mu \in I^X : \mu \geq \lambda, \mu \text{ is a } r\text{-feo set}\}$ is called the r-fuzzy e-closure of λ.

Definition 1.7 ([16])**.** A function $f : (X, \tau) \to (Y, \eta)$ is said to be fuzzy e-closed if $f(\lambda)$ is r-fec set in Y for each $\lambda \in I^X, r \in I_0$ with $\tau(\overline{1}-\lambda) \geq r$.

Definition 1.8 ([13])**.** Let (X, τ) be a fts and $x_t \in Pt(X)$. We denote

$$\mathcal{Q}_\tau(x_t, r) = \{\mu \in I^X | x_t q \mu, \tau(\mu) \geq r\},$$
$$\mathcal{R}_\tau(x_t, r) = \{\mu \in I^X | x_t q \mu, \mu \text{ is } r\text{-fro}\}.$$

Definition 1.9 ([2])**.** Let (X, τ) be a fts, $\lambda \in I^X$ and $r \in I_0$. We denote

$$eT_\tau(\lambda, r) = \{\mu \in I^X | \lambda \leq I_\tau(\mu, r), \mu \text{ is } r\text{-fec}\}.$$

2. Fuzzy weakly e-closed functions

In the following section, we extend the concept of fuzzy e-closed function introduced by Sobana et al. [16] to the fuzzy topological spaces in Sostak's sense.

Definition 2.1. A function $f : (X, \tau) \to (Y, \eta)$ is said to be fuzzy weakly e-closed if $eT_\tau(f(I_\tau(\lambda, r)), r) \leq f(\lambda)$ for each $\lambda \in I^X, r \in I_0$ with $\tau(\overline{1} - \lambda) \geq r$.

Clearly, every fuzzy weakly e-closed function is fuzzy e-closed, but the converse is not true in general.

Example 2.2. Let $X = \{a, b, c\}$, $\lambda_1, \lambda_2, \lambda_3 \in I^X$, $\mu_1, \mu_2 \in I^Y$ be defined as follows,

$$\begin{array}{lll} \lambda_1(a) = 0.3, & \lambda_2(a) = 0.6, & \lambda_3(a) = 0.6, \\ \lambda_1(b) = 0.4, & \lambda_2(b) = 0.5, & \lambda_3(b) = 0.6, \\ \lambda_1(c) = 0.5, & \lambda_2(c) = 0.5, & \lambda_3(c) = 0.5, \end{array}$$

$$\begin{array}{ll} \mu_1(a) = 0.6, & \mu_2(a) = 0.6, \\ \mu_1(b) = 0.5, & \mu_2(b) = 0.5, \\ \mu_1(c) = 0.5, & \mu_2(c) = 0.4. \end{array}$$

Define fuzzy topologies $\tau, \eta : I^X \to I$ as follows.

$$\tau(\lambda) = \begin{cases} 1 & \text{if } \lambda = \overline{0} \text{ or } \overline{1}. \\ \frac{1}{2} & \text{if } \lambda = \lambda_1, \\ \frac{1}{2} & \text{if } \lambda = \lambda_2, \\ \frac{1}{2} & \text{if } \lambda = \lambda_3, \\ 0 & \text{otherwise,} \end{cases} \qquad \eta(\lambda) = \begin{cases} 1 & \text{if } \lambda = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{2} & \text{if } \lambda = \mu_1, \\ \frac{1}{2} & \text{if } \lambda = \mu_2, \\ 0 & \text{otherwise.} \end{cases}$$

For $r = 1/2$, then the identity function $id_X : (X, \tau) \to (X, \eta)$ is fuzzy e-closed function but not fuzzy weakly e-closed. Since the fuzzy set λ is defined as

$$\lambda(a) = 0.4, \qquad \lambda(b) = 0.4, \qquad \lambda(c) = 0.5,$$

and $\tau(\overline{1} - \lambda) \geq r$, $f(\lambda)$ is r-fuzzy e-closed set in Y. But

$$f(\lambda) = \lambda, \qquad f(I_\tau(\lambda, r)) = \lambda_1, \qquad eT_\eta\left(f(I_\tau(\lambda, r)), r\right) = \mu_2,$$

and hence

$$eT_\eta(f(I_\tau(\lambda, r)), r) \nleq f(\lambda).$$

Theorem 2.3. *For a function $f : (X, \tau) \to (Y, \eta)$, the following conditions are equivalent:*

(1) *f is fuzzy weakly e-closed.*
(2) *$eT_\eta(f(\lambda), r) \leq f(C_\tau(\lambda, r))$, $\forall \lambda \in I^X$, $r \in I_0$ with $\tau(\lambda) \geq r$.*

Proof.(1)⇒(2): Let $\lambda \in I^X$, $r \in I_0$ with $\tau(\lambda) \geq r$. Then

$$\begin{aligned} eT_\eta\left(f(\lambda), r\right) &= eT_\eta\left(f(I_\tau(\lambda, r)), r\right) \\ &\leq eT_\eta\left(f(I_\tau(C_\tau(\lambda, r), r)), r\right) \\ &\leq f\left(C_\tau(\lambda, r)\right). \end{aligned}$$

(2)⇒(1): Let $\lambda \in I^X$, $r \in I_0$ with $\tau(\overline{1} - \lambda) \geq r$. Then

$$\begin{aligned} eT_\eta\left(f(I_\tau(\lambda, r)), r\right) &\leq f\left(C_\tau(I_\tau(\lambda, r), r)\right) \\ &\leq f\left(C_\tau(\lambda, r)\right) \\ &\leq f(\lambda). \end{aligned}$$

□

Theorem 2.4. *For a function $f : (X, \tau) \to (Y, \eta)$, the following conditions are equivalent:*

(1) *f is fuzzy weakly e-closed.*
(2) *$eT_\eta(f(I_\tau(\lambda, r)), r) \leq f(\lambda)$, $\forall \lambda \in I^X$, $r \in I_0$ with $\tau(\overline{1} - \lambda) \geq r$.*
(3) *$eT_\eta(f(I_\tau(\lambda, r)), r) \leq f(\lambda)$, $\forall \lambda \in I^X$, $r \in I_0$ with λ is r-$f\delta pc$.*
(4) *$eT_\eta(f(I_\tau(\lambda, r)), r) \leq f(\lambda)$, $\forall \lambda \in I^X$, $r \in I_0$ with λ is r-$f\delta sc$.*

Proof.(2)⇒(3): Let λ be r-fδ pc set. i.e., $C_\tau(\delta\text{-}I_\tau(\lambda, r), r) \leq \lambda$. Since $\tau\left(\overline{1} - C_\tau(\delta\text{-}I_\tau(\lambda, r), r)\right) \geq r$, by (2), we have

$$eT_\eta\left(f(I_\tau(C_\tau(\delta\text{-}I_\tau(\lambda, r), r), r)), r\right) \leq f(C_\tau(\delta\text{-}I_\tau(\lambda, r), r)).$$

It implies

$$\begin{aligned} eT_\eta(f(I_\tau(\lambda, r)), r) &\leq eT_\eta(f(I_\tau(C_\tau(\delta\text{-}I_\tau(\lambda, r), r), r)), r) \\ &\leq f(C_\tau(\delta\text{-}I_\tau(\lambda, r), r)) \\ &\leq f(\lambda). \end{aligned}$$

Other cases are easily proved. □

Lemma 2.5. *Let (X, τ) be a fts. For each $\lambda \in I^X$ and $r \in I_0$. If λ is r-fδpo, then $C_\tau(\lambda, r) = eT_\tau(\lambda, r)$.*

Theorem 2.6. *For a function $f : (X, \tau) \to (Y, \eta)$ the following conditions are equivalent:*

(1) *f is fuzzy weakly e-closed.*
(2) *$eT_\eta(f(I_\tau(C_\tau(\lambda, r), r)), r) \leq f(C_\tau(\lambda, r))$, $\forall \lambda \in I^X$, $r \in I_0$.*
(3) *$eT_\eta(f(I_\tau(C_\tau(\lambda, r), r)), r) \leq f(eT_\tau(\lambda, r))$, $\forall \lambda \in I^X$, $r \in I_0$.*
(4) *$eT_\eta(f(\lambda), r) \leq f(C_\tau(\lambda, r))$, if λ is r-fδpo.*

Proof. It is clear that (1)⇒(2)⇒(4)⇒(2) and (1)⇒(3). (3)⇒(4) if λ is r-fδpo, by Lemma 2.5, $eT_\tau(\lambda, r) = C_\tau(\lambda, r)$. □

Theorem 2.7. *If (Y, η) is r-fuzzy regular space and $r \in I_0$, then for a function $f : (X, \tau) \to (Y, \eta)$ the following conditions are equivalent:*

(1) *f is fuzzy weakly e-closed.*
(2) *$eT_\eta(f(\lambda), r) \leq f(C_\tau(\lambda, r))$, $\forall \lambda \in I^X$, $r \in I_0$ with λ is r-feo.*
(3) *For each $\mu \in I^Y$ and each $\lambda \in I^X$, $r \in I_0$ with $\tau(\lambda) \geq r$ and $f^{-1}(\mu) \leq \lambda$, there exists r-feo set $\nu \in I^Y$ with $\mu \leq \nu$ and $f^{-1}(\nu) \leq C_\tau(\lambda, r)$.*
(4) *For each fuzzy point y_p in Y and each $\lambda \in I^X$, $r \in I_0$ with $\tau(\lambda) \geq r$ with $f^{-1}(y_p) \leq \lambda$, there exists r-feo set $\nu \in I^Y$ containing y_p and $f^{-1}(\nu) \leq C_\tau(\lambda, r)$.*

Proof. It is clear that (1) ⇒ (2) and (3) ⇒ (4).

(2) ⇒ (3): Let $\mu \in I^Y$ and let $\lambda \in I^X$, $r \in I_0$ with $\tau(\lambda) \geq r$ and $f^{-1}(\mu) \leq \lambda$. Then

$$f^{-1}(\mu)\overline{q}C_\tau(\overline{1} - C_\tau(\lambda, r), r),$$

and consequently

$$\mu\overline{q}f(C_\tau(\overline{1} - C_\tau(\lambda, r), r)).$$

Since $\overline{1} - C_\tau(\lambda, r)$ is r-fro,

$$\mu\overline{q}eT_\eta\left(f(\overline{1} - C_\tau(\lambda, r)), r\right),$$

by (2). Let

$$\nu = \overline{1} - eT_\eta \left(f(\overline{1} - C_\tau(\lambda, r)), r \right).$$

Since (Y, η) is r-fuzzy regular, by Corollary 3.3 in [2], we have

$$\begin{aligned} eT_\eta(\overline{1} - \nu) &= C_\tau(\overline{1} - \nu) \\ &= \overline{1} - \nu. \end{aligned}$$

Then ν is feo with $\mu \leq \nu$ and

$$\begin{aligned} f^{-1}(\nu) &\leq \overline{1} - f^{-1}(eT_\eta(f(\overline{1} - C_\tau(\lambda, r)), r)) \\ &\leq \overline{1} - f^{-1}f(\overline{1} - C_\tau(\lambda, r)) \\ &\leq C_\tau(\lambda, r). \end{aligned}$$

(4) ⇒ (1): Let $\mu \in I^X$, $r \in I_0$ with $\tau(\overline{1} - \mu) \geq r$ and let $y_p \in \overline{1} - f(\mu)$. Since $f^{-1}(y_p) \leq \overline{1} - \mu$, there exists r-feo ν in Y with $y_p \in \nu$ and

$$\begin{aligned} f^{-1}(\nu) &\leq C_\tau(\overline{1} - \mu, r) \\ &= \overline{1} - I_\tau(\mu, r) \end{aligned}$$

by (4). Therefore $\nu \overline{q} f(I_\tau(\mu, r))$. Since $eT_\eta(\overline{1} - \nu, r) = \overline{1} - \nu$, we have

$$\begin{aligned} eT_\eta(f(I_\tau(\mu, r)), r) &\leq eT_\eta(\overline{1} - \nu, r) \\ &= \overline{1} - \nu. \end{aligned}$$

So,

$$y_p \in \nu \leq \overline{1} - eT_\eta(f(I_\tau(\mu, r)), r).$$

Thus

$$eT_\eta(f(I_\tau(\mu, r)), r) \leq f(\lambda).$$

□

Note that the fact of Y being r-fuzzy regular space in Theorem 2.7 is only used in (2) ⇒ (3).

Definition 2.8. Let (X, τ) be a fts, $\mu \in I^X$, $x_t \in Pt(X)$, $r \in I_0$. Then μ is called r-open $\mathcal{E}_\tau$-neighborhood of x_t if $x_t q \mu$ which μ is r-feo. We denote

$$\mathcal{E}_\tau(x_t, r) = \left\{ \mu \in I^X : x_t q \mu, \mu \text{ is } r\text{-feo} \right\}.$$

Theorem 2.9. *If $f : (X, \tau) \to (Y, \eta)$ is fuzzy weakly e-closed, then for each fuzzy point y_t in Y and each $\mu \in Q_\tau \left(f^{-1}(y)_t, r \right)$, there exists $\lambda \in \mathcal{E}_\eta(y_t, r)$ such that $f^{-1}(\lambda) \leq C_\tau(\mu, r)$.*

Proof. Let $\mu \in Q_\tau(f^{-1}(y)_t, r)$ with $f(x) = y$. Then $\mu(x) + t > 1$ and hence there exists $p > 0$ such that $\mu(x) \geq 1 - t + p > 1 - t$. Put $s = 1 - t + p$. For each fuzzy point y_s and $\tau(\mu) \geq r$ with $f^{-1}(y_s) \leq \mu$, by Theorem 2.7 (4), there exists r-feo ν containing y_s in Y such that $f^{-1}(\nu) \leq C_\tau(\mu, r)$. Now $\nu(y) \geq \nu(x) \geq s = 1 - t + p > 1 - t$. Hence $\nu(y) > 1 - t$. Thus $\nu \in \mathcal{E}_\eta(y_t, r)$. □

Theorem 2.10. *If $f : (X, \tau) \to (Y, \eta)$ is fuzzy weakly e-closed and if for each $\lambda \in I^X$, $r \in I_0$ with $\tau(\overline{1} - \lambda) \geq r$ and $y_p q(\overline{1} - f(\lambda))$, there exists $\mu \in Q_\tau(f^{-1}(y)_p, r)$ such that $f^{-1}(y)_p q\mu \leq C_\tau(\mu, r) \leq \overline{1} - \lambda$, then f is fuzzy e-closed.*

Proof. Let $\lambda \in I^X$, $r \in I_0$ with $\tau(\overline{1} - \lambda) \geq r$ and let $y_p q(\overline{1} - f(\lambda))$. By hypothesis, there exists $\mu \in Q_\tau(f^{-1}(y)_p, r)$ such that

$$f^{-1}(y)_p \leq \mu \leq C_\tau(\mu, r) \leq \overline{1} - \lambda.$$

Since f is fuzzy weakly e-closed, by Theorem 2.9, There exists $\nu \in \mathcal{E}_\eta(y_p, r)$ with $y_p \in \nu$ and $f^{-1}(\nu) \leq C_\tau(\mu, r)$. Therefore, we obtain $f^{-1}(\nu)\overline{q}\lambda$ and hence $\nu\overline{q}f(\lambda)$, this shows that $y_p \notin eT_\eta(f(\lambda, r))$. Thus, $p > f(\lambda)(y)$ implies $p > eT_\eta(f(\lambda), r)(y)$. □

Therefore, Now we give the following:

Definition 2.11. Let $f : (X, \tau) \to (Y, \eta)$ be a function from a fts (X, τ) into a fts (Y, η). The function f is called:

(1) fuzzy contra-closed (FC-closed, for short) if for $\lambda \in I^X$ we have $\tau_2(f(\lambda)) \geq \tau_1(\overline{1} - \lambda)$,
(2) fuzzy contra e-open if $f(\lambda)$ is r-fec subset of Y for each $\lambda \in I^X$, $r \in I_0$ with $\tau(\lambda) \geq r$.

Theorem 2.12. (1) *If $f : (X, \tau) \to (Y, \eta)$ is FP-closed and FC-closed, then f is fuzzy weakly e-closed.*
(2) *If $f : (X, \tau) \to (Y, \eta)$ is fuzzy contra e-open, then f is fuzzy weakly e-closed.*

Proof. (1) Let $\lambda \in I^X$, $r \in I_0$ with $\tau(\overline{1} - \lambda) \geq r$. Since f is FP-closed,

$$\begin{aligned} eT_\eta(I_\eta(f(\lambda), r), r) &= C_\eta(I_\eta(f(\lambda), r), r) \\ &\leq f(\lambda), \end{aligned}$$

and since f is FC-closed $\eta(f(\lambda)) \geq r$. Therefore

$$\begin{aligned} eT_\eta(f(I_\tau(\lambda, r)), r) &= eT_\eta(f(\lambda), r) \\ &= eT_\eta(I_\eta(f(\lambda), r), r) \\ &\leq f(\lambda). \end{aligned}$$

(2) Let $\lambda \in I^X$, $r \in I_0$ with $\tau(\overline{1} - \lambda) \geq r$. Then

$$eT_\eta(f(I_\tau(\lambda, r)), r) = f(I_\tau(\lambda, r)) \leq f(\lambda).$$

□

3. Conclusion

Šostak's fuzzy topology has been recently of major interest among fuzzy topologies. In this paper, we have introduced and characterized fuzzy weakly e-closed functions in fuzzy topological spaces and the relationship between these mappings are investigated. Also, we have investigated some properties of them.

Acknowledgment. The authors are greatful to the referees for their useful comments and suggestions. The authors thank the referees for his suggestions in improving this paper.

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[1] Department of Mathematics, Kandaswami Kandar's College, P-velur, Tamil Nadu-638 182, India.

E-mail address: vckkc3895@gmail.com

[2] Research Scholar (Part Time), Department of Mathematics, Kandaswami Kandar's College, P-velur, Tamil Nadu-638 182, India.

E-mail address: pspmaths@gmail.com