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CERTAIN SUBCLASSES OF BI-UNIVALENT FUNCTIONS ASSOCIATED WITH THE AGHALARY-EBADIAN-WANG OPERATOR

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ABSTRACT. In this paper, we introduce and investigate two new subclasses of the functions class Σ of bi-univalent functions defined in the open unit disk, which are associated with the Aghalary-Ebadian-Wang operator. We estimate the coefficients $|a_2|$ and $|a_3|$ for functions in these new subclasses. Several consequences of the result are also pointed out.

1. Introduction

Let $\mathcal{A}$ denotes the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the open unit disk

$$\mathbb{U} = \{z : z \in \mathbb{C} \quad \text{and} \quad |z| < 1\}$$

and normalized by the conditions $f(0) = 0$ and $f'(0) = 1$. Further, by $\mathcal{S}$ we shall denoted the class of all functions in $\mathcal{A}$ which are univalent in $\mathbb{U}$. Some of the most important and well-investigated subclasses of the univalent functions class $\mathcal{S}$ include (for example) the class $\mathcal{S}^*(\alpha)$ of starlike functions of order α in $\mathbb{U}$ and the class $K(\alpha)$ of convex functions of order α in $\mathbb{U}$. The Koebe one-quarter theorem [1] ensures that the image of $\mathbb{U}$ under every univalent function $f \in \mathcal{A}$ contains a disk of radius $1/4$. Thus every univalent function f has an inverse f^{-1} satisfying

$$f^{-1}(f(z)) = z, \quad (z \in \mathbb{U})$$

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and

$$f\left(f^{-1}(w)\right)=w, \qquad \left(|w|<r_0(f), \quad r_0(f)\geqslant 1/4\right),$$

where

$$f^{-1}(w)=w-a_2w^2+\left(2a_2^2-a_3\right)w^3-\left(5a_2^3-5a_2a_3+a_4\right)w^4+\cdots.$$

A function $f\in\mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ if both $f(z)$ and $f^{-1}(z)$ are univalent in $\mathbb{U}$. Let Σ denote the class of bi-univalent functions in $\mathbb{U}$ given by (1.1). Also, let $\mathcal{P}$ denotes the class consisting of Caratheodory functions $p(z)$, such that

$$p(z)=1+\sum_{k=2}^{\infty}a_kz^k, \qquad \Re\{p(z)\}>0, \quad z\in\mathbb{U}. \tag{1.2}$$

For two functions $f_j(z),(j=1,2)$, given by

$$f_j(z)=z+\sum_{k=2}^{\infty}a_{k,j}z^k, \quad (j=1,2),$$

the Hadamard product (or convolution) of $f_1(z)$ and $f_2(z)$ is given by

$$\begin{aligned}(f_1*f_2)(z)&:=z+\sum_{k=2}^{\infty}a_{k,1}a_{k,2}z^k\\&=(f_2*f_1)(z), \quad (z\in\mathbb{U}).\end{aligned}$$

The Pochhammer symbol (or the *shifted factorial*) in terms of the Gamma function is given by,

$$\begin{aligned}(\kappa)_n&:=\frac{\Gamma(\kappa+n)}{\Gamma(\kappa)}\\&=\begin{cases}1 & (n=0;\ \kappa\in\mathbb{C}\setminus\{0\}),\\ \kappa(\kappa+1)\ldots(\kappa+n-1) & (n\in\mathbb{N}=\{1,2,\ldots\};\ \kappa\in\mathbb{C}).\end{cases}\end{aligned}$$

Then define the function $\phi_a^{\lambda}(b,c;z)$, by

$$\phi_a^{\lambda}(b,c;z):=1+\sum_{n=1}^{\infty}A_n\psi_nz^n, \quad (z\in\mathbb{U}), \tag{1.3}$$

where

$$A_n:=\left(\frac{a}{a+n}\right)^{\lambda}, \qquad \psi_n:=\frac{(b)_n}{(c)_n}$$

and

$b\in\mathbb{R};\ c\in\mathbb{R}\setminus\mathbb{Z}_0^-;\ a\in\mathbb{C}\setminus\mathbb{Z}_0^- \quad (\mathbb{Z}_0^-:=\{0,-1,-2,-3,\cdots\});\ \lambda\geqslant 0.$

By using the function $\phi_a^\lambda(b,c;z)$ given by (1.2), Aghalary-Ebadian-Wang [1], introduced the familiar convolution operator $L_a^\lambda(b,c;\beta)$ as follows;

$$L_a^\lambda(b,c;\beta)f(z) := \phi_a^\lambda(b,c;z) * \left(\frac{f(z)}{z}\right)^\beta,$$

where $f \in \mathcal{A}; \beta \in \mathbb{C} \setminus \{0\}; z \in \mathbb{U}$. In 1967, Lewin [8] investigated the bi-univalent function class Σ and showed that $|a_2| < 1/51$. On the other hand, Brannan and Clunie [2] (see also [3, 4, 13]) and Netanyahu [10] made an attempt to introduce various, subclasses of the bi-univalent function class Σ and obtained non-sharp coefficient estimates on the first two coefficients $|a_2|$ and $|a_3|$ of (1.1). But the coefficient problem for each of the following Taylor-Maclaurin coefficients

$$|a_n|, \quad (n \in \mathbb{N}\backslash\{1,2\}); \quad \mathbb{N} = \{1,2,3,\ldots\}$$

is still an open problem. Following Brannan and Taha [4] many researchers (see [6, 7, 9, 11, 12, 14, 15]), have recently introduced and investigated several interesting subclasses of the bi-univalent function class Σ and they have found non-sharp estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$. Making use of the certain convolution operator $L_a^\lambda(b,c;\beta)$, we introduce the following two new subclasses of the function class Σ.

Definition 1.1. A function $f(z)$ given by (1.1) is said to be in the class $\mathcal{M}_\Sigma^L(\mu,\alpha)$ if the following conditions are satisfied $f \in \Sigma$ and

$$\left|\arg\left(\frac{z\big(zL_a^\lambda(b,c;\beta)f(z)\big)'}{(1-\alpha)z + \alpha z L_a^\lambda(b,c;\beta)f(z)}\right)\right| < \frac{\mu\pi}{2}, \tag{1.4}$$

where $0 < \mu \le 1; 0 \le \alpha \le 1; z \in \mathbb{U}$; and

$$\left|\arg\left(\frac{w\left(wL_a^\lambda(b,c;\beta)g(w)\right)'}{(1-\alpha)w + \alpha w L_a^\lambda(b,c;\beta)g(w)}\right)\right| < \frac{\mu\pi}{2}, \tag{1.5}$$

where $0 < \mu \le 1; 0 \le \alpha \le 1; w \in \mathbb{U}$; and

$$g(w) = w - a_2w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2a_3 + a_4)w^4 + \cdots, \tag{1.6}$$

that is, the extension of f^{-1} in $\mathbb{U}$.

Definition 1.2. A function $f(z)$ given by (1.1) is said to be in the class $\mathcal{N}_\Sigma^L(\gamma,\alpha)$ if the following conditions are satisfied:

$$f \in \Sigma \quad \text{and} \quad \Re\left(\frac{z\left(zL_a^\lambda(b,c;\beta)f(z)\right)'}{(1-\alpha)z + \alpha z L_a^\lambda(b,c;\beta)f(z)}\right) > \gamma,$$

where $0 \le \gamma < 1; 0 \le \alpha \le 1; z \in \mathbb{U}$; and

$$\Re\left(\frac{w\left(wL_a^\lambda(b,c;\beta)g(w)\right)'}{(1-\alpha)w+\alpha wL_a^\lambda(b,c;\beta)g(w)}\right) > \gamma,$$

where $0 \le \gamma < 1; 0 \le \alpha \le 1; w \in \mathbb{U}$; and the function g is given by (1.6). The object of the present paper is to find estimates on the coefficients $|a_2|$ and $|a_3|$ for functions in the above-defined subclasses $\mathcal{M}_\Sigma^L(\mu,\alpha)$ and $\mathcal{N}_\Sigma^L(\gamma,\alpha)$ of the function class Σ with the techniques used earlier by Srivastava et al. [12]. In order to derive our main result, we shall need the following Lemma.

Lemma 1.3 ([1]). *If $p \in \mathcal{P}$, then $|c_k| \le 2$ for each k, where $\mathcal{P}$ is the family of all functions p that is introduced in (1.3).*

2. Coefficient Bounds for the Function Class $\mathcal{M}_\Sigma^L(\mu,\alpha)$

We begin by finding the estimates on the coefficients $|a_2|$ and $|a_3|$ for functions in the class $\mathcal{M}_\Sigma^L(\mu,\alpha)$.

Theorem 2.1. *Let the function $f(z)$ of the form* (1.1) *be in the class $\mathcal{M}_\Sigma^L(\mu,\alpha)$, $(0 < \mu \le 1; 0 \le \alpha \le 1)$. Then*

$$|a_2| \le \tfrac{2}{|\beta|}\sqrt{\frac{(6-\alpha)}{(2-\alpha)(3-\alpha)}\frac{\mu}{|\psi_2||A_2|}} \tag{2.1}$$

and

$$|a_3| \le 2\left(\frac{\mu}{(2-\alpha)|\beta||\psi_1||A_1|}\right)^2 + 2\left(\frac{\mu}{(3-\alpha)|\beta||\psi_2||A_2|}\right). \tag{2.2}$$

Proof. It follows from (1.4) and (1.5) that there exist functions $p, q \in \mathcal{P}$ such that

$$\frac{z\left(zL_a^\lambda(b,c;\beta)f(z)\right)'}{(1-\alpha)z+\alpha zL_a^\lambda(b,c;\beta)f(z)} = [p(z)]^\mu \tag{2.3}$$

We can write (2.3) as

$$\frac{\left(zL_a^\lambda(b,c;\beta)f(z)\right)'}{(1-\alpha)+\alpha L_a^\lambda(b,c;\beta)f(z)} = [p(z)]^\mu$$

or as

$$\frac{(zh(z))'}{(1-\alpha)+\alpha h(z)} = [p(z)]^\mu,$$
$$h(z) = L_a^\lambda(b,c;\beta),$$
$$f(z) = 1 + z + a_2z^2 + \cdots .$$

Therefore,

$$1 + 2z + 3a_2z^2 + \cdots = \left\{1 - \alpha + \alpha z + \alpha a_2 z^2 + \cdots\right\} \times \left\{1 + \mu p_1 z + \left(\mu(\mu-1)\frac{p_1}{2} + \mu p_2\right) z^2 + \cdots\right\}.$$

and

$$\frac{w\left(wL_a^\lambda(b,c;\beta)g(w)\right)'}{(1-\alpha)w + \alpha w L_a^\lambda(b,c;\beta)g(w)} = [q(w)]^\mu, \tag{2.4}$$

where $p(z)$ and $q(w)$ in $\mathcal{P}$ and have the following forms

$$p(z) = 1 + p_1 z + p_2 z^2 + \cdots \tag{2.5}$$

and

$$q(w) = 1 + q_1 w + q_2 w^2 + \cdots , \tag{2.6}$$

respectively. Now, equating the coefficients in (2.3) and (2.4), we get

(2.7)

$$\begin{aligned}&(2-\alpha)\beta a_2 A_1 \psi_1\\ &\quad= \mu p_1 (3-\alpha)\beta\left[(\beta-1)a_2^2 + 2a_3\right]\psi_2 A_2/2 - \alpha(2-\alpha)\beta^2 a_2^2 A_1^2 \psi_1^2\\ &\quad= \left[2p_2\mu + \mu(\mu-1)p_1^2\right]/2\end{aligned}$$

and

(2.8)

$$\begin{aligned}&(2-\alpha)\beta a_2 A_1 \psi_1\\ &\quad= \mu q_1 (3-\alpha)\beta\left[(\beta-1)a_2^2 + 2(a_2^2 - a_3)\right]\psi_2 A_2/2 - \alpha(2-\alpha)\beta^2 a_2^2 A_1^2 \psi_1^2\\ &\quad= \left[2q_2\mu + \mu(\mu-1)q_1^2\right]/2.\end{aligned}$$

From (2.7) and (2.8) we find that;

$$p_1 = -q_1, p_2 = q_2, \tag{2.9}$$

$$\beta^2 a_2^2 \psi_1^2 A_1^2 = \frac{\mu^2 p_1^2}{(2-\alpha)^2} \tag{2.10}$$

and

$$\beta^2 a_2^2 \psi_1^2 A_1^2 = \frac{\mu^2 q_1^2}{(2-\alpha)^2}. \tag{2.11}$$

Now from (2.7), (2.10) and (2.8), (2.11), we obtain:

$$\frac{1}{2}(3-\alpha)\beta\left[(\beta-1)a_2^2 + 2a_3\right]\psi_2 A_2 - \frac{\alpha\mu^2 p_1^2}{(2-\alpha)} = \left[p_2\mu + \frac{\mu(\mu-1)}{2}p_1^2\right]. \tag{2.12}$$

and

$$\frac{1}{2}(3-\alpha)\beta\left[(\beta-1)a_2^2+2a_3\right]\psi_2 A_2-\frac{\alpha\mu^2 q_1^2}{(2-\alpha)} = \left[q_2\mu+\frac{\mu(\mu-1)}{2}q_1^2\right]. \tag{2.13}$$

Adding (2.12) and (2.13), we get,

$$\beta^2(3-\alpha)a_2^2\psi_2 A_2 = \mu(p_2+q_2)+\frac{(2+\alpha)\mu^2-(2-\alpha)\mu}{2(2-\alpha)}(p_1^2+q_1^2). \tag{2.14}$$

Substituting (2.9) into (2.14), we obtain;

$$\beta^2(3-\alpha)a_2^2\psi_2 A_2=\mu(p_2+q_2)+\frac{(2+\alpha)\mu^2-(2-\alpha)\mu}{(2-\alpha)}p_1^2. \tag{2.15}$$

Applying Lemma (1.2) for the coefficients p_2, q_2 and p_1 we immediately have;

$$|a_2|^2\leq\frac{4\mu+4\frac{(2+\alpha)\mu^2-(2-\alpha)\mu}{2-\alpha}}{|\beta^2|(3-\alpha)|\psi_2||A_2|}.$$

Since $0<\mu\leq 1$, we know $\mu^2\leq\mu$, then

$$|a_2|\leq\frac{2}{|\beta|}\sqrt{\frac{(6-\alpha)}{(2-\alpha)(3-\alpha)}\frac{\mu}{|\psi_2||A_2|}}.$$

This gives the bound on $|a_2|$ as asserted in (2.1).

Next, in order to find the bound on $|a_3|$, by subtracting (2.13) from (2.12) we get,

$$(\alpha-3)\beta a_2^2\psi_2 A_2+2(3-\alpha)\beta a_3\psi_2 A_2=\mu(p_2-q_2)+\frac{\mu(\mu-1)}{2}(p_1^2-q_1^2). \tag{2.16}$$

From (2.10) we find that

$$a_2^2=\frac{\mu^2 p_1^2}{(2-\alpha)^2\beta^2\psi_1^2 A_1^2}. \tag{2.17}$$

It follows from (2.10), (2.16) and (2.17) that

$$a_3=\frac{\mu^2 p_1^2}{2(2-\alpha)^2\beta^2\psi_1^2 A_1^2}+\frac{\mu(p_2-q_2)}{(3-\alpha)\beta\psi_2 A_2}.$$

This completes the proof of Theorem 2.1. □

It is easy to observe that the function

$$\varphi : [0,1] \longrightarrow \mathbb{R}, \qquad \varphi(\alpha) = \frac{6-\alpha}{(2-\alpha)(3-\alpha)}$$

is an increasing function and has a maximum value at $\alpha = 1$, which is an equals 2/5, therefore we get the following corollary of Theorem 2.1:

Corollary 2.2. *Let the function $f(z)$ given by* (1.1) *be in the following class:*

$$\mathcal{M}_{\Sigma}^{L}(\mu,\alpha), \quad (0 < \mu \leq 1; 0 \leq \alpha \leq 1).$$

Then

$$|a_2| \leq \frac{1}{|\beta|}\sqrt{\frac{10\mu}{|\psi_2||A_2|}}$$

and

$$|a_3| \leq 2\left(\frac{\mu}{(2-\alpha)|\beta||\psi_1||A_1|}\right)^2 + 2\left(\frac{\mu}{(3-\alpha)|\beta||\psi_2||A_2|}\right).$$

Taking $\mathcal{M}_{\Sigma}^{L}(\mu,\alpha)$ $(0 < \mu \leq 1;\ b = c, \beta = 1, \alpha = 1, a = 1, \lambda = 1$ in Corollary 2.2, we obtain $0 \leq \alpha \leq 1)$.in the following corollary:

Corollary 2.3. *Let the function $f(z)$ given by* (1.1) *be in the following class:*

$$\mathcal{M}_{\Sigma}^{L}(\mu,1), \quad \left(0 < \mu \leq 1; L = L_1^1(b,b;1)\right).$$

Then

$$|a_2| \leq \sqrt{30\mu}$$

and

$$|a_3| \leq 8\mu^2 + 3\mu \leq 11\mu.$$

3. Coefficient Bounds for the Function Class $\mathcal{N}_{\Sigma}^{L}(\mu,\alpha)$

In this section, we find the estimates on the coefficients $|a_2|$ and $|a_3|$ for functions in the class $\mathcal{N}_{\Sigma}^{L}(\mu,\alpha)$

Theorem 3.1. *Let the function $f(z)$ given by* (1.1) *be in the following class:*

$$\mathcal{N}_{\Sigma}^{L}(\gamma,\alpha), \quad (0 \leq \gamma < 1; 0 \leq \alpha \leq 1).$$

Then

$$|a_2| \leq \frac{2}{|\beta|}\sqrt{\frac{(1-\gamma)(2+\alpha)}{(3-\alpha)(2-\alpha)|\psi_2||A_2|}}$$

and

$$|a_3| \leq 2\left(\frac{(1-\gamma)}{(2-\alpha)|\beta||\psi_1||A_1|}\right)^2 + 2\left(\frac{(1-\gamma)}{(3-\alpha)|\beta||\psi_2||A_2|}\right).$$

Proof. It follows from (2.3) and (2.4) that there exist $p, q \in \mathcal{P}$ such that

$$\frac{z\left(zL_a^\lambda(b,c;\beta)f(z)\right)'}{(1-\alpha)z+\alpha zL_a^\lambda(b,c;\beta)f(z)} = \gamma + (1-\gamma)p(z) \tag{3.1}$$

and

$$\frac{w\left(wL_a^\lambda(b,c;\beta)g(w)\right)'}{(1-\alpha)w+\alpha wL_a^\lambda(b,c;\beta)g(w)} = \gamma + (1-\gamma)q(w). \tag{3.2}$$

where $p(z)$ and $q(w)$ in $\mathcal{P}$ and have the form (2.5) and (2.6), respectively.

Equating coefficient in (3.1) and (3.2), we get

(3.3)

$$\begin{aligned}(2-\alpha)\beta a_2 A_1\psi_1&\\ &= (1-\gamma)p_1\frac{1}{2}(3-\alpha)\beta\left[(\beta-1)a_2^2+2a_3\right]\psi_2A_2-\alpha(2-\alpha)\beta^2a_2^2A_1^2\psi_1^2\\ &= (1-\gamma)p_2-(2-\alpha)\beta p_1^2.\end{aligned}$$

and

(3.4)

$$\begin{aligned}(2-\alpha)\beta a_2 A_1\psi_1&\\ &= (1-\gamma)q_1\frac{1}{2}(3-\alpha)\beta\left[(\beta-1)a_2^2+2(a_2^2-a_3)\right]\psi_2A_2-\alpha(2-\alpha)\beta^2a_2^2A_1^2\psi_1^2\\ &= (1-\gamma)q_2-(2-\alpha)\beta q_1^2.\end{aligned}$$

From (3.3)and (3.4) we find that

$$p_1 = -q_1, \tag{3.5}$$

and implies

$$\beta^2a_2^2\psi_1^2A_1^2 = \frac{(1-\gamma)^2p_1^2}{(2-\alpha)^2} \tag{3.6}$$

and

$$\beta^2a_2^2\psi_1^2A_1^2 = \frac{(1-\gamma)^2q_1^2}{(2-\alpha)^2}. \tag{3.7}$$

Also, from (3.6), we get

$$a_2^2 = \frac{(1-\gamma)^2p_1^2}{\beta^2\psi_1^2A_1^2(2-\alpha)^2} \tag{3.8}$$

Now from (3.3), (3.6) and (3.4), (3.7) we obtain

$$\begin{aligned}(1-\gamma)p_1\frac{1}{2}(3-\alpha)\beta\left[(\beta-1)a_2^2+2a_3\right]\psi_2A_2-\frac{\alpha(1-\gamma)^2p_1^2}{(2-\alpha)}&\\ = (1-\gamma)p_2-(2-\alpha)\beta p_1^2.&\end{aligned} \tag{3.9}$$

and

(3.10)

$$(1-\gamma)q_1\frac{1}{2}(3-\alpha)\beta\left[(\beta-1)a_2^2+2(a_2^2-a_3)\right]\psi_2A_2-\beta^2a_2^2\psi_1^2A_1^2-\frac{\alpha(1-\gamma)^2q_1^2}{(2-\alpha)}$$
$$=(1-\gamma)q_2-(2-\alpha)\beta q_1^2.$$

Adding (3.9) and (3.10), we get

$$\beta^2(3-\alpha)a_2^2\psi_2A_2=(1-\gamma)(p_2+q_2)+\frac{\alpha(1-\gamma)^2}{(2-\alpha)}(p_1^2+q_1^2). \tag{3.11}$$

Substituting (3.4) into (3.11), we obtain:

$$\beta^2(3-\alpha)a_2^2\psi_2A_2=(1-\gamma)(p_2+q_2)+\frac{\alpha(1-\gamma)^2}{(2-\alpha)}(2p_1^2)$$

Applying Lemma 1.2 for the coefficients p_2, q_2 and p_1 we immediately have

$$|a_2|^2\le\frac{4(2-\alpha)(1-\gamma)+8\alpha(1-\gamma)^2}{|\beta|^2(3-\alpha)(2-\alpha)|\psi_2||A_2|}.$$

Next, in order to find the bound on $|a_3|$, by subtracting (3.10) from (3.9), we get

$$(\alpha-3)\beta a_2^2\psi_2A_2+2(3-\alpha)\beta a_3\psi_2A_2=(1-\gamma)(p_2-q_2). \tag{3.12}$$

Substituting the value of a_2^2 from (3.8) into (3.12), we obtain

$$a_3=\frac{(1-\gamma)^2p_1^2}{2(2-\alpha)^2\beta^2\psi_1^2A_1^2}+\frac{(1-\gamma)(p_2-q_2)}{2(3-\alpha)\beta\psi_2A_2}.$$

Applying Lemma 1.2 for the coefficients p_2, q_2 and p_1 we immediately have

$$|a_3|\le 2\left(\frac{(1-\gamma)}{(2-\alpha)|\beta||\psi_1||A_1|}\right)^2+2\left(\frac{(1-\gamma)}{(3-\alpha)|\beta||\psi_2||A_2|}\right).$$

This complete the proof of Theorem 3.1. □

Now we consider the function

$$\nu:[0,1]\longrightarrow\mathbb{R},\qquad \nu(\alpha)=\frac{(2+\alpha)}{(3-\alpha)(2-\alpha)}$$

which is an increasing function and has a maximum value at $\alpha=1, equals$ with $1/5$, therefore we obtain the following corollary:

Corollary 3.2. *Let the function $f(z)$ given by* (1.1) *be in the following class:*

$$\mathcal{N}_\Sigma^L(\gamma,\alpha),\quad (0\le\gamma<1;0\le\alpha\le1).$$

Then

$$|a_2| \leq \frac{1}{|\beta|}\sqrt{\frac{6(1-\gamma)}{|\psi_2||A_2|}}$$

and

$$|a_3| \leq 2\left(\frac{(1-\gamma)}{(2-\alpha)|\beta||\psi_1||A_1|}\right)^2 + 2\left(\frac{(1-\gamma)}{(3-\alpha)|\beta||\psi_2||A_2|}\right).$$

putting $b = c, \beta = 1, \alpha = 1, a = 1, \lambda = 1$ in Corollary 3.2, we get the following corollary:

Corollary 3.3. *Let the function $f(z)$ given by (1.1) be in the following class:*

$$\mathcal{N}_{\Sigma}^{L}(\gamma, 1), \quad \left(0 \leq \gamma < 1; L = L_1^1(b, b; 1)\right).$$

Then

$$|a_2| \leq 3\sqrt{2(1-\gamma)}$$

and

$$|a_3| \leq 8(1-\gamma)^2 + 3(1-\gamma) \leq 11(1-\gamma).$$

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