Sahand Communications in Mathematical Analysis (SCMA) Vol. 16 No. 1 (2019), 47-56
http://scma.maragheh.ac.ir
DOI: 10.22130/scma.2018.77684.355

p-adic Dual Shearlet Frames

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Abstract. We introduced the continuous and discrete p-adic shearlet systems. We restrict ourselves to a brief description of the p-adic theory and shearlets in real case. Using the group G_p consist of all p-adic numbers that all of its elements have a square root, we defined the continuous p-adic shearlet system associated with $L^2\left(Q_p^2\right)$. The discrete p-adic shearlet frames for $L^2\left(Q_p^2\right)$ is discussed. Also we prove that the frame operator S associated with the group G_p of all with the shearlet frame $SH\left(\psi;\Lambda\right)$ is a Fourier multiplier with a function in terms of $\widehat{\psi}$. For a measurable subset $H \subset Q_p^2$, we considered a subspace $L^2\left(H\right)^{\vee}$ of $L^2\left(Q_p^2\right)$. Finally we give a necessary condition for two functions in $L^2\left(Q_p^2\right)$ to generate a p-adic dual shearlet tight frame via admissibility.

1. Introduction

D. Labate, G. Kutyniok and others developed the concept of shearlets [2, 5, 6]. It is a well known fact that the shearlet system has better efficiency than two dimensional wavelets. We introduced the p-adic shearlet systems on $L^2\left(Q_P^2\right)$ and characterized some conditions for a discrete p-adic shearlet system to be a frame. Finally, we obtained a necessary condition for a function in $L^2\left(Q_p^2\right)$ with its dual to generate a dual p-adic shearlet tight frame.

The field of p-adic numbers were introduced by K. Hensel in 1897. We restrict ourselves to a brief description of the p-adic theory and for details we refer the readers to [7]. For a prime p, the p-adic norm $|\cdot|_p$

2010 *Mathematics Subject Classification.* 42C40, 43A70, 43A99.
Key words and phrases. p-adic numbers, Dual frame, p-adic shearlet system, p-adic dual tight frame.
Received: 19 December 2017, Accepted: 24 January 2018.
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satisfies the strong triangle inequality

$$|x+y|_p \leq \max\left(|x|_p, |y|_p\right),$$

and is defined as follows, $|0|_p$=0; and if $x \neq 0$ is a rational number of the form $x = p^\gamma \frac{m}{n}$, where $\gamma = \gamma(x) \in \mathbb{Z}$ and the integers m, n are not divisible by p, then $|x|_p = p^{-\gamma}$. The completion of Q with respect to this norm is called the field of p-adic numbers, denoted by Q_p. Any p-adic number $x \neq 0$ can be uniquely represented in the canonical form

$$x = p^\gamma \sum_{j=0}^{\infty} x_j p^j,$$

where $\gamma = \gamma(x) \in \mathbb{Z}$, $x_j \in \{0, 1, \ldots, p-1\}$, $x_0 \neq 0$ and one has

$$|x|_p = p^{-\gamma}.$$

The fractional part of a number $x \in Q_p$ is defined by

$$\{x\}_p = p^\gamma \sum_{j=0}^{-\gamma-1} x_j p^j.$$

The above definition is equivalent with

$$\{x\}_p = \begin{cases} 0, & \text{if } \gamma(x) \geq 0 \text{ or } x = 0, \\ p^\gamma\left(x_0 + x_1 p + \cdots + x_{|\gamma|-1} p^{|\gamma|-1}\right), & \text{if } \gamma(x) < 0. \end{cases}$$

The disc of radius p^N with the center at a point $a \in Q_p$, $N \in \mathbb{Z}$ is denoted by $B_N(a)$ and its boundary is denoted by $S_N(a)$, i.e.

$$B_N(a) = \left\{x \in Q_p : |x-a|_p \leq p^N\right\},$$
$$S_N(a) = \left\{x \in Q_p : |x-a|_p = p^N\right\}.$$

It is a well known fact that the disc $B_N(a)$ and the circle $S_N(a)$ are both open and close sets in Q_p and that the space Q_p is locally-compact. Note that the disc $B_0(0)$ (or B_0) coincides with the ring of p-adic integers $\mathbb{Z}_p = \left\{x \in Q_p : \{x\}_p = 0\right\}$. In [7], it is proved that every disc $B_N(a)$ is compact, so $\mathbb{Z}_p$ is compact. Also we set $I_p = \left\{x \in Q_p : \{x\}_p = x\right\}$. It is known that $Q_p = \bigcup_{x \in I_p} B_0(x)$ [7], which implies I_p is a discrete subset of Q_p.

The space $Q_p^n = Q_p \times Q_p \times \cdots \times Q_p$ consists of points $x = (x_1, \ldots, x_n)$, $x_j \in Q_p$. The norm on Q_p^n is defined as follows:

$$|x|_p = \max_{1 \leq j \leq n} |x_j|_p.$$

If $a = (a_1, \ldots, a_n)$, then $B_N(a) = B_N(a_1) \times \cdots \times B_N(a_n)$. A complex-valued function f defined on Q_p is called locally-constant if for any $x \in Q_p$ there exists an integer $l(x) \in \mathbb{Z}$ such that $f(x+y) = f(x)$, $y \in B_{l(x)}(0)$. We denote the linear space of locally-constant compactly supported functions (so-called test functions) by $D(Q_p)$, (or D). The Fourier transform of $\varphi \in D\left(Q_p^d\right)$ is defined as

$$F(\varphi)(\xi) = \widehat{\varphi}(\xi) = \int_{Q_p^d} \chi_p(\xi \cdot x)\, \varphi(x)\, d^d x, \quad \xi \in Q_p^d,$$

where "$\cdot$" denotes the inner product in Q_p^d and $\chi_p(\xi \cdot x) = \chi_p(\xi_1 x_1) \cdots \chi_p(\xi_d x_d)$ and $\chi_p(\xi_j x_j) = e^{2\pi i \{\xi_j x_j\}_p}$, for $j = 1, \ldots, d$. Since Q_p is a locally compact group, it possesses the Haar measure dx such that $\int_{\mathbb{Z}_p} dx = 1$. This Haar measure satisfies:

$$d(x+a) = dx \text{ and } d(ax) = |a|_p\, dx \text{ for } a \in Q_p \setminus \{0\}.$$

Details can be found in [7].

Theorem 1.1. *[7] The Fourier transform maps $L^2(Q_p)$ onto $L^2(Q_p)$ one-to-one and continuously.*

2. Shearlets on $L^2\left(Q_p^2\right)$

Let $a \in \mathbb{Z}$. If the equation $x^2 \equiv a (\bmod\ p)$ has a solution $x \in \mathbb{Z}$, then a is called quadratic residue modulo p. In the follwing lemma [7], we need the Legendre symbol defined by

$$\left(\frac{a}{p}\right) = \begin{cases} 1, & \text{if } a \text{ is quadratic residue modulo } p, \\ -1, & \text{if } a \text{ is quadratic non-residue modulo } p. \end{cases}$$

Lemma 2.1. *The equation*

$$x^2 = a,\ a = p^{\gamma(a)}(a_0 + a_1 p + \cdots), \quad 0 \le a_i < p,\ a_0 \ne 0,$$

has a solution $x \in Q_p$ if and only if

1) *$\gamma(a)$ is even,*
2) *for $p \ne 2$, $\left(\frac{a_0}{p}\right) = 1$, and for $p = 2$, $a_1 = a_2 = 0$.*

Set $A_a = \begin{pmatrix} a^{-1} & 0 \\ 0 & a^{-\frac{1}{2}} \end{pmatrix}$ for $a \in G_p$, where G_p is the group that all of its elements have a square root and $S_s = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}$ for $s \in Q_p$.

For $f \in L^2(Q_p)$ the translation and dilation operators are defined by $T_y(f)(\cdot) = f(\cdot - y)$ and $D_a f(\cdot) = a^{\frac{1}{2}} f(a\cdot)$, resp. and we have

$$F\left[f\left(a\cdot+b\right)\right](\xi)=|a|_p^{-1}\chi_p\left(-\frac{b}{a}\xi\right)F[f]\left(\frac{\xi}{a}\right),\quad a\neq 0,b\in Q_p.$$

Also for $f\in L^2\left(Q_p^d\right)$ we have [7]

$$F\left(D_Af\right)(\xi)=|\det A|_p^{-1}F[f]\left(A^{-T}\xi\right),$$

where A is a $d\times d$ dilation matrix and $D_Af(\cdot)=|\det A|_p^{\frac{1}{2}}f(A)$.

Definition 2.2. The continuous p-adic shearlet system associated with $\psi\in L^2\left(Q_p^2\right)$ is defined as follows:

$$SH(\psi)=\left\{\psi_{a,s,t}=|a|_p^{-\frac{3}{4}}\psi\left(A_aS_s(x-t)\right):a\in G_p,s\in Q_p,t\in Q_p^2\right\}.$$

We define the p-adic shearlet group $\mathbb{S}=G_p\times Q_p\times Q_p^2$, equipped with multiplication given by

$$(a,s,t)\left(a',s',t'\right)=\left(aa',s+\sqrt{a}s',t+S_s^{-1}A_a^{-1}t'\right).$$

It is obvious that $\frac{dadsdt}{|a|_p^3}$ is a left Haar measure of this group.

Definition 2.3. The continuous p-adic shearlet transform of $f\in L^2\left(Q_p^2\right)$ is defined as follows

$$f\mapsto SH_\psi f(a,s,t)=\langle f,\psi_{a,s,t}\rangle,\quad (a,s,t)\in\mathbb{S}.$$

In the following, we define an admissible p-adic shearlet.

Definition 2.4. The function $\psi\in L^2\left(Q_p^2\right)$ is called an admissible p-adic shearlet if

$$C_\psi=\int_{Q_p}\int_{Q_p}\frac{\left|\widehat{\psi}\left(\xi_1,\xi_2\right)\right|^2}{|\xi_1|_p^2}d\xi_2d\xi_1<\infty.$$

Set $J\subseteq\mathbb{Z}$ and $c\in Q_p$ such that $c\neq 0$. Now we consider the discrete subset of $\mathbb{S}$ of the form

$$\Lambda=\left\{\left(a_j,s_{j,d},S_{s_{j,d}}^{-1}A_{a_j}^{-1}cb\right):j\in J,d\in I_p,b\in I_p^2\right\},\quad a_j\in G_p,s_{j,d}\in Q_p.$$

Let $\psi\in L^2\left(Q_p^2\right)$. Then the discrete p-adic shearlet system is defined as follows

$$SH(\psi,\Lambda)=\left\{\psi_{j,d,b}=T_{S_{s_{j,d}}^{-1}A_{a_j}^{-1}cb}D_{A_{a_j}}D_{S_{s_{j,d}}}\psi:\left(a_j,s_{j,d},S_{s_{j,d}}^{-1}A_{a_j}^{-1}cb\right)\in\Lambda\right\}.$$

As one can see in [1], a discrete p-adic shearlet system $\{\psi_{j,d,b}\}$ is called a shearlet frame, if there exist constants $0<A\leq B<\infty$ such that for all $f \in L^2\left(Q_p^2\right)$

$$A\,||f||^2 \leq \sum_{j,d} \sum_{b} |\langle f, \psi_{j,d,b}\rangle|^2 \leq B\,||f||^2\,. \tag{2.1}$$

The following theorem gives sufficient conditions on the sequence $\Lambda \subseteq \mathbb{S}$ and the function $\psi \in L^2\left(Q_p^2\right)$ such that $SH\left(\psi, \Lambda\right)$ forms a frame.

Theorem 2.5. *[3] Let $c \neq 0$ be fixed and Λ defined as above. Let $\psi \in D\left(Q_p^2\right)$ and set*

$$\phi\left(\omega\right) = ess \sup_{\xi \in Q_p^2} \sum_{j,d} \left|\widehat{\psi}\left(A_{a_j}^{-1} S_{s_{j,d}}^{-T}\xi\right)\right| \left|\widehat{\psi}\left(A_{a_j}^{-1} S_{s_{j,d}}^{-T}\xi + \omega\right)\right|, \quad \omega \in Q_p^2.$$

If there exist $0 < \alpha \leq \beta < \infty$ such that

$$\alpha \leq \sum_{j,d} \left|\widehat{\psi}\left(A_{a_j}^{-1} S_{s_{j,d}}^{-T}\xi\right)\right|^2 \leq \beta \;\; a.e\; \xi \in Q_p^2,$$

and

$$\sum_{b \in I_p^2, b \neq 0} \left(\phi\left(\frac{1}{c}b\right)\phi\left(-\frac{1}{c}b\right)\right)^{\frac{1}{2}} =: \gamma < \alpha,$$

then $SH\left(\psi, \Lambda\right)$ is a frame for $L^2\left(Q_p^2\right)$ with frame bounds C and D satisfying

$$\frac{1}{|c|_p^2}\left(\alpha - \gamma\right) \leq C \leq D \leq \frac{1}{|c|_p^2}\left(\beta + \gamma\right).$$

As an example, let $p = 2$ and $c \neq 0$ such that $|c|_2 < 2^{-1}$. Define the discrete subset Λ by

$$\Lambda = \left\{\left(2^{2j}, d2^j, S_{d2^j}^{-1} A_{2^{2j}}^{-1} cb\right) : j \in \mathbb{Z}, \quad d \in I_p, b \in I_p^2\right\},$$

and $\psi \in D\left(Q_p^2\right)$ by

$$\widehat{\psi}\left(\xi_1, \xi_2\right) = \widehat{\psi_1}\left(\frac{\xi_2}{\xi_1}\right)\left(\widehat{\psi_2}\left(\xi_1\right) + \widehat{\psi_2}\left(2\xi_1\right)\right),$$

where $\widehat{\psi_1}\left(\omega\right) = \Omega\left(|\omega|_2\right)$ and $\widehat{\psi_2}\left(\omega\right) = \delta\left(|\omega|_2 - 1\right)$ and the function δ is defined as:

$$\delta\left(|\omega|_p - p^\gamma\right) = \begin{cases} 1, & \text{if } \; \omega \in S_\gamma\left(0\right), \\ 0, & o.w. \end{cases}$$

Then by [3, Corollary 4.3], the shearlet system $SH\left(\psi, \Lambda\right)$ forms a p-adic tight frame for $L^2\left(Q_2^2\right)$. See also [3, Example 4.4].

3. Dual shearlet frames on $L^2\left(Q_p^2\right)$

A discrete shearlet system $\{\psi_{j,d,b}\}$ forms a Bessel sequence if only the right hand side inequality in (2.1) holds. Two functions ψ and $\widetilde{\psi}$ generate dual shearlet frames if $\{\psi_{j,d,b}\}$ and $\left\{\widetilde{\psi}_{j,d,b}\right\}$ are Bessel sequences and for all $f \in L^2\left(Q_p^2\right)$, we have

$$f = \sum_{j,d} \sum_{b} \langle f, \psi_{j,d,b} \rangle \widetilde{\psi}_{j,d,b}.$$

We say that ψ with $\widetilde{\psi}$ generate dual shearlet tight frames if $\{\psi_{j,d,b}\}$ and $\left\{\widetilde{\psi}_{j,d,b}\right\}$ are Bessel sequences and for some non-zero constant B [4], we have

$$B \langle f, g \rangle = \sum_{j,d} \sum_{b} \langle f, \psi_{j,d,b} \rangle \left\langle \widetilde{\psi}_{j,d,b}, g \right\rangle. \tag{3.1}$$

In this section, we characterize the p-adic dual shearlet tight frames and give a necessary condition for two functions to generate p-adic dual shearlet tight frames. Let $\{\psi_{j,d,b}\}$ be a frame. The frame operator

$$S : L^2\left(Q_p^2\right) \to L^2\left(Q_p^2\right),$$

is defined as follows

$$Sf = \sum_{j,d,b} \langle f, \psi_{j,d,b} \rangle \psi_{j,d,b}, \text{ for all } f \in L^2\left(Q_p^2\right).$$

This operator is positive, self adjoint, invertible and the canonical dual shearlet frame is $\left\{S^{-1}\psi_{j,d,b}\right\}$.

Theorem 3.1. *The frame operator S associated with the shearlet frame $SH(\psi, \Lambda)$ is a Fourier multiplier with the function*

$$\Delta(\xi) = \sum_{j,k} \left| \widehat{\psi}\left(A_{a_j}^{-1} S_{s_{j,d}}^{-T} \xi\right)\right|^2.$$

Proof. Set $\psi\left(A_{a_j} S_{s_{j,d}} \cdot - cb\right) = \psi_{j,d,b}$ and let $f, g \in D\left(Q_p^2\right)$. Then

(3.2)

$$\begin{aligned}\left\langle \widehat{Sf}, \widehat{g} \right\rangle &= \left\langle \sum_{j,d,b} \langle f, \psi_{j,d,b} \rangle \widehat{\psi_{j,d,b}}, \widehat{g} \right\rangle \\ &= \sum_{j,d,b} \langle f, \psi_{j,d,b} \rangle \left\langle \widehat{\psi_{j,d,b}}, \widehat{g} \right\rangle \\ &= \sum_{j,d,b} \left\langle \widehat{f}, \widehat{\psi_{j,d,b}} \right\rangle \left\langle \widehat{\psi_{j,d,b}}, \widehat{g} \right\rangle\end{aligned}$$

$$= \sum_{j,d,b} \int_{Q_p^2} \int_{Q_p^2} \widehat{f}(\xi) \overline{\widehat{\psi}\left(A_{a_j}^{-1} S_{s_{j,d}}^{-T} \xi\right)} \widehat{\psi}\left(A_{a_j}^{-1} S_{s_{j,d}}^{-T} \omega\right) \overline{\widehat{g}(\omega)} \chi_p \left(\left(S_{s_{j,d}}^{-1} A_{a_j}^{-1} cb\right) \cdot (\omega - \xi)\right) d\xi d\omega$$

$$= \sum_{j,d} \int_{Q_p^2} \widehat{f}(\xi) \widehat{g}(\xi) \left|\widehat{\psi}\left(A_{a_j}^{-1} S_{s_{j,d}}^{-T} \xi\right)\right|^2.$$

Since $f, g, \psi \in D\left(Q_p^2\right)$, so we can use the Fubini theorem. Hence the equation (3.2) is equivalent to

$$\sum_{j,d} \int_{Q_p^2} \widehat{f}(\xi) \widehat{g}(\xi) \left|\widehat{\psi}\left(A_{a_j}^{-1} S_{s_{j,d}}^{-T} \xi\right)\right|^2 = \int_{Q_p^2} \widehat{f}(\xi) \widehat{g}(\xi) \sum_{j,d} \left|\widehat{\psi}\left(A_{a_j}^{-1} S_{s_{j,d}}^{-T} \xi\right)\right|^2 = \left\langle \Delta \widehat{f}, \widehat{g} \right\rangle.$$

We know that $D\left(Q_p^2\right)$ is dense in $L^2\left(Q_p^2\right)$, so we have

$$\left\langle \widehat{Sf}, \widehat{g} \right\rangle = \left\langle \Delta \widehat{f}, \widehat{g} \right\rangle,$$

for $f, g \in L^2\left(Q_p^2\right)$, and this means $\widehat{Sf} = \Delta \widehat{f}$.

□

Theorem 3.2. *If $\{\psi_{j,d,b}\}$ forms a Bessel sequence for $D\left(Q_p^2\right)$ with bound B, then*

$$\sum_{j,d} \left|\widehat{\psi}\left(cA_{a_j}^{-1} S_{s_{j,d}}^{-T}\right)\right|^2 \le B.$$

Proof. Using (2.1) we have

$$\sum_{j,d} \sum_{b \in I_p^2} \left|\left\langle \widehat{f}, \widehat{\psi}_{j,d,b} \right\rangle\right|^2 \le B \left\|\widehat{f}\right\|^2, \quad \forall f \in D\left(Q_p^2\right).$$

By Parseval identity we can write

$$\sum_{b \in I_p^2} \left|\left\langle \widehat{f}, \widehat{\psi}_{j,d,b} \right\rangle\right|^2$$

$$= \sum_{b} \left|\int_{Q_p^2} \widehat{f}(\xi) \chi_p \left(\left(-S_{s_{j,d}}^{-1} A_{a_j}^{-1} cb\right) \cdot \xi\right) \overline{\widehat{\psi}}\left(A_{a_j}^{-1} S_{s_{j,d}}^{-T} \xi\right) d\xi\right|^2$$

$$= \sum_{b} \left|\int_{Q_p^2} \widehat{f}\left(\frac{1}{c} S_{s_{j,d}}^{T} A_{a_j} \omega\right) \overline{\widehat{\psi}}(\omega) \chi_p(b \cdot \omega) d\omega\right|^2$$

$$= \sum_{b} \left|\sum_{i \in I_p^2} \int_{B_0(i)} \widehat{f}\left(\frac{1}{c} S_{s_{j,d}}^{T} A_{a_j} \omega\right) \overline{\widehat{\psi}}(\omega) \chi_p(b \cdot \omega) d\omega\right|^2$$

$$= \sum_{b} \left| \sum_{i \in I_p^2} \int_{B_0(i)} \widehat{f}\left(\frac{1}{c} S_{s_{j,d}}^T A_{a_j} (\omega + i)\right) \overline{\widehat{\psi}} (\omega + i)\, \chi_p (b \cdot \omega)\, d\omega \right|^2$$

$$= \int_{B_0} \left| \sum_{i \in I_p^2} \widehat{f}\left(\frac{1}{c} S_{s_{j,d}}^T A_{a_j} (\omega + i)\right) \overline{\widehat{\psi}} (\omega + i) \right|^2 .$$

Then we have

$$\sum_{j,d} \int_{B_0} \left| \sum_{i \in I_p^2} \widehat{f}\left(\frac{1}{c} S_{s_{j,d}}^T A_{a_j} (\omega + i)\right) \overline{\widehat{\psi}} (\omega + i) \right|^2 \leq B \left\| \widehat{f} \right\|^2 . \tag{3.3}$$

Consider $\upsilon \in Q_p^2$ and the function $\widehat{f}$ as

$$\widehat{f}(\xi) = \frac{1}{p^M} \chi_{B_M(\upsilon)} (\xi) .$$

For any positive integer N, M, by (3.3), we obtain

$$\sum_{d \in I_p} \sum_{|j| \leq N} \int_{B_M\left(cA_{a_j}^{-1} S_{s_{j,d}}^{-T}\right)} \left| \widehat{\psi}(\omega) \right|^2 d\omega \leq B.$$

Hence the result follows by taking $N \to \infty$ and $M \to \infty$. □

Let H be a measurable subset of Q_p^2. We consider the subspace $L^2(H)^{\vee}$ of $L^2\left(Q_p^2\right)$ as

$$L^2(H)^{\vee} = \left\{ f \in L^2\left(Q_p^2\right) : \operatorname{supp} \widehat{f} \subseteq H \right\} .$$

Now we have the following main result.

Theorem 3.3. *Let ψ and $\widetilde{\psi}$ be admissible shearlets. If ψ with $\widetilde{\psi}$ in $L^2(H)^{\vee}$ generate a dual shearlet tight frame in $L^2(H)^{\vee}$ with bound B, then we have*

$$\sum_{j,d} \overline{\widehat{\psi}} \left(\frac{1}{c} A_{a_j}^{-1} S_{s_{j,d}}^{-T} \xi\right) \widehat{\widetilde{\psi}} \left(\frac{1}{c} A_{a_j}^{-1} S_{s_{j,d}}^{-T} \xi\right) = B \chi_H (\xi) \ \text{a.e.}.$$

Proof. We can write

$$\sum_{j,d} \sum_{b \in I_p^2} \langle f, \psi_{j,d,b} \rangle \left\langle \widetilde{\psi}_{j,d,b}, g \right\rangle$$

$$= \sum_{j,d} \int_{B_0} \left[\widehat{f}\left(\frac{1}{c} S_{s_{j,d}}^T A_{a_j} \cdot\right), \widehat{\psi} \right] (\eta) \left[\widehat{\widetilde{\psi}}, \widehat{g}\left(\frac{1}{c} S_{s_{j,d}}^T A_{a_j} \cdot\right) \right] (\eta)\, d\eta,$$

where the bracket product is defined by

$$[f,g](\eta)=\sum_{b\in I_p^2} f(\eta+b)\,\overline{g}(\eta+b).$$

Since ψ with $\widetilde{\psi}$ generate a dual shearlet tight frame then by (3.1), we have

$$B\left\langle \widehat{f},\widehat{g}\right\rangle=\sum_{j,d}\int_{B_0}\left[\widehat{f}\left(\frac{1}{c}S^T_{s_{j,d}}A_{a_j}\cdot\right),\widehat{\psi}\right](\eta)\left[\widehat{\widetilde{\psi}},\widehat{g}\left(\frac{1}{c}S^T_{s_{j,d}}A_{a_j}\cdot\right)\right](\eta)\,d\eta.$$

Fix $d\in I_p$ and set

$$M^j:=S^T_{s_{j,d}}A_{a_j}=\begin{pmatrix} a_j^{-1} & 0\\ s_{j,d}a_j^{-1} & a_j^{-\frac{1}{2}}\end{pmatrix}.$$

Fix $\xi\in H^\circ$ (where H° denotes the interior of H). For for any $k\in\mathbb{Z}$ there exists a unique $i_k\in I_p^2$ such that

$$\xi\in D_k(\xi,i_k):=\left\{M^j(x+i_k):x\in B_0\right\}.$$

Now applying a technique similar to the proof of [4, Theorem 2.2], the result follows. □

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