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Identification of Initial Taylor-Maclaurin Coefficients for Generalized Subclasses of Bi-Univalent Functions

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Abstract. In the present work, the author determines some coefficient bounds for functions in a new class of analytic and bi-univalent functions, which are introduced by using of polylogarithmic functions. The presented results in this paper one the generalization of the recent works of Srivastava et al. [26], Frasin and Aouf [13] and Siregar and Darus [25].

1. Introduction

Let A stand for class of the functions of the form

$$\psi(z) = z + \sum_{k=2}^{\infty} a_k z^k, \tag{1.1}$$

which are analytic in $E = \{z \in \mathbb{C} :| z |< 1\}$. The familiar generalized polilogarithmic function $G(n; z)$ is signified by

$$G(n; z) = \sum_{k=1}^{\infty} \frac{z^k}{k^n}, \quad (n \in \mathbb{C},\, z \in E).$$

Note that

$$G(-1; z) = \frac{z}{1-z^2},$$

is Koebe function. For more details about polylogarthms in the theory of univalent functions, see Ponnusamy and Sabapath [20] and Ponnusamy

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[19]. For $n, \tau \in \mathbb{N}_0 = \{0, 1, 2, 3, \dots\}$ Al-Shaqsi and Darus [24] defined the linear operator

$$D_\tau^n \psi(z) = z + \sum_{k=2}^{\infty} k^n \frac{(k+\tau-1)!}{\tau!\,(k-1)!} a_k z^k, \quad (z \in E). \tag{1.2}$$

If we take $\tau = 0$ in equation (1.2), then we obtain

$$\begin{aligned} D_0^n \psi(z) &= D^n \psi(z) \\ &= z + \sum_{k=2}^{\infty} k^n a_k z^k, \end{aligned}$$

which gives the Sălăgean's differential operator [23]. For $n = 0$

$$\begin{aligned} D_\tau^0 \psi(z) &= D^\delta \psi(z) \\ &= z + \sum_{k=2}^{\infty} C(\delta, k) a_k z^k, \end{aligned} \tag{1.3}$$

where $C(\delta, k) = \binom{k+\delta-1}{\delta}$, $\delta \in \mathbb{N}_0$, which gives the Ruscheweyh derivative operator [22]. It is obvius that the operator D_τ^n includes two well known derivative operators. Also we have

$$D_0^1 \psi(z) = D_1^0 \psi(z) = z\psi'(z).$$

Let $\mathcal{S}$ denote the class of all functions in A consisting of functions which are univalent in E. The analytic and univalent functions play very important roles in the theory of geometric functions. Since univalent functions are one-to-one, they have inverse and the inverse functions do not need to be defined on the entire unit disc E. The Koebe one quarter theorem [12] guarantees that the image of E under any ψ from $\mathcal{S}$ containes a disc of radius $\frac{1}{4}$. Thus, every $\psi \in \mathcal{S}$ has an inverse function ψ^{-1} such that

$$\psi^{-1}(\psi(z)) = z, \quad (z \in E),$$

and

$$\psi\left(\psi^{-1}(w)\right) = w, \quad \left(\mid w \mid < r_0(\psi)\ ; r_0(\psi) \geq \tfrac{1}{4}\right),$$

where

$$\begin{aligned} \psi^{-1}(w) &= \phi(w) \\ &= w - a_2 w^2 + \left(2a_2^2 - a_3\right) w^3 - \left(5a_2^3 - 5a_2 a_3 + a_4\right) w^4 + \cdots. \end{aligned} \tag{1.4}$$

A function $\psi \in A$ is said to be bi-univalent in E if both $\psi(z)$ and $\psi^{-1}(z)$ are univalent in E. Let σ denote the class of bi-univalent functions in E given by (1.1). The Koebe function is not a member of σ. The concept of bi-univalent functions and their coefficient estimates, obtaining the bound $|a_2| < 1.51$, was studied by Lewin [14]. Soon after, Brannan and Clunie [9] conjectured that $|a_2| \leq \sqrt{2}$ for $\psi \in \sigma$. Then, Netanyahu [17] proved that $\max |a_2| = \frac{4}{3}$. In recent years, Srivastava et al. [26], Frasin and Aouf [13], and Xu et al. [28–30] investigated several subclasses of the bi-univalent function class σ and found estimates on the Taylor-Maclourin coefficients $|a_2|$ and $|a_3|$ for functions in these subclasses. Many authors investigated the coefficients $|a_2|$ and $|a_3|$ for different subclasses of bi-univalent functions (see [5–8, 10, 11, 15, 16, 21, 23, 26, 27]).

The target of this study is obtaining the appropriate classes of σ involving polylogarythm is functions. We investigate estimates on the coefficients $|a_2|$ and $|a_3|$ of functions in these new subclasses by employing the methods used earlier by Srivastava et al. [26] and by Frasin and Aouf [13].

First of all, we present the following lemma to prove our main result

Lemma 1.1 ([18]). *If $h \in \mathbb{P}$ then $| c_k | \leq 2$ for each k, where $\mathbb{P}$ is the family of all functions h analytic in E for which $Re(h(z)) > 0, h(z) = 1 + c_1 z + c_2 z^2 + c_3 z^3 + \cdots$ for $z \in E$.*

2. Coefficient Bounds for the Function Class $B_\sigma^{\tau,n}(\zeta, \eta)$

Definition 2.1. The class $B_\sigma^{\tau,n}(\zeta, \eta)$ of functions $\psi(z)$ determined by the equality (1.1) consists of those functions $\psi(z)$ that satisfy the following conditions $\psi \in \sigma$,

$$Re\left(\frac{(1-\zeta)D_\tau^n \psi(z) + \zeta D_\tau^{n+1}\psi(z)}{z}\right) > \eta, \tag{2.1}$$

where $0 \leq \eta < 1$, $\tau, n \in \mathbb{N}_0, \zeta \geq 1, z \in E$ and

$$Re\left(\frac{(1-\zeta)D_\tau^n \phi(w) + \zeta D_\tau^{n+1}\phi(w)}{w}\right) > \eta, \tag{2.2}$$

where $0 \leq \eta < 1$, $\tau, n \in \mathbb{N}_0, \zeta \geq 1, w \in E$ and the function ϕ is given by (1.4).

Remark 2.2. If we choose $n = 0, \zeta = 1$ and $\tau = 0$ in Definition 2.1 , then the class $B_\sigma^{\tau,n}(\zeta, \eta)$ reduces to the class $H_\sigma(\eta), (0 \leq \eta < 1)$ introduced and studied by Srivastava et al. [26].

Remark 2.3. If we choose $n = 0$ and $\tau = 0$ in Definition 2.1, then the class $B_\sigma^{\tau,n}(\zeta, \eta)$ reduces to the class $B_\sigma(\eta, \zeta), (0 \leq \eta < 1, \zeta \geq 1)$ introduced and studied by Frasin and Aouf [13].

Remark 2.4. If we choose $\zeta = 1$ in Definition 2.1, then the class $B_\sigma^{\tau,n}(\zeta, \eta)$ reduces to the class $M_\sigma^n(\zeta, \eta)(0 \leq \eta < 1, \zeta \geq 1)$ introduced and studied by Siregar and Darus [25].

Theorem 2.5. *Let $\psi(z)$ given by (1.1) be in the class $B_{\sigma}^{\tau,n}(\zeta,\eta)$ $0<\eta<1$ and $\tau, n\in\mathbb{N}_0, \zeta\geq 1$. Then*

$$|a_2| \leq 2\sqrt{\frac{(1-\eta)}{3^n(\tau+1)(\tau+2)(2\zeta+1)}}, \tag{2.3}$$

and

$$|a_3| \leq \frac{4(1-\eta)}{3^n(\tau+1)(\tau+2)(2\zeta+1)} + \frac{4(1-\eta)^2}{2^{2n}(\tau+1)^2(\zeta+1)^2}, \tag{2.4}$$

Proof. Considering relations (2.1) and (2.2), there exist functions ϑ and $\theta\in\mathbb{P}$ such that

$$\left(\frac{(1-\zeta)D_\tau^n\psi(z)+\zeta D_\tau^{n+1}\psi(z)}{z}\right) = \eta+(1-\eta)\vartheta(z), \quad (z\in E), \tag{2.5}$$

and

$$\left(\frac{(1-\zeta)D_\tau^n\phi(w)+\zeta D_\tau^{n+1}\phi(w)}{w}\right) = \eta+(1-\eta)\theta(w), \quad (w\in E), \tag{2.6}$$

where $\vartheta(z)$ and $\theta(w)$ in $\mathbb{P}$ in the forms are

$$\vartheta(z) = 1+\vartheta_1 z+\vartheta_2 z^2+\vartheta_3 z^3+\cdots, \tag{2.7}$$

and

$$\theta(\omega) = 1+\theta_1\omega+\theta_2\omega^2+\theta_3\omega^3+\cdots, \tag{2.8}$$

By equating the coefficients of equations (2.5) and (2.6), we get

$$2^n(\tau+1)(\zeta+1)a_2 = (1-\eta)\vartheta_1, \tag{2.9}$$

$$\frac{3^n(\tau+1)(\tau+2)(2\zeta+1)}{2}a_3 = (1-\eta)\vartheta_2, \tag{2.10}$$

$$-2^n(\tau+1)(\zeta+1)a_2 = (1-\eta)\theta_1, \tag{2.11}$$

and

$$\frac{3^n(\tau+1)(\tau+2)(2\zeta+1)}{2}(2a_2^2-a_3) = (1-\eta)\theta_2. \tag{2.12}$$

Considering (2.9) and (2.11), we have

$$\vartheta_1 = -\theta_1,$$

and

$$2^{2n+1}(\tau+1)^2(\zeta+1)^2a_2^2 = (1-\eta)^2(\vartheta_1^2+\theta_1^2). \tag{2.13}$$

Also, from (2.10) and (2.12), we find that,

$$3^n(\tau+1)(\tau+2)(2\zeta+1)a_2^2 = (1-\eta)(\vartheta_2+\theta_2).$$

Therefore, we have

$$\left|a_2^2\right| \leq \frac{(1-\eta)\left(|\vartheta_2|+|\theta_2|\right)}{3^n(\tau+1)(\tau+2)(2\zeta+1)}.$$

Applying Lemma 1.1, we get the desired result on the coefficient $|a_2|$ as propounded in (2.3). Next, to find the bound on $|a_3|$, by using relations (2.12) and (2.10), we get

$$3^n(\tau+1)(\tau+2)(2\zeta+1)\left(a_3-a_2^2\right)=(1-\eta)(\vartheta_2-\theta_2),$$

which, upon the value of a_2^2 from (2.13), we have

$$3^n(\tau+1)(\tau+2)(2\zeta+1)a_3 = \frac{3^n(2\zeta+1)(\tau+1)(\tau+2)(1-\eta)^2(\vartheta_1^2+\theta_1^2)}{2^{2n+1}(\tau+1)^2(\zeta+1)^2} + (1-\eta)(\vartheta_2-\theta_2).$$

Then, we obtain

$$a_3=\frac{(1-\eta)(\vartheta_2-\theta_2)}{3^n(2\zeta+1)(\tau+1)(\tau+2)}+\frac{(1-\eta)^2(\vartheta_1^2+\theta_1^2)}{2^{2n+1}(\tau+1)^2(\zeta+1)^2}.$$

Applying Lemma 1.1 for the coefficients ϑ_1, θ_1, ϑ_2 and θ_2, we get

$$|a_3| \leq \frac{(1-\eta)\left(|\vartheta_2|+|\theta_2|\right)}{3^n(2\zeta+1)(\tau+1)(\tau+2)}+\frac{(1-\eta)^2(|\vartheta_1|^2+|\theta_1|^2)}{2^{2n+1}(\tau+1)^2(\zeta+1)^2},$$

which is the desired estimate on the coefficient $|a_3|$ as propounded in (2.4). □

If we choose $n=0, \zeta=1$ and $\tau=0$ in Theorem 2.5, then we reduce the result by Srivastava et all [26], as follows.

Corollary 2.6 ([26]). *Let the function $\psi(z)$ given by* (1.1) *be in the class $H_\sigma(\eta)$, $(0 \leq \eta < 1)$. Then*

$$|a_2| \leq \sqrt{\frac{2(1-\eta)}{3}},$$

and

$$|a_3| \leq \frac{(1-\eta)(5-3\eta)}{3}.$$

If we choose $n=0$ and $\tau=0$ in Theorem 2.5, then we reduce the result by Frasin and Aouf [13] as follows.

Corollary 2.7 ([13]). *Let the function $\psi(z)$ given by* (1.1) *be in the class $B_\sigma(\eta)$ $\ (0 \leq \eta < 1), \zeta \geq 1$. Then*

$$|a_2| \leq \sqrt{\frac{2(1-\eta)}{2\zeta+1}},$$

and

$$|a_3| \leq \frac{4(1-\eta)^2}{(\zeta+1)^2} + \frac{2(1-\eta)}{(2\zeta+1)}.$$

If we choose $\zeta = 1$ in Theorem 2.5, then we reduce the result by Siregar and Darus [25] as follows.

Corollary 2.8 ([25])**.** *Let the function $\psi(z)$ given by* (1.1) *be in the class $M_\sigma^n(\tau,\eta)(0 \leq \eta < 1)$. Then*

$$|a_2| \leq \sqrt{\frac{4(1-\eta)}{3^{n+1}(\tau+1)(\tau+2)}},$$

and

$$|a_3| \leq \frac{(1-\eta)^2}{2^{2n}(\tau+1)^2} + \frac{4(1-\eta)}{3^{n+1}(\tau+1)(\tau+2)}.$$

3. Coefficient Bounds for the Function Class $H_\sigma^{\tau,n}(\zeta,\alpha)$

Definition 3.1. The class $H_\sigma^{\tau,n}(\zeta,\alpha)$, of the functions $\psi(z)$ determined by the equality (1.1), consists of those functions $\psi(z)$ that satisfy the following conditions $\psi \in \sum$,

$$\left|\arg\left(\frac{(1-\zeta)D_\tau^n\psi(z)+\zeta D_\tau^{n+1}\psi(z)}{z}\right)\right| < \frac{\alpha\pi}{2}, \tag{3.1}$$

where $0 < \alpha \leq 1$, $\tau, n, \in \mathbb{N}_0, \zeta \geq 1$ $z \in E$ and

$$\left|\arg\left(\frac{(1-\zeta)D_\tau^n\phi(w)+\zeta D_\tau^{n+1}\phi(w)}{w}\right)\right| < \frac{\alpha\pi}{2}, \tag{3.2}$$

where $0 < \alpha \leq 1$, $\tau, n \in \mathbb{N}_0, \zeta \geq 1$, $w \in E$ and the function ϕ is given by the equality (1.3).

Remark 3.2. If we choose $n = 0, \zeta = 1$ and $\tau = 0$ in Definition 3.1, then the class $H_\sigma^{\tau,n}(\zeta,\alpha)$ reduces to the class $H_\sigma(\alpha), (0 < \alpha \leq 1)$ introduced and studied by Srivastava et al. [26].

Remark 3.3. If we choose $n = 0$ and $\tau = 0$ in Definition 3.1, then the class $H_\sigma^{\tau,n}(\zeta,\alpha)$ reduces to the class $B_\sigma(\alpha,\zeta), (0 < \alpha \leq 1)$ introduced and studied by Frasin and Aouf [13].

Remark 3.4. If we choose $\zeta = 1$ in Definition 3.1, then the class $H_\sigma^{\tau,n}(\zeta,\alpha)$ reduces to the class $M_\sigma^n(\zeta,\alpha), (0 < \alpha \leq 1)$ introduced and studied by Siregar and Darus [25].

Theorem 3.5. *Let $\psi(z)$ given by (1.1) be in the class $H_{\sigma}^{\tau,n}(\zeta,\alpha), 0 < \alpha \leq 1$ and $\zeta \geq 1, \tau, n, \in \mathbb{N}_0$. Then*

(3.3)
$$|a_2| \leq \frac{2\alpha}{\sqrt{|3^n(\tau+1)(\tau+2)(2\zeta+1)\alpha - 2^{2n}(\tau+1)^2(\zeta+1)^2(\alpha-1)|}},$$

and

$$| a_3 |\leq \frac{4\alpha}{3^n(\tau+1)(\tau+2)(2\zeta+1)} + \frac{4\alpha^2}{2^{2n}(\tau+1)^2(\zeta+1)^2}.$$

Proof. If we write argument inequalities in (3.1) and (3.2), then we have

$$\left(\frac{(1-\zeta)D_{\tau}^{n}\psi(z) + \zeta D_{\tau}^{n+1}\psi(z)}{z}\right) = [\vartheta(z)]^{\alpha}, \quad (z \in E), \tag{3.4}$$

and

$$\left(\frac{(1-\zeta)D_{\tau}^{n}\phi(w) + \zeta D_{\tau}^{n+1}\phi(w)}{w}\right) = [\theta(w)]^{\alpha}, \quad (w \in E), \tag{3.5}$$

where $\vartheta(z)$ and $\theta(w)$ are in $\mathbb{P}$ and have the forms (2.7) and (2.8). Now, equating the coefficients in equations (3.4) and (3.5), we have

$$2^n(\tau+1)(\zeta+1)a_2 = \alpha\vartheta_1, \tag{3.6}$$

$$\frac{3^n(\tau+1)(\tau+2)(2\zeta+1)}{2}a_3 = \alpha\vartheta_2 + \frac{\alpha(\alpha-1)}{2}\vartheta_1^2, \tag{3.7}$$

$$-2^n(\tau+1)(\zeta+1)a_2 = \alpha\theta_1 \tag{3.8}$$

and

$$\frac{3^n(\tau+1)(\tau+2)(2\zeta+1)}{2}(2a_2^2 - a_3) = \alpha\theta_2 + \frac{\alpha(\alpha-1)}{2}\theta_1^2. \tag{3.9}$$

From (3.6) and (3.8), we have

$$\vartheta_1 = -\theta_1, \tag{3.10}$$

and

$$2^{2n+1}(\tau+1)^2(\zeta+1)^2a_2^2 = \alpha^2(\vartheta_1^2 + \theta_1^2). \tag{3.11}$$

Also from (3.7), (3.9) and (3.11), we obtain

$$\begin{aligned} 3^n(\tau+1)(\tau+2)(2\zeta+1)a_2^2 &= \alpha(\vartheta_2+\theta_2) + \frac{\alpha(\alpha-1)}{2}(\vartheta_1^2+\theta_1^2) \\ &= \alpha(\vartheta_2+\theta_2) + \frac{(\alpha-1)2^{2n}(\tau+1)^2(\zeta+1)^2a_2^2}{\alpha}. \end{aligned}$$

Therefore, we have

$$a_2^2 = \frac{\alpha^2(\vartheta_2+\theta_2)}{3^n(\tau+1)(\tau+2)(2\zeta+1)\alpha - 2^{2n}(\tau+1)^2(\zeta+1)^2(\alpha-1)}.$$

Applying Lemma 1.1. for the coefficients ϑ_2 and θ_2, we have

$$|a_2| \leq \frac{2\alpha}{\sqrt{|3^n(\tau+1)(\tau+2)(2\zeta+1)\alpha - 2^{2n}(\tau+1)^2(\zeta+1)^2(\alpha-1)|}}.$$

This gives the bound on $|a_2|$ as propounded in inequality (3.3). Next in order to find the bound on $|a_3|$, using relations 3.8 and 3.10, we find that

$$3^n(\tau+1)(\tau+2)(2\zeta+1)(a_3-a_2^2) = \alpha(\vartheta_2-\theta_2)+\frac{\alpha(\alpha-1)}{2}(\vartheta_1^2-\theta_1^2). \tag{3.12}$$

It follows from (3.9), (3.10) and (3.12) that

$$\begin{aligned} 3^n(\tau+1)(\tau+2)(2\zeta+1)a_3 &= \alpha(\vartheta_2-\theta_2)+3^n(\tau+1)(\tau+2)(2\zeta+1)a_2^2 \\ &= \alpha(\vartheta_2-\theta_2)+3^n(\tau+1)(\tau+2)(2\zeta+1) \\ &\quad \times \frac{\alpha^2(\vartheta_1^2+\theta_1^2)}{2^{2n+1}(\tau+1)^2(\zeta+1)^2}. \end{aligned}$$

Then, equivalently, we get

$$a_3 = \frac{\alpha(\vartheta_2-\theta_2)}{3^n(\tau+1)(\tau+2)(2\zeta+1)} + \frac{\alpha^2(\vartheta_1^2+\theta_1^2)}{2^{2n+1}(\tau+1)^2(\zeta+1)^2}.$$

Applying Lemma 1.1 once again for the coefficients ϑ_1, ϑ_2, θ_1, θ_2, we readily obtain

$$|a_3| \leq \frac{4\alpha}{3^n(\tau+1)(\tau+2)(2\zeta+1)} + \frac{4\alpha^2}{2^{2n}(\tau+1)^2(\zeta+1)^2}.$$

This completes the proof of Theorem 3.5. □

If we choose $n=0, \zeta=1$ and $\tau=0$ in Theorem 3.5, then we reduce the result given by Srivastava et al. [26] as follow

Corollary 3.6 ([26])**.** *Let the function $\psi(z)$ given by (1.1) be in the class $H_\sigma(\eta)(0 \leq \eta < 1)$. Then*

$$|a_2| \leq \alpha\sqrt{\frac{2}{\alpha+2}},$$

and

$$|a_3| \leq \frac{\alpha(3\alpha+2)}{3}.$$

If we choose $n=0$ and $\tau=0$ in Theorem 3.5, then we reduce the result given by Frasin and Aouf [13] as follows.

Corollary 3.7 ([13])**.** *Let the function $\psi(z)$ given by (1.1) be in the class $B_\sigma(\eta), (0 \leq \eta < 1)$ and $\zeta \geq 1$. Then*

$$a_2 \,| \leq \frac{2\alpha}{\sqrt{(\zeta+1)^2+\alpha(1+2\zeta-\zeta^2)}},$$

and

$$|a_3| \leq \frac{4\alpha^2}{(\zeta+1)^2} + \frac{2\alpha}{(2\zeta+1)}.$$

If we choose $\zeta = 1$ in Theorem 3.5, then we reduce the result given by Siregar and Darus [25] as follows.

Corollary 3.8 ([25])**.** *Let the function $\psi(z)$ given by* (1.1) *be in the class $M_{\sigma}^{n}(\zeta,\alpha), (0 < \alpha \leq 1)$ and $\zeta \geq 0$. Then*

$$|a_2| \leq \alpha\sqrt{\frac{4}{3^{n+1}\alpha(\tau+1)(\tau+2) - 2^{n+2}(\alpha-1)(\tau+1)^2}},$$

and

$$|a_3| \leq \frac{\alpha^2}{2^{2n}(\tau+1)^2} + \frac{4\alpha}{3^{n+1}(\tau+1)(\tau+2)}.$$

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