Sahand Communications in Mathematical Analysis (SCMA) Vol. 14 No. 1 (2019), 173-183
http://scma.maragheh.ac.ir
DOI: 10.22130/scma.2018.74414.315

A New Model for the Secondary Goal in DEA

Harun Kinaci[1] and Vadoud Najjari[2*]

Abstract. The purpose of the current paper is to propose a new model for the secondary goal in DEA by introducing secondary objective function. The proposed new model minimizes the average of the absolute deviations of data points from their median. Similar problem is studied in a related model by Liang et al. (2008), which minimizes the average of the absolute deviations of data points from their mean. By using two well known data sets, which are also used by Liang et al.(2008), and Greene (1990) we compare the results of the proposed new model and several other models.

1. Introduction

Data envelopment analysis (DEA) is a nonparametric method for evaluating the relative efficiency of decision-making units (DMUs) on the basis of multiple inputs and outputs. In recent years DEA has had important role in application of many branches such as energy (Alp and Sözen, 2011; Sözen et al., 2010), banking (Mercan et al., 2003), sport (Alp, 2006; Anderson and Sharp, 1997) etc.

Cross-efficiency in DEA is a method that could be utilized to identify good overall performers and effectively rank DMUs (Sexton et al. 1986). Cross-efficiency methods evaluate the performance of a DMU with respect to the optimal input and output weights of other DMUs. Anderson et al. (2002) summarize the advantages of cross evaluation as follows:

(i) it provides a unique ordering of the DMUs;

2010 *Mathematics Subject Classification.* 62H05, 62H20.
Key words and phrases. Data envelopment analysis, Cross-efficiency, linear programming, Secondary goal.
Received: 30 October 2017, Accepted: 09 April 2018.
* Corresponding author.

(ii) it eliminates unrealistic weight schemes without requiring the elicitation of weight restrictions from application area experts.

Cross-efficiency evaluation has long been suggested as an alternative method for ranking DMUs in DEA. As it discussed in Doyle and Green (1994), usefulness of cross efficiency possibly reduces as the DEA optimal weights are not unique. Sexton et al. (1986) and Doyle and Green (1994) propose the use of secondary goals to deal with the non-uniqueness issue. They investigate two models which the first one identifies optimal weights that maximize not only the efficiency of a particular DMU under evaluation but also the average efficiency of other DMUs. The second model seeks weights that minimize the average efficiency of those other units.

Liang et al. (2008) have extend the model of Doyle and Green (1994) by introducing various secondary objective functions, which are seen by models (2.1), (2.2) and (2.3) in Section 3. It is observed that each secondary objective function represents an efficiency evaluation criterion and these new models obtain a better picture of cross-efficiency stability with respect to multiple DEA weights.

There is another related study by Wang and Chin (2010b) which proposes a neutral DEA model for cross-efficiency evaluation. The neutral DEA model for each DMU determines one set of input and output weights without being aggressive or benevolent to the other DMUs. In this way the computed cross-efficiencies are more neutral, neither aggressive nor benevolent. Numerical examples illustrate applications of the neutral DEA model and the cross-weight evaluation in DEA ranking. By another study Wang and Chin (2010a) propose some new alternative models for DEA cross-efficiency evaluation to provide more methodological options for DMUs. These alternative models are illustrated with two numerical examples and results are believed to be more realistic than those obtained by the alternative secondary goals suggested by Liang et al. (2008).

Jahanshahloo et. al. (2011) are proposed symmetric weights assignment technique (SWAT). Finaly, Soltanifar and Shahghobadi (2013) extend of some models about secondary goal (Dimitrov and Sutton (2010), Liang et. al. (2008), Liu and Peng (2008) and Obata and Ishii (2003)) and they propose a new model based on a voting model.

There is other study about secondary goal by Tohidi et al. (2013). Their model includes secondary goal for decreasing the number of zero weights and they use symmetric weights for computing the element of cross efficiency matrix. So they focus on generates more acceptable result in the ranking process of DMUs. Other study by Wu et al. (2016) incorporates a target identification model to get reachable targets for all DMUs. Besides their proposed models, they considered the DMUs willingness to get close their desirable cross efficiency target and to avoid their undesirable cross-efficiency targets simultaneously. Lin et al. (2016) proposed an iterative method for determining weights in cross efficiency evaluation to reduce the number of zero weights and ensure the unique weight set for inputs and outputs data. In the cross evaluation it is suggested to read study by Örkcü and Bal (2011) which proposes goal programming models for second stage of the cross evaluation.

The main endeavor in the current paper is to propose a new model by introducing secondary objective function, which is based on median. Our main motivation is the related study by Liang et al. (2008). The proposed third model (model (2.3)) of their study, minimizes the average of the absolute deviations of data points from their mean. As median is a more robust statistic than mean, we suggest to use median instead of mean in the proposed model by Liang et al. (2008). By using two well known data sets, which are also used by Liang et al.(2008), and Greene (1990) we compare the results of the proposed new model and several other models.

The structure of this study is as follows: The next section discusses some related preliminaries. In Section 3, we introduce a new approach for the secondary goal in DEA. Section 4 illustrates the new method by two data set. Finally, conclusions are given in Section 5.

2. Preliminaries

In this section we aim to review three models which were proposed by Liang et al. (2008) to extend the model of Doyle and Green (1994). Suppose we have a set of n DMUs, and each DMUj produces s different outputs from m different inputs. The ith input and rth output of DMUj $(j = 1, 2, \ldots, n)$ are denoted by x_{ij} $(i = 1, 2, \ldots, m)$ and y_{rj} $(r = 1, 2, \ldots, s)$, respectively. Let the (CCR) inefficiency of DMUd be α_d^*. Firstly we consider the following model, where the secondary goal is to minimize the sum of "inefficiencies".

I. Minimizing total deviation from the ideal point

$$\min \sum_{j}^{n} \alpha_j' \tag{2.1}$$

subject to

$$\sum_{r=1}^{s} u_r^d y_{rj} - \sum_{i=1}^{m} v_i^d x_{ij} + \alpha_j' = 0, \quad j = 1, \ldots, n$$

$$\sum_{i=1}^{m} v_i^d x_{id} = 1,$$
$$\sum_{r=1}^{s} u_r^d y_{rd} = 1 - \alpha_d^*,$$
$$v_i^d, u_r^d, \alpha_j^{'} \geq 0, \quad \forall i, j, r.$$

This model by minimizing the sum of the d-inefficiencies $\alpha_j^{'}$, ($j = 1, 2, \ldots, n$) is intuitively appealing, and in the spirit of all DMUs attempting to maximize their respective performances.

II. Minimizing the maximum d-efficiency score

This model minimizes the maximal d-inefficiency $\alpha_j^{'}$ which is related to maximizing the minimal efficiency among n efficiencies as in Troutt (1997).

$$\begin{aligned} &\min \theta \\ &\text{subject to} \\ &\sum_{r=1}^{s} u_r^d y_{rj} - \sum_{i=1}^{m} v_i^d x_{ij} + \alpha_j^{'} = 0, \quad j = 1, \ldots, n \\ &\sum_{i=1}^{m} v_i^d x_{id} = 1, \\ &\sum_{r=1}^{s} u_r^d y_{rd} = 1 - \alpha_d^*, \\ &v_i^d, u_r^d, \alpha_j^{'} \geq 0, \quad \forall i, j, r. \\ &\theta \geq \alpha_j^{'}, \quad j = 1, \ldots, n \end{aligned} \tag{2.2}$$

where $\theta = \max \alpha_j^{'}$, for all $j = 1, \ldots, n$.

III. Minimizing the mean absolute deviation

$$\begin{aligned} &\min \frac{1}{n} \sum_{j=1}^{n} |\alpha_j^{'} - \bar{\alpha}^{'}| \\ &\text{subject to} \\ &\sum_{r=1}^{s} u_r^d y_{rj} - \sum_{i=1}^{m} v_i^d x_{ij} + \alpha_j^{'} = 0, \quad j = 1, \ldots, n \\ &\sum_{i=1}^{m} v_i^d x_{id} = 1, \end{aligned} \tag{2.3}$$

$$\sum_{r=1}^{s} u_r^d y_{rd} = 1 - \alpha_d^*$$
$$u_r^d, v_i^d, \alpha_j^{'} \geq 0, \quad \forall i, j, r,$$

where

$$\bar{\alpha}^{'} = \frac{1}{n}\sum_{j=1}^{n} \alpha_j^{'}.$$

The objective function has to be linear, so it can be rewritten as follows:

$$\min \frac{1}{n}\sum_{j=1}^{n}(a_j^{'} + b_j^{'}) \tag{2.4}$$

subject to

$$\sum_{r=1}^{s} u_r^d y_{rj} - \sum_{i=1}^{m} v_i^d x_{ij} + \alpha_j^{'} = 0, \quad j = 1, \ldots, n,$$
$$\sum_{i=1}^{m} v_i^d x_{id} = 1,$$
$$\sum_{r=1}^{s} u_r^d y_{rd} = 1 - \alpha_d^*$$
$$a_j^{'} - b_j^{'} = \alpha_j^{'} - \frac{1}{n}\sum_{j=1}^{n} \alpha_j^{'}, \quad j = 1, \ldots, n,$$
$$u_r^d, v_i^d, \alpha_j^{'}, a_j^{'}, b_j^{'} \geq 0, \quad \forall i, j, r,$$

for details see Liang et al. (2008).

3. A New Model in the Secondary Goal

The purpose of the current section is to extend the model of Doyle and Green (1994) by introducing a new secondary objective function. As median is a more robust statistic than mean, we use average of the absolute deviation from median, instead of the absolute deviation of mean which is used by Liang et al. (2008). The new model is proposed as below:

$$\min \frac{1}{n}\sum_{j=1}^{n} |md_\alpha - \alpha_j^{'}| \tag{3.1}$$

subject to

$$\sum_{r=1}^{s} u_r^d y_{rj} - \sum_{i=1}^{m} v_i^d x_{ij} + \alpha_j^{'} = 0, \quad j = 1, \ldots, n,$$

$$\sum_{i=1}^{m} v_i^d y_{id} = 1,$$
$$\sum_{r=1}^{s} u_r^d y_{rd} = 1 - \alpha_d^*$$
$$u_r^d, v_i^d, \alpha_j^{'} \geq 0, \quad \forall i, j, r,$$

where, $md_\alpha = median(\alpha_j^{'}, j = 1, \ldots, n)$ and other variables are defined same as in the model (2.4). Let $w = \frac{n+1}{2}$ while n is even, and $w = \frac{n}{2}$ if n is odd, also α_j^{srt}, $(j = 1, \ldots, n)$ be ascending sorted deviations $(\alpha_j^{'})$. Then the model (3.1) can be expressed equivalently in the following form:

$$\min \frac{1}{n} \left(\sum_{j=1}^{w} \left(md_\alpha - \alpha_j^{srt} \right) + \sum_{j=w+1}^{n} \left(\alpha_j^{srt} - md_\alpha \right) \right) \tag{3.2}$$

subject to

$$\sum_{r=1}^{s} u_r^d y_{rj} - \sum_{i=1}^{m} v_i^d x_{ij} + \alpha_j^{'} = 0, \quad j = 1, \ldots, n$$
$$\sum_{i=1}^{m} v_i^d y_{id} = 1,$$
$$\sum_{r=1}^{s} u_r^d y_{rd} = 1 - \alpha_d^*$$
$$u_r^d, v_i^d, \alpha_j^{'} \geq 0 \forall i, j, r,$$

hereby objective function in the model (3.2) is linear. Therefore, minimizing the objective function decreases the efficiency difference among DMUs, which to some extent demonstrates an equalitarian principle. This criterion would apply to the same settings as the model (2.3), and it tries to make all DMUs to being equally efficient.

4. Application

In this section two well known data sets, which are also used by Liang et al.(2008) and Greene (1990), are used to compare results of the proposed new model (3.2) by several other models. Table 1, consists of six nursing homes, where the input and output variables are defined as follows:
StHr(x1): staff hours per day, including nurses, physicians, etc.
Supp(x2): supplies per day, measured in thousands of dollars.
MCPD(y1): total medicare-plus medicaid-reimbursed patient days.

PPPD(y2): total privately paid patient days. These data sets firstly discussed by Sexton et al. (1986).

Table 2 shows cross efficiency scores of the discussed models on the data set in Table 1 and Table 3 provides ranking of the nursing homes in the discussed models. By Table 2 and Table 3 it is seen that the proposed new model in selecting efficient DMUs and ranking of the DMUs has a similar result by the Models (2.1, 2.2, 2.3). In Table 3 results of the new model is same as results of the Models (2.1, 2.3). But there are differences in the result of DMUd and DMUf by the new model and Model (2.2).

Table 1. Nursing homes

DMU	Inputs		Outputs	
	StHr(x_1)	Supp(x_2)	MCPD(y_1)	PPPD(y_2)
a	1,5	0,2	1,4	0,35
b	4	0,7	1,4	2,1
c	3,2	1,2	4,2	1,05
d	5,2	2	2,8	4,2
e	3,5	1,2	1,9	2,5
f	3,2	0,7	1,4	1,5

Table 2. Cross efficiency scores on the nursing homes

DMU	CCR	Model(I)	Model(II)	Model(III)	New Model
a	1	1	1	1	1
b	1	0,9547	0,9616	0,9547	0,9773
c	1	0,8864	0,8759	0,8864	0,8580
d	1	1	1	1	1
e	0,9775	0,9742	0,9748	0,9742	0,9758
f	0,8675	0,8465	0,8499	0,8465	0,8570

Table 3. Ranking of the nursing homes by the mentioned models

DMU	CCR	Model(I)	Model(II)	Model(III)	New Model
a	5	5	5	5	5
b	3	1	1	1	1
c	6	6	6	6	6
d	2	2	3	2	2
e	4	4	4	4	4
f	1	3	2	3	3

Table 4, consists of 13 open coastal Chinese cities and 5 Chinese special economic zones in 1989. Two inputs and three outputs were chosen to

TABLE 4. Chines cities

DMU	Inputs		Outputs		
	x1	x2	y1	y2	y3
Dalian	2874,8	16738	160,89	80800	5092
Qinhuangdao	946,3	691	21,14	18172	6563
Tianjin	6854	43024	375,25	144530	2437
Qingdao	2305,1	10815	176,68	70318	3145
Yantai	1010,3	2099	102,12	55419	1225
Weihai	282,3	757	59,17	27422	246
Shanghai	17478,6	116,9	1029,09	351390	14604
Lianyungang	661,8	2024	30,07	23550	1126
Ningbo	1544,2	3218	160,58	59406	2230
Wenzhou	428,4	574	53,69	47504	430
Guangzhou	6228,1	29842	258,09	151356	4649
Zhanjiang	697,7	3394	38,02	45336	1555
Beihai	106,4	367	7,07	8236	121
Shenzhen	4539,3	45809	116,46	56135	956
Zhuhai	957,8	16947	29,2	17554	231
Shantou	1209,2	15741	65,36	62341	618
Xiamen	972,4	23822	54,52	25203	513
Hainan	2192	10943	25,24	40267	895

characterize the technology of those cities/zones as follows:
(x1): Investment in fixed assets by stateowned enterprises (10,000 RMB), where RMB is the Chinese monetary unit.
(x2): Foreign funds actually used (10,000 dollar);
Output 1 (y1): Total industrial output value (based on fixed prices of 1980) (10,000 RMB);
Output 2 (y2): Total value of retail sales (10,000 RMB);
Output 3 (y3): Handling capacity of coastal ports (10,000 tones).
These data set for the first time discussed by Zhu (1998).

Table 5 provides cross efficiency scores and ranking of the Chines cities by the discussed models. In Table 5 it is seen that results of Model (2.1), Model (2.2) and Model (2.3) on Chines cities data set are the same. It means that in these models the cross-efficiency scores are unique or stable, for details see Liang et al. (2008). Moreover, there are small differences between results of the new model and Models (2.1), (2.2) and (2.3). Table 6 consists of correlations between ranking of the Chines cities by the given models. It is seen that there is 99.79% compatibility between results of the Model(I,II,III) and the new model.

5. CONCLUSION

This study proposed a new model by introducing secondary objective function, which is based on median. As median is a more robust statistic

TABLE 5. Cross efficiency scores and ranking of the Chines cities by the discussed models

DMU	CCR	Ranking	Model(I,II,III)	Ranking	New Model	Ranking
Dalian	0,4691	11	0.4461	10	0,4320	10
Qinhuangdao	1	1	1	1	1	1
Tianjin	0,2779	15	0.2436	15	0,2331	15
Qingdao	0,5022	8	0.4522	9	0,4361	9
Yantai	0,6311	7	0.605	6	0,6119	6
Weihai	1	1	0.9722	2	0,9722	2
Shanghai	1	12	0.3047	12	0,3820	12
Lianyungang	0,4959	9	0.4545	8	0,4566	8
Ningbo	0,6545	6	0.5693	7	0,5626	7
Wenzhou	1	1	0.887	3	0,9383	3
Guangzhou	0,3010	14	0.2788	14	0,2756	13
Zhanjiang	0,7866	4	0.6576	4	0,6698	4
Beihai	0,7514	5	0.6086	5	0,6351	5
Shenzhen	0,1382	18	0.1288	18	0,1241	18
Zhuhai	0,1867	17	0.1665	16	0,1623	16
Shantou	0,4704	10	0.3877	11	0,3946	11
Xiamen	0,3059	13	0.2819	13	0,2672	14
Hainan	0,1953	16	0.1513	17	0,1575	17

TABLE 6. Spearman rank correlation between ranking of the Chines cities by the mentioned models

Models	CCR	Model(I,II,III)	New Model
CCR	1	0.9887	0.9867
Model(I,II,III)	0.9887	1	0.9979
New Model	0.9867	0.9979	1

than mean, hence suggested to use median instead of mean in the model proposed by Liang et al. (2008). By using two well known data sets, which are also used by Liang et al. (2008) and Greene (1990) results of the new model compared by results of several other models. Calculations show that there are compatibility results by the new model and the Liang et al.'s model.

Acknowledgment. The authors are grateful to the anonymous referees and editors whose deep comments helped to improve the original version of this contribution.

REFERENCES

1. İ. Alp, *Performance of evaluation of Goalkeepers of World Cup*, G.U. Journal of Science, 19 (2006), pp. 119-125.

2. İ. Alp and A. Sözen, *Efficiency Assessment of Turkey's Carbonization Index, Energy Sources*, Part A: Recovery, Utilization, and Environmental Effects, 33 (2011), pp. 1678-1691.
3. T.R. Anderson, K.B. Hollingsworth, and L.B. Inman, *The fixed weighting nature of a cross-evaluation model*, J. of Prod. Anal., 18 (2002), pp. 249-255.
4. T. Anderson and G. Sharp, *A New Measure of Baseball Batters Using DEA*, Ann. of Oper. Res., 73 (1997), pp. 141-155.
5. J. Doyle and R. Green, *Efficiency and cross efficiency in DEA: Derivations, meanings and the uses*, J. of the Oper. Res. Soc., 45 (1994), pp. 567-578.
6. W. Greene, *A Gamma-distributed Stochastic Frontier Model*, Journal of Econometrics, J. of Econ., 46 (1990), pp. 141-163.
7. L. Liang, J. Wu, W.D. Cook, and J.Zhu, *Alternative secondary goals in DEA cross-efficiency evaluation*, Int. J. Prod. Econ., 113 (2008), pp. 1025-1030.
8. R. Lin, Z. Chen, and W. Xiong, *An Iterative Method for Determining Weights in Cross Efficiency Evaluation*, Comp. and Ind. Engineering, 101 (2016), pp. 91-102.
9. M. Mercan, A. Reisman, R. Yoalan, and A.B. Emel, *The Effect of Scale and Mode of Ownership on the Financial Performance of the Turkish Banking Sector: Results of A DEA Based Analysis*, Soc.-Econ. Plan. Sciences, 37 (2003), pp. 185-202.
10. H.H. Örkcü and H. Bal, *Goal Programming Approaches for Data Envelopment Analysis Cross-Efficiency Evaluation*, Applied Math. and Comp., 218 (2011), pp. 346-356.
11. G. Tohidi, M. Khodadadi, and E. Rostamian, *Improving the Selection Symmetric Weights as a Secondary Goal in DEA Cross-Efficiency Evaluation*, Int. J. of Math. Modelling and Computations, 3 (2013), pp. 149-155.
12. M.D. Troutt, *Derivation of the maximin efficiency ratio model from the maximum decisional efficiency principle*, Ann. of Oper. Research, 73 (1997), pp. 323-338.
13. T.R. Sexton, *The methodology of data envelopment analysis*, In Silkman, R.H. (Ed.) Measuring efficiency: An assessment of Data Envelopment Analysis, San Francisco: Jossey-Bass, (1986), pp. 7-29.
14. A. Sözen and İ. Alp, and A. Özdemir, *Assessment Of Operational And Environmental Performance Of The Thermal Power Plants In Turkey By Using Data Envelopment Analysis*, E. Policy (Soc. A), 38 (2010), pp. 6194-6203.
15. Y.M. Wang and K.S. Chin, *Some alternative models for DEA cross-efficiency evaluation*, Int. J. of Prod. Econ., 128 (2010a), pp. 332-338.
16. Y.M. Wang and K.S. Chin, *A neutral DEA model for cross-efficiency evaluation and its extension*, Expert Sys. with App., 37 (2010b), pp. 3666-3675.
17. J. Wu, J. Chu, J. Sun, Q. Z, and L. Liang, *Extended Secondary Goal Models for Weights Selection in DEA Cross-Efficiency Evaluation*, Comp. and Ind. Engineering, 93 (2016), pp. 143-151.
18. J. Zhu, *Data envelopment analysis vs. principal component analysis: An illustrative study of economic performance of Chinese cities*, E. J. of Oper. Res., 111 (1998), pp. 50-61.

[1] Erciyes University, Faculty of Economics and Administrative Sciences, Business Department, Kayseri, Turkey.
E-mail address: hkinaci@erciyes.edu.tr

[2] Young Researchers and Elite Club, Maragheh branch, Islamic Azad University, Maragheh, Iran.
E-mail address: vnajjari@iau-maragheh.ac.ir; vnajjari@gazi.edu.tr