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A Class of Hereditarily $\ell_p(c_0)$ Banach Spaces

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Abstract. We extend the class of Banach sequence spaces constructed by Ledari, as presented in "A class of hereditarily ℓ_1 Banach spaces without Schur property" and obtain a new class of hereditarily $\ell_p(c_0)$ Banach spaces for $1 \leq p < \infty$. Some other properties of this spaces are studied.

1. Introduction

We follow the same notations and terminology as in [5]. Let Y be a subspace of X. Then we say that X contains Y hereditarily if every infinite dimensional subspace of X contains an isomorphic copy of Y. Thus, if X hereditarily contains Y, then we naturally expect to have the interior properties of X to be close to those of Y. Any exception may be of interest. For example, it is well known that ℓ_1 possesses the Schur property, while there are hereditarily ℓ_1 Banach spaces without the Schur property [1, 2, 3, 4, 7].

In this paper, we use $\ell_{w,p}$ spaces to introduce and study a new class of hereditarily $\ell_p(c_0)$ spaces. Indeed, if $p_1 > p_2 > \cdots > 1$, the subspace Z_p for $p \in [1, \infty) \cup \{0\}$ of

$$X_p = \left(\sum_{n=1}^{\infty} \oplus \ell_{w,p_n} \right)_p ,$$

is hereditarily $\ell_p(c_0)$. Other properties of these spaces are investigated. In this article, we show that under some conditions for $p \in [1, \infty) \cup \{0\}$,

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the natural operator from $\ell_{w,p}$ to Z_p is unbounded. Also the natural operator from Z_p to $\ell_{w,p}$ is unbounded.

Let $w = (w_n)$ be a fixed nonnegative real sequence. We recall the definition of $\ell_{w,p}\,(1 \le p < \infty)$, the weighted ℓ_p Banach sequence space. We know

$$\ell(w,p) = \left\{ x = (x_1, x_2, \ldots) : x_i \in \mathbb{R}, \sum_{i=1}^{\infty} w_i \left|x_i\right|^{p} < \infty \right\}.$$

For any $x \in \ell(w,p)$, define

$$\|x\|_{w,p} = \left(\sum_{i=1}^{\infty} w_i |x_i|^p \right)^{\frac{1}{p}}.$$

For any i, let $e_i = \left(\underbrace{0, \ldots, 0}_{i-1}, \left(\frac{1}{w_i}\right)^{\frac{1}{p}}, 0, \ldots \right)$. We know that $\{e_i : i \in \mathbb{N}\}$ is a normalized basis for $\ell(w,p)$. Now we go through the construction of the spaces X_p analogous of the space of Popov. Let $w = (w_n)$ be a fixed sequence, and $(\ell_{w,p_n})_{n=1}^{\infty}$ a sequence of Banch spaces as above with $\infty > p_1 > p_2 > \cdots > 1$. The direct sum of these spaces in the sense of ℓ_p is defined as the linear space

$$X_p = \left(\sum_{n=1}^{\infty} \oplus \ell_{w,p_n} \right)_p,$$

with $p \in [1,\infty)$ which is the space of all sequences $x = \left(x^1, x^2, \ldots\right)$, $x^n \in \ell_{w,p_n}$, $n = 1, 2, \ldots$, with

$$\|x\|_p = \left(\sum_{n=1}^{\infty} \|x^n\|_{w,p_n}^p \right)^{\frac{1}{p}} < \infty.$$

The direct sum of the spaces (ℓ_{w,p_n}) in the sense of c_0 is the linear space

$$X_0 = \left(\sum_{n=1}^{\infty} \oplus \ell_{w,p_n} \right)_0,$$

of all sequences $x = \left(x^1, x^2, \ldots\right)$, $x^n \in \ell_{w,p_n}$, $n = 1, 2, \ldots$, for which $\lim_n \|x^n\|_{w,p_n} = 0$ with norm

$$\|x\|_0 = \max_n \|x^n\|_{w,p_n}.$$

The construction and idea of the proof follow from [8], but the nature of these spaces is different. So for similar results, we omit the details of the proofs.

In fact these spaces are a rich class of spaces which depend on the sequences $w = (w_i)$ and (p_n) as above. Fix a sequence $w = (w_i)$ of reals which satisfies the above conditions and a sequence (p_n) of reals with $\infty > p_1 > p_2 > \cdots > 1$. Consider the sequence space X_p as above. For each $n \geq 1$, denote by $(\overline{e_{i,n}})_{i=1}^{\infty}$ the unit vector basis of ℓ_{w,p_n} and by $(e_{i,n})_{i=1}^{\infty}$ its natural copy in X_p:

$$e_{i,n} = \left(\underbrace{0, \ldots, 0}_{n-1}, \overline{e_{i,n}}, 0, \ldots \right) \in X_p.$$

Let $\delta_n > 0$ and $\Delta = (\delta_n)$ such that

$$\sum_{n=1}^{\infty} \delta_n^p = 1, \quad \text{if } p \geq 1,$$

and $\lim_n \delta_n = 0$ and $\max_n \delta_n = 1$ if $p = 0$. For each $i \geq 1$ put

$$z_i = \sum_{n=1}^{\infty} \delta_n e_{i,n}.$$

Then

$$\begin{aligned} \|z_i\|_p &= \left(\sum_{n=1}^{\infty} \|\delta_n e_{i,n}\|_{w,p_n}^p \right)^{\frac{1}{p}} \\ &= \left(\sum_{n=1}^{\infty} \delta_n^p \right)^{\frac{1}{p}} \\ &= 1, \end{aligned}$$

since $\|e_{i,n}\|_{w,p_n} = 1$ and

$$\|z_i\|_0 = \max_n \|\delta_n e_{i,n}\|_{w,p_n} = 1.$$

It is clear that for any sequence $(t_i)_{i=1}^m$ of scalars,

$$\left\| \sum_{i=1}^m t_i z_i \right\|_p^p = \sum_{n=1}^{\infty} \delta_n^p \left\| \sum_{i=1}^m t_i e_{i,n} \right\|_{w,p_n}^p, \quad \text{if} \quad 1 \leq p < \infty,$$

and

$$\left\| \sum_{i=1}^m t_i z_i \right\|_0 = \max_n \delta_n \left\| \sum_{i=1}^m t_i e_{i,n} \right\|_{w,p_n}, \quad \text{if} \quad p = 0.$$

Let Z_p be the closed linear space of $(z_i)_{i=1}^{\infty}$. For each $I \subseteq \mathbb{N}$ the projection P_I denotes the natural projection of X_p on to $[e_{i,n} : i \in \mathbb{N}, n \in I]$. Denote also $Q_n = P_{\{n,n+1,\ldots\}}$.

We recall the main properties of $\ell_{w,p}$ $(1 \leq p < \infty)$ and Z_1 spaces [4].

Theorem 1.1. *For $1 \leq p < \infty$, $\ell(w,p)$ is hereditarily isometrically isomorphic to ℓp,*

$$\left\| \sum_{i=1}^{n} t_i v_i \right\|_{w,p}^{p} = \sum_{i=1}^{n} |t_i|^p.$$

Theorem 1.2. *Let $w_i \geq 1$ for any $i \in \mathbb{N}$ and $w = (w_i)$. For $1 \leq p_{n+1} \leq p_n < \infty$, $\ell(w,p_{n+1}) \subseteq \ell(w,p_n)$. In particular $\|x\|_{w,p_n} \leq \|x\|_{w,p_{n+1}}$.*

Theorem 1.3. *Z_1 is a hereditarily ℓ_1 Banach space which fails the Schur property.*

2. The Result

Now, we show that Z_p is hereditarily $\ell_p(c_0)$ for $p \in [1,\infty) \cup \{0\}$. But first, we collect some basic facts about our spaces in the following lemmas.

Lemma 2.1. *Let E_0 be an infinite dimensional subspace of Z_p, $n,m,j \in \mathbb{N}(n > 1)$, and $\varepsilon > 0$. Then there are $\{x_i\}_{i=1}^m \subset E_0$ and $\{u_i\}_{i=1}^m \subset Z_p$ such that the k'th component of u_i is of the form*

$$u_{i,k} = \delta_k \sum_{s=j_i+1}^{j_{i+1}} a_{i,s} v_s,$$

where $j = j_1 < j_2 < \cdots < j_{m+1}$. The v_i's are obtained from the proof of Theorem 1.1, for $p = p_n$ such that

$$\sum_{s=j_i+1}^{j_{i+1}} |a_{i,s}|^{p_{n-1}} = 1, \qquad \|u_i - x_i\| < \frac{\varepsilon}{m} \|u_i\|,$$

for each $i = 1,\ldots,m$.

Proof. Put $E_1 = E_0 \cap [z_i]_{i=j+1}^{\infty}$. Since E_0 is infinite dimensional and $[z_i]_{i=j+1}^{\infty}$ has finite codimension in Z_p, E_1 is infinite dimensional as well. Put $j_1 = j$ and choose any $\bar{x}_1 \in E_1 \setminus \{0\}$ such that the k'th component of $\bar{x}_1$ has the form

$$\overline{x_{1,k}} = \delta_k \sum_{s=j_1+1}^{\infty} \overline{a_{1,s}} v_s.$$

Take $\bar{x}_1$ and use Lemma 2.2 of [8] to obtain x_1 and u_1 with above properties and continue the procedure of that lemma to construct the desired sequence. $\square$

Lemma 2.2. *Let E_0 be an infinite dimensional subspace of Z_p, $j,n \in \mathbb{N}$, and $\varepsilon > 0$. Then, there exist $x \in E_0$, $x \neq 0$, and $u \in Z_p$ such that*

(i) $\|Q_n u\| \geq (1-\varepsilon)\|u\|$,

(ii) $\|x-u\| < \varepsilon \|u\|$.

Proof. Choose $m \in \mathbb{N}$ so that

$$\frac{1}{\delta_n} m^{\frac{1}{p_{n-1}} - \frac{1}{p_n}} < \varepsilon.$$

Using Lemma 2.1, choose $\{x_i\}_{i=1}^m \subseteq E_0$ and $\{u_i\}_{i=1}^m \subseteq Z_p$ so that satisfy the claims of lemma and put

$$x = \sum_{i=1}^m x_i \textit{ and } u = \sum_{i=1}^m u_i.$$

First, we prove (ii). We know that $\|u_i\| \leq \|u\|$ for $i = 1, \ldots, m$ and

$$\begin{aligned}
\|x-u\| &\leq \sum_{i=1}^m \|x_i - u_i\| \\
&< \sum_{i=1}^m \frac{\varepsilon \|u_i\|}{m} \\
&\leq \sum_{i=1}^m \frac{\varepsilon \|u\|}{m} \\
&= \varepsilon \|u\|.
\end{aligned}$$

To prove (i), we first show that

$$\|u\| - \|Q_n u\| < m^{\frac{1}{p_{n-1}}}.$$

Anyway, $\|u\| - \|Q_n u\| \leq \left\|p_{\{1,\ldots,n-1\}} u\right\|$. Hence, by Theorem 1.1 and Theorem 1.2, for $p \geq 1$, we have

$$\begin{aligned}
(\|u\| - \|Q_n u\|)^p &\leq \sum_{k=1}^{n-1} \delta_k^p \left\| \sum_{i=1}^m \sum_{s=j_i+1}^{j_{i+1}} a_{i,s} v_s \right\|_{w,p_k}^p \\
&\leq \sum_{k=1}^{n-1} \delta_k^p \left\| \sum_{i=1}^m \sum_{s=j_i+1}^{j_{i+1}} a_{i,s} v_s \right\|_{w,p_{n-1}}^p \\
&= \sum_{k=1}^{n-1} \delta_k^p \left(\sum_{i=1}^m \sum_{s=j_i+1}^{j_{i+1}} |a_{i,s}|^{p_{n-1}} \right)^{\frac{p}{p_{n-1}}} \\
&= \sum_{k=1}^{n-1} \delta_k^p \left(\sum_{i=1}^m 1 \right)^{\frac{p}{p_{n-1}}}
\end{aligned}$$

$$= m^{\frac{p}{p_{n-1}}} \sum_{k=1}^{n-1} \delta_k^p$$
$$< m^{\frac{p}{p_{n-1}}}.$$

And for $p = 0$, we have

$$\begin{aligned}
\|u\| - \|Q_n u\| &\leq \max_{1\leq k<n} \delta_k \left\| \sum_{i=1}^{m} \sum_{s=j_i+1}^{j_{i+1}} a_{i,s} v_s \right\|_{w,p_k} \\
&\leq \max_{1\leq k<n} \delta_k \left\| \sum_{i=1}^{m} \sum_{s=j_i+1}^{j_{i+1}} a_{i,s} v_s \right\|_{w,p_{n-1}} \\
&= \max_{1\leq k<n} \delta_k \left(\sum_{i=1}^{m} \sum_{s=j_i+1}^{j_{i+1}} |a_{i,s}|^{p_{n-1}} \right)^{\frac{1}{p_{n-1}}} \\
&= \max_{1\leq k<n} \delta_k \left(\sum_{i=1}^{m} 1 \right)^{\frac{1}{p_{n-1}}} \\
&= \max_{1\leq k<n} \delta_k m^{\frac{1}{p_{n-1}}} \\
&\leq m^{\frac{1}{p_{n-1}}}.
\end{aligned}$$

On the other hand, for $p \geq 1$, we have

$$\begin{aligned}
\|u\|^p &\geq \delta_n^p \left\| \sum_{i=1}^{m} \sum_{s=j_i+1}^{j_{i+1}} a_{i,s} v_s \right\|_{w,p_n}^p \\
&= \delta_n^p \left(\sum_{i=1}^{m} \sum_{s=j_i+1}^{j_{i+1}} |a_{i,s}|^{p_n} \right)^{\frac{p}{p_n}} \\
&\geq \delta_n^p \left(\sum_{i=1}^{m} \left(\sum_{s=j_i+1}^{j_{i+1}} |a_{i,s}|^{p_{n-1}} \right)^{\frac{p_n}{p_{n-1}}} \right)^{\frac{p}{p_n}} \\
&= \delta_n^p \left(\sum_{i=1}^{m} 1 \right)^{\frac{p}{p_n}} \\
&= \delta_n^p m^{\frac{p}{p_n}}.
\end{aligned}$$

And for $p = 0$, we can write

$$\begin{aligned}
\|u\| &= \max_{k\in\mathbb{N}} \delta_k \left\| \sum_{i=1}^{m} \sum_{s=j_i+1}^{j_{i+1}} a_{i,s} v_s \right\|_{w,p_k} \\
&\geq \delta_n \left\| \sum_{i=1}^{m} \sum_{s=j_i+1}^{j_{i+1}} a_{i,s} v_s \right\|_{w,p_n} \\
&= \delta_n \left(\sum_{i=1}^{m} \sum_{s=j_i+1}^{j_{i+1}} |a_{i,s}|^{p_n} \right)^{\frac{1}{p_n}} \\
&\geq \delta_n \left(\sum_{i=1}^{m} \left(\sum_{s=j_i+1}^{j_{i+1}} |a_{i,s}|^{p_{n-1}} \right)^{\frac{p_n}{p_{n-1}}} \right)^{\frac{1}{p_n}} \\
&= \delta_n \left(\sum_{i=1}^{m} 1 \right)^{\frac{1}{p_n}} \\
&= \delta_n m^{\frac{1}{p_n}}.
\end{aligned}$$

Thus, anyway $\|u\| \geq \delta_n m^{\frac{1}{p_n}}$, and hence,

$$1 - \frac{\|Q_n u\|}{\|u\|} \leq \frac{m^{\frac{1}{p_{n-1}}}}{\delta_n m^{\frac{1}{p_n}}} = \frac{1}{\delta_n} m^{\frac{1}{p_{n-1}} - \frac{1}{p_n}} < \varepsilon,$$

and $\|Q_n u\| \geq (1-\varepsilon)\|u\|$. □

The following theorem is the main result of this paper:

Theorem 2.3. (i) *The Banach space Z_p is hereditarily ℓ_p for $1 \leq p < \infty$.*
(ii) *The space Z_0 is hereditarily c_0.*

Proof. For the proof of our main results we need the following Lemma an Theorem from Popov [8] (see Lemma 2.4 and Theorem 2.5). □

Lemma 2.4. *Suppose $\varepsilon > 0$ and ε_s for $s \in \mathbb{N}$ are such that*

(i) $2\varepsilon_s \leq \varepsilon$ *if* $p = 1$,
(ii) $\sum_{s=1}^{\infty} (2\varepsilon_s)^q \leq \varepsilon^q$ *if* $1 \leq p < \infty$ *where* $\frac{1}{p} + \frac{1}{q} = 1$,
(iii) $\sum_{s=1}^{\infty} (2\varepsilon_s) \leq \varepsilon$ *if* $p = 0$.

If for given vectors $\{u_s\}_{s=1}^{\infty} \subset S(Z_p)$, *where* $Z_p = Z_p(P)$, *there is a sequence of integers* $1 \leq n_1 < n_2 < \cdots$ *such that, for each* $s \in \mathbb{N}$, *one has*

(i) $\|u_s - Q_{n_s} u_s\| \leq \varepsilon_s$,
(ii) $\|Q_{n_{s+1}} u_s\| \leq \varepsilon_s$,

then $\{u_s\}_{s=1}^{\infty}$ *is* $(1+\varepsilon)(1-3\varepsilon)^{-1}$*-equivalent to the unit vector basis of* ℓ_p *(respectively,* c_0*).*

Theorem 2.5. *The Banach space* $Z_p = Z_p(P)$ *is hereditarily* ℓ_p *if* $1 \leq p < \infty$ *and is hereditarily* c_0 *if* $p = 0$.

The proos of Lemma 2.4 and Theorem 2.5 are based on the definition of Q_i and the norm on Z_p. In fact by the conditions of this lemma and for any sequence $(a_s)_{s=1}^m$ of scalars, it follows that

$$(1-3\varepsilon)\left(\sum_{s=1}^{m}|a_s|^p\right)^{\frac{1}{p}} \leq \left\|\sum_{s=1}^{m} a_s u_s\right\| \leq (1+\varepsilon)\left(\sum_{s=1}^{m}|a_s|^p\right)^{\frac{1}{p}},$$

for $1 \leq p < \infty$, and

$$(1-3\varepsilon)\max_{1\leq s<m}|a_s| \leq \left\|\sum_{s=1}^{m} a_s u_s\right\| \leq (1+\varepsilon)\max_{1\leq s<m}|a_s|,$$

for $p = 0$. Then by using the stability properties of the bases ([5], p. 5) and Lemma 2.2, we conclude the proofs.

Definition 2.6. Let X be an arbitrary Banach space. Then

a) X has the nowhere Schur property if X contains no infinite dimensional closed subspace with the Schur property.
b) X has the nowhere dual Schur property if X contains no infinite dimensional closed subspace such that its dual has the Schur property.

Definition 2.7. A Banach space X has the Schur property if every weak convergent sequence is norm convergent.

Theorem 1.3 in this paper and Theorem 1.3 of [6] have the following consequence.

Theorem 2.8. Z_1 *possesses the nowhere dual Schur property.*

3. Operators

Definition 3.1. Let X and Y be any of the spaces $\ell_{w,p}\,(1 \leq p < \infty)$, or $Z_p\,(1 \leq p < \infty)$ with their natural bases $(x_n)_{n=1}^{\infty}$ and $(y_n)_{n=1}^{\infty}$, respectively. The formal (possibly unbounded) operator $T : X \longrightarrow Y$ which extends by linearity and continuity the equality $Tx_n = y_n$ is called the natural operator from X to Y.

Theorem 3.2. *Let $p \in [1,\infty)$ and $p_1 > p_2 > \cdots > 1$.*

(i) *If $\inf_n p_n < p$, then the natural operator from $\ell_{w,p}$ to Z_p is unbounded.*

(ii) *If $\inf_n p_n \geq p$, then the natural operator from Z_p to $\ell_{w,p}$ is unbounded.*

Proof. For constant scalars $a_1 = a_2 = \cdots = a_m = 1$, we have

$$\left\|\sum_{i=1}^{m} z_i\right\|_p^p = \sum_{n=1}^{\infty} \delta_n^p \left\|\sum_{i=1}^{m} e_{i,n}\right\|_{w,p_n}^p = \sum_{n=1}^{\infty} \delta_n^p m^{\frac{p}{p_n}},$$

if $1 \leq p < \infty$.

On the other hand,

$$\left\|\sum_{i=1}^{m} e_i\right\|_{w,p}^p = m,$$

if $1 \leq p < \infty$.

Therefore, for $1 \leq p < \infty$, we have

$$\|T\|^p \geq \frac{\left\|\sum\limits_{i=1}^{m} Te_i\right\|_p^p}{\left\|\sum\limits_{i=1}^{m} e_i\right\|_{w,p}^p} = \frac{\left\|\sum\limits_{i=1}^{m} z_i\right\|_p^p}{\left\|\sum\limits_{i=1}^{m} e_i\right\|_{w,p}^p} = \sum_{n=1}^{\infty} \delta_n^p m^{\frac{p}{p_n}-1}.$$

If $\inf_n p_n < p$, then there exists n_0 such that $p_{n_0} < p$, and hence,

$$\|T\|^p \geq \sum_{n=1}^{\infty} \delta_n^p m^{\frac{p}{p_n}-1} \geq \delta_{n_0}^p m^{\frac{p}{p_{n_0}}-1} \to \infty,$$

as $m \to \infty$.

Now assume that $\inf_n p_n \geq p$. In this case, we have $\frac{p}{p_n} - 1 < 0$ for each n. Given $\varepsilon > 0$, we choose n_o so that

$$\sum_{n=n_0}^{\infty} \delta_n^p < \frac{\varepsilon}{2}.$$

Then we choose m_0 such that

$$\left(\max_{1\leq n\leq n_0} \delta_n\right)^p m^{\frac{p}{p_{n_0}}-1} < \frac{\varepsilon}{2n_0},$$

for $m \geq m_0$. So, for such m, we have

$$\|T\|^p \geq \frac{1}{\sum\limits_{n=1}^{\infty} \delta_n^p m^{\frac{p}{p_n}-1}}$$

$$
\begin{aligned}
&= \frac{1}{\sum_{n=1}^{n_0} \delta_n^p m^{\frac{p}{p_n}-1} + \sum_{n=n_0+1}^{\infty} \delta_n^p m^{\frac{p}{p_n}-1}} \\
&\geq \frac{1}{\sum_{n=1}^{n_0} \left(\max_{1 \leq n < n_0} \delta_n \right)^p m^{\frac{p}{p_n}-1} + \sum_{n=n_0+1}^{\infty} \delta_n^p m^{\frac{p}{p_n}-1}} \\
&\geq \frac{1}{\frac{\varepsilon}{2} + \frac{\varepsilon}{2}} \to \infty,
\end{aligned}
$$

as $m \to \infty$. □

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