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On Regular Generalized δ-closed Sets in Topological Spaces

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Abstract. In this paper a new class of sets called regular generalized δ-closed set (briefly rgδ-closed set)is introduced and its properties are studied. Several examples are provided to illustrate the behaviour of these new class of sets.

1. Introduction and Preliminaries

In 1970, Levine[6] introduced the concept of generalized closed set and discussed the properties of sets, closed and open maps, compactness, normal and separation axioms. In this paper, a new class of closed sets called regular generalized δ-closed set is introduced to prove that the class forms a toplogy. The notion of regular generalized δ-closed set and its different characterizations are given in this paper. Throughout this paper (X, τ) and (Y, σ) represent the non-empty topological spaces on which no seperation axioms are assumed, unless otherwise mentioned. Let $A \subseteq X$, the closure of A and interior of A will be denoted by $cl(A)$ and $int(A)$ respectively, union of all δ-open sets of X contained in A is called δ-interior of A and it is denoted by $\delta int(A)$, the intersection of all δ- closed sets of X containing A is called δ-closure of A and it is denoted by $\delta cl(A)$.

We shall require the following known definitions.

Definition 1.1. *A subset A of a topological space (X, τ) is called*

(i) a pre-open set[10] if $A \subseteq \text{int}(cl(A))$.
(ii) a semi-open set[6]if $A \subseteq cl(\text{int}(A))$.
(iii) a α-open set[11] if $A \subseteq \text{int}(cl(\text{int}(A)))$.
(iv) a b-open set [1]if $A \subseteq cl(\text{int}(A)) \cup \text{int}(cl(A))$.

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(v) a generalized closed set (briefly g-closed) [8]if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
(vi) a generalized α-closed set (briefly $g\alpha$-closed)[7] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α open in X.
(vii) a generalized b closed set (briefly gb-closed)[12] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
(viii) a generalized semi-pre closed set (briefly gsp-closed)[9] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
(ix) a generalized pre-closed set (briefly gp-closed)[11] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
(x) a generalized semi-closed set (briefly gs-closed)[2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
(xi) a semi generalized closed set (briefly sg-closed) [3]if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.
(xii) a generalized αb-closed set (briefly $g\alpha b$-closed)[13] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in X.
(xiii) a generalized pre regular closed set (briefly gpr-closed)[4] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
(xiv) a semi generalized b-closed set (briefly sgb-closed)[5] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.
(xv) strongly generalized closed set (briefly g^*-closed)[14]if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X.
(xvi) δ-closed set [15] if $A = Cl_\delta(A)$, where $Cl_\delta(A)$=$\{x \in X : \text{int}(cl(U)) \cap A \neq \phi, U \in \tau and x \in U\}$.
(xvii) regular generalized closed set(briefly rg-closed) [16] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

The complements of the above mentioned closed sets are their corresponding open sets.

2. Regular Generalized δ-Closed Sets.

In this section a new class sets called regular generalized δ-closed set is introduced and some of its properties are investigated.

Definition 2.1. A subset A of a topological space (X, τ) is called regular generalized δ-closed set (briefly $rg\delta$-closed set) if $\delta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

Theorem 2.2. *Every closed set is an $rg\delta$-closed set.*

Proof. Let A be any closed set in X such that $A \subseteq U$, where U is regular open. Since $\delta cl(A) \subseteq cl(A) = A$. Therefore $\delta cl(A) \subseteq U$. Hence A is an $rg\delta$-closed set in X. □

The converse of the above theorem need not be true as seen from the following example.

Example 2.3. Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. The set $\{a, b\}$ is $rg\delta$-closed set but not a closed set.

Theorem 2.4. *Every δ-closed set is an $rg\delta$-closed set.*

Proof. Let A be any δ-closed set in X such that U be any regular open set containing A. Since A is δ-closed, $\delta cl(A) = A$. Therefore $\delta cl(A) \subseteq U$. Hence A is an $rg\delta$-closed set. □

The converse of the above theorem need not be true as seen from the following example.

Example 2.5. In example2.3, the set $\{a, b\}$ is an $rg\delta$-closed set but not a δ-closed set.

Theorem 2.6. *Every α-closed set is an $rg\delta$-closed set.*

Proof. Let A be any α-closed set in X and U be any regular open set containing A. Since A is α-closed, $\delta cl(A) \subseteq \alpha cl(A) \subseteq U$. Therefore $\delta cl(A) \subseteq U$. Hence A is an $rg\delta$-closed set. □

The converse of the above theorem need not be true as seen from the following example.

Example 2.7. Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$. The set $\{a, c\}$ is an $rg\delta$-closed set but not an α-closed set.

Theorem 2.8. *Every semi-closed set is an $rg\delta$-closed set.*

Proof. Let A be any semi-closed set in X and U be any regular open set containing A. Since A is semi-closed, $\delta cl(A) \subseteq scl(A) \subseteq U$. Therefore $\delta cl(A) \subseteq U$. Hence A is an $rg\delta$-closed set. □

The converse of the above theorem need not be true as seen from the following example.

Example 2.9. In example 2.7, the set $\{a, c\}$ is an $rg\delta$-closed set but not a semi-closed set.

Theorem 2.10. *Every pre-closed set is an $rg\delta$-closed set.*

Proof. Let A be any pre-closed set in X and U be any regular open set containing A. Since A is pre-closed, $pcl(A) \subseteq \delta cl(A) \subseteq U$. Therefore $\delta cl(A) \subseteq U$. Hence A is an $rg\delta$-closed set. □

The converse of the above theorem need not be true as seen from the following example.

Example 2.11. In example 2.7, the set $\{a, c\}$ is an $rg\delta$-closed set but not a pre-closed set.

Theorem 2.12. *Every g^*-closed set is an $rg\delta$-closed set.*

Proof. Let A be any g^*-closed set in X and U be any regular open set containing A. Since A is g^*-closed, $g^*cl(A) \subseteq \delta cl(A) \subseteq U$. Therefore $\delta cl(A) \subseteq U$. Hence A is an $rg\delta$-closed set. □

The converse of the above theorem need not be true as seen from the following example.

Example 2.13. In example 2.7, the set $\{c\}$ is an $rg\delta$-closed set but not a g^*-closed set.

Theorem 2.14. *Every gpr-closed set is an $rg\delta$-closed set.*

Proof. Let A be any *gpr*-closed set in X and U be any regular open set containing A. Since every pre-open sets are δ-open sets, $pcl(A) \subseteq \delta cl(A) \subseteq U$. Therefore $\delta cl(A) \subseteq U$. Hence A is an $rg\delta$-closed set. □

The converse of the above theorem need not be true as seen from the following example.

Example 2.15. In example 2.3, the set $\{a, b\}$ is an $rg\delta$-closed set but not a *gpr*-closed set.

Theorem 2.16. *Every $rg\delta$-closed set is a gb-closed set.*

Proof. Let A be any $rg\delta$-closed set in X such that U be any regular open set containing A. Since every regular open set is an open set, $bcl(A) \subseteq \delta cl(A) \subseteq U$. Hence A is a *gb*-closed set. □

The converse of the above theorem need not be true as seen from the following example.

Example 2.17. In example 2.3, the set $\{b\}$ is a *gb*-closed set but not an $rg\delta$-closed set.

Theorem 2.18. *Every $rg\delta$-closed set is a gsp-closed set.*

Proof. Let A be any $rg\delta$-closed set in X such that U be any regular open set containing A. Since every regular open set is an open set, $\delta cl(A) \subseteq spcl(A) \subseteq U$. Therefore $spcl(A) \subseteq U$. Hence A is a *gsp*-closed set. □

The converse of the above theorem need not be true as seen from the following example.

Example 2.19. In example 2.7, the set $\{c\}$ is a *gsp*-closed set but not a $rg\delta$-closed set.

Theorem 2.20. *Every $g\alpha$-closed set is an $rg\delta$-closed set.*

Proof. Let A be a $g\alpha$-closed set in X such that U be any regular open set containing A. Since every regular open set is an open set, $\delta cl(A) \subseteq \alpha cl(A) \subseteq U$. Therefore $\delta cl(A) \subseteq U$. Hence A is an $rg\delta$-closed set. □

The converse of the above theorem need not be true as seen from the following example.

Example 2.21. In example 2.7, the set $\{a, c\}$ is an $rg\delta$-closed set but not a $g\alpha$-closed set.

Theorem 2.22. *Every $rg\delta$-closed set is a $g\alpha b$-closed set.*

Proof. Let A be an $rg\delta$-closed set in X such that U be any regular open set containing A. Since every regular open set is an α-open set, $\delta cl(A) \subseteq U$ and $A \subseteq U$, U is α-open. Hence A is an $g\alpha b$-closed set. □

The converse of the above theorem need not be true as seen from the following example.

Example 2.23. Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$. The set $\{b\}$ is a $g\alpha b$-closed set but not an $rg\delta$-closed set.

Theorem 2.24. *Every sgb-closed set is an $rg\delta$-closed set.*

Proof. Let A be any sgb-closed set in X such that U be any semi open set containing A. Since every semi-open set is a regular-open set, therefore $\delta cl(A) \subseteq U$ and $A \subseteq U$, U is regular-open. Hence A is an $rg\delta$-closed set. □

The converse of the above theorem need not be true as seen from the following example.

Example 2.25. In example 2.7, the set $\{a, c\}$ is an $rg\delta$-closed set but not a sgb-closed set.

Theorem 2.26. *Every gp-closed set is an $rg\delta$-closed set.*

Proof. Let A be a gp-closed set in X such that U be any regular open set containing A. Since every regular open set is an open set, $\delta cl(A) \subseteq pcl(A) \subseteq U$. Therefore $\delta cl(A) \subseteq U$. Hence A is an $rg\delta$-closed set. □

The converse of the above theorem need not be true as seen from the following example.

Example 2.27. In example 2.7, the set $\{a, c\}$ is an $rg\delta$-closed set but not a gp-closed set.

Example 2.28. In example 2.3, the set $\{a, b\}$ is an $rg\delta$-closed set but not a gp-closed set.

Remark 2.29. The $rg\delta$-closed sets and rg-closed sets are independent from each other as seen from the following examples.

Example 2.30. Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$. The set $\{c\}$ is an $rg\delta$-closed set but not a rg-closed set and the set $\{b\}$ is an rg-closed set but not a rgδ-closed set.

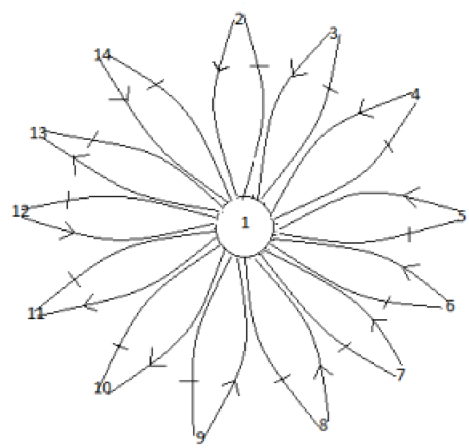
Example 2.31. Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$. The set $\{b\}$ is an $rg\delta$-closed set but not a rg-closed set and the set $\{c\}$ is an rg-closed set but not a rgδ-closed set.

Remark 2.32. The $rg\delta$-closed sets and sg-closed sets are independent from each other as seen from the following examples.

Example 2.33. Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. The set $\{b\}$ is an sg-closed set but not a $rg\delta$-closed set.

Example 2.34. Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$. The set $\{a, c\}$ is an $rg\delta$-closed set but not a sg-closed set.

Remark 2.35. By the above results we have the following diagram.



1.$rg\delta$-closed set	8.gpr-closed set
2.closed set	9.gb-closed set
3.δ-closed set	10.gsb-closed set
4.α-closed set	11.$g\alpha$-closed set
5.Semi-closed set	12.$g\alpha b$-closed set
6.Pre-closed set	13.sgb-closed set
7.g^*-closed set	14.gp-closed set

Remark 2.36. The $rg\delta$-closed sets and gs-closed sets are independent from each other as seen from the following examples.

Example 2.37. Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{b\}, \{a, c\}, \{b, c\}\}$. The set $\{a, c\}$ is an $rg\delta$-closed set but not a gs-closed set.

Example 2.38. Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. The set $\{a\}$ is a gs-closed set but not an $rg\delta$-closed set.

3. Characteristics of $rg\delta$-closed Sets.

Theorem 3.1. *If A and B are $rg\delta$-closed sets in X then $A \cup B$ is an $rg\delta$-closed set in X.*

Proof. Let A and B be $rg\delta$-closed sets in X and U be any regular open set containing A and B. Therefore $cl(A) \subseteq U$, $cl(B) \subseteq U$. Since $A \subseteq U$, $B \subseteq U$ then $A \cup B \subseteq U$. Hence $cl(A \cup B) = cl(A) \cup cl(B) \subseteq U$. Therefore $A \cup B$ is $rg\delta$-closed set in X. □

Theorem 3.2. *A set A is $rg\delta$-closed set iff $\delta cl(A)$-A contains no non-empty regular closed set.*

Proof. Necessity: Let F be a regular closed set in X such that $F \subseteq \delta cl(A)$-A. Then $A \subseteq X$-F. Since A is an $rg\delta$-closed set and X-F is regular open then $\delta cl(A) \subseteq X$-F. (i.e $F \subseteq X$-$\delta cl(A)$). Then $F \subseteq (X - \delta cl(A)) \cap \delta cl(A)$-$A$. Therefore $F = \phi$.

Sufficiency: Let us assume that $\delta cl(A)$-A contains no non empty regular closed set. Let $A \subseteq U$ and U be regular-open. Suppose that $\delta cl(A)$ is not contained in U, then $\delta cl(A) \cap U^c$ is non empty regular closed set of $\delta cl(A)$-A which is a contradiction. Therefore $\delta cl(A) \subseteq U$. Hence A is an $rg\delta$-closed set. □

Theorem 3.3. *The intersection of any two subsets of $rg\delta$-closed sets in X is an $rg\delta$-closed set in X.*

Proof. Let A and B be two subsets of an $rg\delta$-closed set. Assume $A, B \subseteq U$, where U is regular-open. Then $\delta cl(A) \subseteq U$, $\delta cl(B) \subseteq U$. Therefore $\delta cl(A \cap B) \subseteq U$. Since A and B are $rg\delta$-closed sets, $A \cap B$ is an $rg\delta$-closed set. □

Theorem 3.4. *If A is an $rg\delta$-closed set in X and $A \subseteq B \subseteq \delta cl(A)$, then B is an $rg\delta$-closed set in X.*

Proof. Since $B \subseteq \delta cl(A)$, we have $\delta cl(B) \subseteq \delta cl(A)$ then $\delta cl(B)$-$B \subseteq \delta cl(A)$-A.By theorem 3.2, $\delta cl(A)$-A contains no non empty regular closed set. Hence $\delta cl(B)$-B contains no non empty regular closed set. Therefore B is an $rg\delta$-closed set in X. □

Theorem 3.5. *Let $A \subseteq Y \subseteq X$ be an $rg\delta$-closed set in X. Then A is an $rg\delta$-closed set relative to Y.*

Proof. Given that $A \subseteq Y \subseteq X$ and A is an $rg\delta$-cosed set in X. To prove that A is an $rg\delta$-closed set relative to Y, let us assume that $A \subseteq Y \cap U$, where U is regular open in X. Since A is an $rg\delta$-closed set, $A \subseteq U$ implies $\delta cl(A) \subseteq U$. It follows that $Y \cap \delta cl(A) \subseteq Y \cap U$. That is A is an $rg\delta$-closed set relative to Y. □

Theorem 3.6. *If A is both regular open and $rg\delta$-closed set in X then A is a regular closed set.*

Proof. Since A is a regular open and $rg\delta$-closed set in X, $\delta cl(A) \subseteq U$. But $A \subseteq \delta cl(A)$. Therefore $A = \delta cl(A)$. Therefore A is a regular closed set. □

Theorem 3.7. *For $x \in X$, the set $X - \{x\}$ is $rg\delta$-closed or regular open.*

Proof. Suppose that $X - \{x\}$ is not regular open, then X is the only regular open set containing $X - \{x\}$. (i.e $\delta cl(X - \{x\}) \subseteq X$). Then $X - \{x\}$ is an $rg\delta$-closed set in X. □

4. Regular Generalized δ-open Sets and Regular Generalized δ-neighborhoods

In this section new class of sets called regular generalized δ-open sets (briefly $rg\delta$-open) and regular generalized δ-neighborhoods (briefly $rg\delta$-nbhd) in topological spaces are introduced and we study some of their properties.

Definition 4.1. A subset A of a topological space (X, τ) is called a regular generalized δ-open set (briefly $rg\delta$-open set) if A^c is $rg\delta$- closed in X. The family of all $rg\delta$-open sets in X is denoted by $rg\delta - O(X)$.

Theorem 4.2. *If A and B are $rg\delta$-open sets in a space X, then $A \cup B$ is also an $rg\delta$-open set in X.*

Proof. If A and B are $rg\delta$-open sets in a space X, then A^c and B^c are $rg\delta$-closed sets in X. By theorem 3.1 $A^c \cup B^c$ is an $rg\delta$-closed set in X(i.e $A^c \cup B^c$=$(A \cap B)^c$ is an $rg\delta$-closed set in X). Therefore $A \cup B$ is an $rg\delta$-open sets in X. □

Remark 4.3. The union of two $rg\delta$-open sets in X is generally not a $rg\delta$-open set in X.

Example 4.4. Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$. The set $A = \{a\}$ and $B = \{b\}$ is an $rg\delta$-open set but $A \cup B$= $\{a, b\}$ is not an $rg\delta$- open set in X.

Remark 4.5. If A and B are $rg\delta$-open sets in X, then $A \cap B$ is not an $rg\delta$-open set in X.

Example 4.6. Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$. The set $A = \{a, c\}$ and $B = \{b, c\}$ is an $rg\delta$- open set but $A \cap B= \{c\}$ is not an $rg\delta$- open set in X.

Theorem 4.7. *If* $\text{int}(B) \subseteq B \subseteq A$ *and* A *is* $rg\delta$*-open in* X*, then* B *is* $rg\delta$*-open in* X*.*

Proof. Suppose that $\text{int}(B) \subseteq B \subseteq A$ and A is $rg\delta$-open in X then $A^c \subseteq B^c \subseteq A^c$. Since A^c is an $rg\delta$-closed set in X by theorem 4.2, B is an $rg\delta$-open sets in X. □

Definition 4.8. Let x be a point in a topological space X. A subset N of X is said to be a $rg\delta$-nbhd of x iff there exists a $rg\delta$-open set G such that $x \in G \subset N$.

Definition 4.9. A subset N of space X is called an $rg\delta$-nbhd of $A \subset X$ iff there exists a $rg\delta$-open set G such that $A \subset G \subset N$.

Theorem 4.10. *Every nbhd* N *of* $x \in X$ *is a* $rg\delta$*-nbhd of* x*.*

Proof. Let N be a nbhd point of $x \in X$. To prove that N is a $rg\delta$-nbhd of x, by definition of nbhd, there exists an open set G such that $x \in G \subset N$. Hence N is an $rg\delta$-nbhd of x. □

Remark 4.11. In general, an $rg\delta$-nbhd of $x \in X$ need not be a nbhd of $x \in X$ as seen from the following example.

Example 4.12. Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$. Then $rg\delta - O(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$. The set $\{a, c\}$ is $rg\delta$-nbhd of point b, since the $rg\delta$-open set $\{b\}$ is such that $b \in \{b\} \subset \{a, b\}$. However, the set $\{a, b\}$ is not a nbhd of the point b, since no open set G exists such that $b \in G \subset \{a, c\}$.

Remark 4.13. The $rg\delta$-nbhd N of $x \in X$ need not be a $rg\delta$-open in X.

Example 4.14. Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$. Then $rg\delta - O(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$. The set $\{c\}$ is an $rg\delta$-open set, but it is an $rg\delta$-nbhd of $\{c\}$. Since $\{c\}$ is an $rg\delta$-open set such that $c \in \{c\} \subset \{b, c\}$.

Theorem 4.15. *If a subset* N *of a space* X *is* $rg\delta$*-open, then* N *is* $rg\delta$*-nbhd of each of all its points.*

Proof. Suppose N is $rg\delta$-open. Let $x \in N$ be an arbitrary point. We claim that N is an $rg\delta$-nbhd of x. Since N is a $rg\delta$-open set and $x \in N \subset N$, it follows that N is an $rg\delta$-nbhd of all of its points. □

Remark 4.16. In general, a $rg\delta$-nbhd of $x \in X$ need not be a nbhd of $x \in X$ as seen from the following example.

Example 4.17. Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$. Then $rgb-O(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$. The set $\{a, b\}$ is an rgb-nbhd of b, since the $rg\delta$-open set $\{b\}$ is such that $b \in \{b\} \subset \{a, b\}$. Also the set $\{a, b\}$ is an $rg\delta$-nbhd point of $\{b\}$, since the $rg\delta$-open set $\{b\}$ is such that $b \in \{b\} \subset \{a, b\}$. (i.e. $\{a, b\}$ is an $rg\delta$-nbhd of each of its points). However the set $\{a, b\}$ is not an $rg\delta$-open set in X.

Theorem 4.18. *Let X be a topological space. If F is an $rg\delta$-closed subset of X and $x \in F^c$ then there exists an $rg\delta$-nbhd N of x such that $N \cap F = \phi$.*

Proof. Let F be an $rg\delta$-closed subset of X and $x \in F^c$. Then F^c is an $rg\delta$-open set of X. So by theorem 4.15 F^c contains an $rg\delta$-nbhd of each of its points. Hence there exists an $rg\delta$-nbhd N of x such that $N \subset F^c$. (i.e $N \cap F = \phi$). □

5. Conclusion

The regular generalised δ-closed set is defined and proved that the class forms a topology. The rgδ-closed set can be used to derive a new decomposition of unity, closed map and open map, homeomorphism, closure and interior and new separation axioms. This idea can be extended to bitopological and fuzzy topological spaces.

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