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New Generalization of Darbo's Fixed Point Theorem via α-admissible Simulation Functions with Application

Hossein Monfared[1], Mehdi Asadi[2] and Ali Farajzadeh[3*]

Abstract. In this paper, at first, we introduce α_μ-admissible, Z_μ-contraction and N_μ-contraction via simulation functions. We prove some new fixed point theorems for defined class of contractions via α-admissible simulation mappings, as well. Our results can be viewed as extension of the corresponding results in this area. Moreover, some examples and an application to functional integral equations are given to support the obtained results.

1. Introduction

Schauder fixed point theorem is one of the useful and important tools in analysis. In 1955, Darbo [5], by using the concept of a measure of non-compactness, proved the fixed point property for known contraction on a closed, bounded and convex subset of Banach spaces. Darbo fixed point plays a key role in nonlinear analysis especially in proving the existence of solutions for a lot of classes of nonlinear equations. Since then, some generalizations of Darbo fixed point theorem have been proved, see [1, 10, 16, 18] and the references therein. Recently, Chen *et al.* [3] proved some new generalizations of Darbo fixed point theorem by using the notion of simulation function that Khojasteh *et al.* [4, 13] proposed it.

In this paper, we investigate the existence of fixed points of certain mappings via α_μ-admissible simulation functions for α-set contraction on a closed, bounded and convex subset of Banach spaces.

Throughout the paper, $\mathbb{N}$, $\mathbb{R}_+$ and $\mathbb{R}$, respectively, denote the set of all positive integers, non-negative real numbers and real numbers. Now,

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let us recall some basic concepts, notations and known results which will be used in the sequel. Let E be a Banach space with the norm $\|.\|$ and ϑ be the zero element in E. The closed ball centered at x with radius r is denoted by $B(x,r)$ and simply by B_r if $x=0$. If X is a nonempty subset of E, then we denote by $\overline{X}$ and $\bar{\mathrm{co}}(X)$ the closure and closed convex hull of X, respectively. Moreover, let M_E be the family of all nonempty bounded subsets of E and N_E be the subfamily consisting of all relatively compact subsets of E. In [2], Banas *et al.* introduced the concept of the measure of non-compactness.

Definition 1.1. A mapping $\mu : M_E \to \mathbb{R}_+$ is said to be a measure of non-compactness in E if it satisfies the following conditions:

(1) The set $\ker\mu = \{\mathrm{X} \in \mathrm{M_E} : \mu(\mathrm{X}) = 0\}$ is nonempty and $\ker\mu \subseteq \mathrm{N_E}$;
(2) $X \subseteq Y \Rightarrow \mu(X) \le \mu(Y)$;
(3) $\mu(\bar{\mathrm{co}}X) = \mu(\bar{X}) = \mu(X)$;
(4) $\mu(\lambda X + (1-\lambda)Y) \le \lambda\mu(X) + (1-\lambda)\mu(Y)$, for all $\lambda \in [0,1]$;
(5) If $\{X_n\}$ is a sequence of closed sets of M_E such that $X_{n+1} \subseteq X_n$ for $n = 1, 2, \dots$ and $\lim_{n\to\infty} \mu(X_n) = 0$, then the intersection set $X_\infty = \cap_{n=1}^{\infty} X_n$ is nonempty.

The set $\ker\mu$ described in (1) of Definition 1.1 is said to be kernel of the measure of non-compactness μ. It is obvious that X_∞ belongs to $\ker\mu$.

Theorem 1.2 (Schauder fixed point Theorem)**.** *Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E, then each continuous and compact map $T : \Omega \to \Omega$ has at least one fixed point in the set Ω.*

The next theorem is an extension of Schauder fixed point Theorem 1.2 by reducing the compactness of the mapping T.

Theorem 1.3 ([3, Darbo fixed point theorem])**.** *Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E and let $T : \Omega \to \Omega$ be a continuous mapping. Assume that there exists a constant $k \in [0,1)$ such that*

$$\mu(TX) \le k\mu(X),$$

for any nonempty subset X of Ω, where μ is a measure of non-compactness defined in E. Then, T has a fixed point in the set Ω.

In order to present the main results, we need the following definitions and preliminary results.

Definition 1.4 ([12, Khan *et al.*])**.** An altering distance function is a continuous and non-decreasing mapping $\varphi : [0,\infty) \to [0,\infty)$ such that $\varphi^{-1}(\{0\}) = \{0\}$.

In [15], see Definition 3.2, the authors slightly modified the definition of simulation function which introduced by Khojasteh et al. [13] and enlarged the family of all simulation functions.

Definition 1.5 ([13]). A function $\sigma : [0,\infty) \times [0,\infty) \to \mathbb{R}$ is said to be simulation if it fulfills:
(σ_1) $\sigma(0,0) = 0$;
(σ_2) $\sigma(t,u) < u - t,$ for all $t, u > 0$;
(σ_3) if $\{t_n\}$, $\{u_n\}$ are sequences in $(0,\infty)$ such that $\lim_{n\to\infty} t_n = \lim_{n\to\infty} u_n > 0$, then

$$\limsup_{n\to\infty} \sigma(t_n, u_n) < 0. \tag{1.1}$$

Let Z be the collection of all simulation functions $\sigma : [0,\infty)\times[0,\infty) \to \mathbb{R}$. It follows from ($\sigma_2$) that

$$\sigma(t,t) < 0, \quad \text{for all} \quad t > 0. \tag{1.2}$$

Definition 1.6. ([3]) A function $\zeta : [0,\infty) \times [0,\infty) \to \mathbb{R}$ is said to be generalized simulation if:

$$\zeta(t,s) \leq s - t, \text{ for all } t, s > 0$$

.

Let N denote the family of all generalized simulation functions $\zeta : [0,\infty) \times [0,\infty) \to \mathbb{R}$.

Definition 1.7 ([6, 7]). Let $f : X \to X$ and $\alpha : X \times X \to (-\infty, +\infty)$ be mappings. We say that f is an α-admissible mapping if $\alpha(x,y) \geq 1$ implies that $\alpha(fx, fy) \geq 1$, for all $x, y \in X$.

In what follows, we recall the notion of (triangular) α-orbital admissible, introduced by Popescu [14], that is inspired by the authors of [17].

Definition 1.8 ([14]). For a fixed mapping $\alpha : M \times M \to [0,\infty)$, we say that a self-mapping $T : M \to M$ is an α-orbital admissible if

$$\alpha(u, Tu) \geq 1 \quad \Rightarrow \quad \alpha(Tu, T^2u) \geq 1.$$

Let $\mathcal{A}$ denote the collection of all α-orbital admissible $T : M \to M$.

In addition, T is called triangular α-orbital admissible if T is α-orbital admissible and

$$\alpha(u,v) \geq 1 \quad \text{and} \quad \alpha(v, Tv) \geq 1 \quad \Rightarrow \quad \alpha(u, Tv) \geq 1.$$

Let $\mathcal{O}$ denote the collection of all triangular α-orbital admissible $T : M \to M$.

Definition 1.9 ([3]). Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E and let $T : \Omega \to \Omega$ be a continuous operator. We say that T is a Z_μ-contraction if there exists a simulation function $\xi \in Z$ such that

$$\xi(\mu(TX), \mu(X)) \geq 0, \tag{1.3}$$

for any nonempty subset X of Ω, where μ is an arbitrary measure of non-compactness.

Now, we observe some useful properties of Z_μ-contractions in Banach spaces.

Remark 1.10 (([3]). If T is a Z_μ-contraction with respect to $\xi \in Z$, then

$$\mu(TX) < \mu(X), \tag{1.4}$$

for any nonempty subset X of Ω. To prove it, applying (σ_2) and (1.3), we have

$$0 \leq \xi(\mu(TX), \mu(X)) < \mu(X) - \mu(TX).$$

Hence, (1.4) holds. We need the following fixed point theorem in the sequel.

Theorem 1.11 ([3]). *Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E and $T : \Omega \to \Omega$ be a continuous operator. If T is a Z_μ-contraction with respect to $\xi \in Z$. Then, T has at least one fixed point in Ω .*

Definition 1.12 ([3]). Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E and let $T : \Omega \to \Omega$ be a continuous operator. We say that T is a N_μ-contraction if there exists $\zeta \in N$ such that

$$\zeta(\mu(TX), \kappa(\mu(X))) \geq 0, \tag{1.5}$$

for any nonempty subset X of Ω, where μ is an arbitrary measure of non-compactness, and $\kappa : [0, \infty) \to \mathbb{R}_+$ is nondecreasing mapping on $\mathbb{R}_+$ such that $\lim_{n\to\infty} \kappa^n(t) = 0$, for each $t > 0$.

Now, some useful properties of N_μ-contractions in the setting of Banach spaces are presented.

Remark 1.13. (1) By the definition of generalized simulation functions, it is obvious that a generalized simulation function must verify $\zeta(r, r) \leq 0$, for all $r > 0$.
(2) If T is N_μ-contraction with respect to $\zeta \in N$, then

$$\mu(TX) \leq \kappa(\mu(X)), \tag{1.6}$$

for any nonempty subset X of Ω. To prove it, applying Definition 1.12, we have

$$0 \leq \xi(\mu(TX), \kappa(\mu(X))) \leq \kappa(\mu(X)) - \mu(TX).$$

Hence, (1.6) holds.

2. Fixed Point Theorems via α-admissible Simulation Functions

In order to prove our fixed point theorems, we need the following related concepts.

Definition 2.1. Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E, $T : \Omega \to \Omega$ be a continuous mapping and $\alpha : \mu(M_E) \times \mu(M_E) \to (-\infty, +\infty)$ be a mapping. We say that T is an α_μ-admissible mapping if

$$\alpha(\mu(X), \mu(Y)) \geq 1 \quad \Rightarrow \quad \alpha(\mu(TX), \mu(TY)) \geq 1,$$

for any nonempty subsets X and Y of Ω, where μ is an arbitrary measure of non-compactness.

Definition 2.2. Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E, $T : \Omega \to \Omega$ be a continuous and α_μ-admissible operator. We say that T is an α_μ-admissible Z_μ-contraction if there exists $\xi \in Z$ such that

$$\xi(\alpha(\mu(X), \mu(TX))\mu(TX), \mu(X)) \geq 0. \tag{2.1}$$

for any nonempty subsets X of Ω, where μ is the measure of non-compactness.

Remark 2.3. If $\alpha(x, y) = 1$, then T turns into a Z_μ-contraction with respect to ξ.

Remark 2.4. If T is an α_μ-admissible Z_μ-contraction with respect to ξ, then

$$\alpha(\mu(X), \mu(TX))\mu(TX) < \mu(X), \tag{2.2}$$

for all $X \subseteq \Omega$ such that $\mu(X) > 0$. To prove the assertion, we assume that $X \subseteq \Omega$. If $\mu(TX) = 0$, then

$$\alpha(\mu(X), \mu(TX))\mu(TX) = 0 < \mu(X).$$

Otherwise, $\mu(TX) > 0$. If $\alpha(\mu(X), \mu(TX)) = 0$, then the inequality is satisfied trivially. So, assume that $\alpha(\mu(X), \mu(TX)) > 0$, applying (σ_2) with (2.1), we derive that

$$0 \leq \xi(\alpha(\mu(X), \mu(TX))\mu(TX), \mu(X)) < \mu(X) - \alpha(\mu(X), \mu(TX))\mu(TX),$$

so (2.2) holds.

Theorem 2.5. *Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E and let $T : \Omega \to \Omega$ be a continuous operator. If T is an α_μ-admissible Z_μ-contraction with respect to $\xi \in \mathbb{Z}$ and there exits $X_0 \subseteq \Omega$ such that X_0 be closed and convex, $TX_0 \subseteq X_0$ and $\alpha(\mu(X_0), \mu(TX_0)) \geq 1$, then T has at least one fixed point in Ω.*

Proof. Let $X_0 \subseteq \Omega$ be such that $\alpha(\mu(X_0), \mu(TX_0)) \geq 1$, and $TX_0 \subseteq X_0$. Define the sequence $\{X_n\}$ as follows:
$X_n = \bar{\text{co}}(TX_{n-1})$, for all $n \geq 1$.
It follows from the induction that

$$X_n \subseteq X_{n-1} \text{ and } TX_n \subseteq X_n.$$

Hence, the hypothesis implies

$$TX_0 \subseteq X_0.$$

Thus,

$$X_1 = \bar{\text{co}}(TX_0) \subseteq \bar{\text{co}}(X_0) = X_0.$$

Now, suppose that $X_{n+1} \subseteq X_n$, therefore we get

$$X_{n+2} = \bar{\text{co}}(TX_{n+1}) \subseteq \bar{\text{co}}(TX_n) = X_{n+1},$$

and

$$TX_{n+1} \subseteq TX_n \subseteq \bar{\text{co}}(TX_n) = X_{n+1}.$$

If there exists natural number n_0 such that $\mu(X_{n_0}) = 0$, then X_{n_0} is compact and $TX_{n_0} \subset X_{n_0}$. Thus, Theorem 1.2 implies that T has a fixed point. Next, we suppose that $\mu(X_n) > 0$, for all $n \geq 0$. Regarding that T is α_μ-admissible, we derive

$$\begin{aligned}\alpha(\mu(X_0), \mu(X_1)) &= \alpha(\mu(X_0), \mu(\bar{\text{co}}(TX_0))) \\ &= \alpha(\mu(X_0), \mu(TX_0)) \geq 1,\end{aligned}$$

which implies that

$$\alpha(\mu(TX_0), \mu(TX_1)) = \alpha(\mu(X_1), \mu(X_2)) \geq 1.$$

Recursively, we obtain that

$$\alpha(\mu(X_n), \mu(X_{n+1})) \geq 1, \text{ for all } n \geq 0. \tag{2.3}$$

On the other hand, by our assumptions and (1.3), we get

$$\begin{aligned}&\xi(\alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1}), \mu(X_n)) \\ &= \xi(\alpha(\mu(X_n), \mu(\bar{\text{co}}(TX_n))\mu(\bar{\text{co}}(TX_n)), \mu(X_n)) \\ &= \xi(\alpha(\mu(X_n), \mu(TX_n))\mu(TX_n), \mu(X_n)) \geq 0.\end{aligned} \tag{2.4}$$

Based on Remark 2.4, we can get

$$\begin{aligned}0 &\leq \xi(\alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1}), \mu(X_n)) \\ &< \mu(X_n) - \alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1}).\end{aligned} \tag{2.5}$$

From (2.3), (2.4) and (2.5), we infer that

$$\mu(X_{n+1}) \leq \alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1}) < \mu(X_n). \tag{2.6}$$

Hence, $\{\mu(X_n)\}$ is a decreasing sequence of positive real numbers. Thus, there exists $r \geq 0$, such that $\mu(X_n) \to r$ as $n \to \infty$. Next, we show that $r = 0$. Suppose to the contrary that $r > 0$. We also have by (2.6):

$$\alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1}) \to r > 0 \text{ as } n \to \infty.$$

Applying the axiom (σ_3) in Definition 1.5 to the sequences:

$$\{t_n = \alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1})\} \text{ and } \{s_n = \mu(X_n)\}$$

(which have the same limit $r > 0$ and verify $t_n < s_n$, for all n), it follows that

$$\limsup_{n\to\infty} \xi(\alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1}), \mu(X_n)) = \limsup_{n\to\infty} \xi(t_n, s_n) < 0,$$

which contradicts (2.4). We get $r = 0$ and hence $\mu(X_n) \to 0$ as $n \to \infty$. Now, since $\{X_n\}$ is a nested sequence, in view of (5) of Definition 1.1, we conclude that $X_\infty = \cap_{n=1}^{\infty} X_n$ is a nonempty, closed and convex subset of Ω. Moreover, we know that X_∞ belongs to kerμ. So, X_n is compact and invariant under the mapping T. Consequently, Theorem 1.2 implies that T has a fixed point in X_∞. Since $X_\infty \subseteq \Omega$, then the proof is complete. □

Corollary 2.6 ([3, Theorem 2.1])**.** *Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E, and $T : \Omega \to \Omega$ be a continuous operator. If T is a Z_μ-contraction with respect to $\xi \in Z$, then T has at least one fixed point in Ω.*

Proof. In Theorem 2.5 let $\alpha(x, y) = 1$. □

3. Fixed Point Theorems via α-admissible Generalized Simulation Mappings

Definition 3.1. Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E, $T : \Omega \to \Omega$ be a continuous and α_μ-admissible mapping. We say that T is an α_μ-admissible N_μ-contraction if there exists $\xi \in \mathbb{N}$ such that

$$\xi(\alpha(\mu(X), \mu(TX))\mu(TX), \kappa(\mu(X))) \geq 0, \tag{3.1}$$

for any nonempty subset X of Ω, where μ is an arbitrary measure of non-compactness and $\kappa : [0, \infty) \to \mathbb{R}_+$ is nondecreasing on $\mathbb{R}_+$ such that $\lim_{n\to\infty} \kappa^n(t) = 0$, for each $t > 0$.

Remark 3.2. If $\alpha(x, y) = 1$, then T turns into a N_μ-contraction with respect to ξ.

Remark 3.3. If T is an α_μ-admissible N_μ-contraction with respect to ξ, then

$$\alpha(\mu(X), \mu(TX))\mu(TX) \leq \kappa(\mu(X)), \tag{3.2}$$

for all $X \subseteq \Omega$ such that $\mu(X) > 0$. To prove the assertion, we assume that $X \subseteq \Omega$. If $\mu(TX) = 0$, then

$$\alpha(\mu(X), \mu(TX))\mu(TX) = 0 \leq \kappa(\mu(X)).$$

Now, we suppose that $\mu(TX) > 0$. If $\alpha(\mu(X), \mu(TX)) = 0$, then the inequality is trivially satisfied. So, assume $\alpha(\mu(X), \mu(TX)) > 0$ and apply (3.1). Hence,

$$\begin{aligned} 0 &\leq \xi(\alpha(\mu(X), \mu(TX))\mu(TX), \kappa(\mu(X))) \\ &\leq \kappa(\mu(X)) - \alpha(\mu(X), \mu(TX))\mu(TX). \end{aligned}$$

So, (3.2) holds.

Next, we prove the following fixed point theorem.

Theorem 3.4. *Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E and $T : \Omega \to \Omega$ be a continuous operator. If T is an α_μ-admissible N_μ-contraction with respect to $\xi \in Z$ and there exits $X_0 \subseteq \Omega$ such that X_0 be closed and convex, $TX_0 \subseteq X_0$ and $\alpha(\mu(X_0), \mu(TX_0)) \geq 1$, then T has at least one fixed point in Ω.*

Proof. Let $X_0 \subseteq \Omega$ be such that $\alpha(\mu(X_0), \mu(TX_0)) \geq 1$ and $TX_0 \subseteq X_0$, then define a sequence $\{X_n\}$ as follows:
$X_n = \bar{\text{co}}(TX_{n-1})$, for all $n \geq 1$.
If there exists natural number n_0 such that $\mu(X_{n_0}) = 0$, then X_{n_0} is compact. Since $TX_{n_0} \subset X_{n_0}$, thus Theorem 1.2 implies that T has a fixed point. Next, we suppose that $\mu(X_n) > 0$, for all $n \geq 0$.
Regarding that T is α_μ-admissible, we derive

$$\begin{aligned} \alpha(\mu(X_0), \mu(X_1)) &= \alpha(\mu(X_0), \mu(\bar{\text{co}}(TX_0))) \\ &= \alpha(\mu(X_0), \mu(TX_0)) \geq 1, \end{aligned}$$

which implies that

$$\alpha(\mu(TX_0), \mu(TX_1)) = \alpha(\mu(X_1), \mu(X_2)) \geq 1.$$

Recursively, we obtain that

$$\alpha(\mu(X_n), \mu(X_{n+1})) \geq 1, \text{ for all } n = 0, 1, \ldots. \tag{3.3}$$

On the other hand, by our assumptions and (1.5), we get

$$\begin{aligned} &\xi(\alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1}), \kappa(\mu(X_n))) \\ &\quad = \xi(\alpha(\mu(X_n), \mu(\bar{\text{co}}(TX_n)))\mu(\bar{\text{co}}(TX_n)), \kappa(\mu(X_n))) \end{aligned} \tag{3.4}$$

$$= \xi(\alpha(\mu(X_n), \mu(TX_n))\mu(TX_n), \kappa(\mu(X_n))) \geq 0.$$

Based on Remark 3.3, we can get

$$\begin{aligned} 0 &\leq \xi(\alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1}), \kappa(\mu(X_n))) \\ &< \kappa(\mu(X_n)) - \alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1}). \end{aligned} \tag{3.5}$$

From (3.3), (3.4) and (3.5), we infer that

$$\mu(X_{n+1}) \leq \alpha(\mu(X_n), \mu(X_{n+1}))\mu(X_{n+1}) < \kappa(\mu(X_n)), \text{ for all } n \in \mathbb{N}. \tag{3.6}$$

Since $\kappa : [0, \infty) \to \mathbb{R}_+$ is nondecreasing, we can get

$$\mu(X_{n+1}) \leq \kappa(\mu(X_n)) \leq \kappa(\kappa(\mu(X_{n-1}))) \leq \cdots \leq \kappa^n(\mu(X_0)). \tag{3.7}$$

In (3.7), letting $n \to \infty$, we deduce

$$\lim_{n\to\infty} \mu(X_{n+1}) = 0.$$

Since $\{X_n\}$ is a nested sequence, in view of (5) of Definition 1.1, we conclude that $X_\infty = \cap_{n=1}^{\infty} X_n$ is a nonempty, closed and convex subset of Ω. Moreover, we know that X_∞ belongs to kerμ. So, X_n is compact and invariant under the mapping T. Consequently, Theorem 1.2 implies that T has a fixed point in X_∞. Since $X_\infty \subseteq \Omega$, then the proof is complete.

□

Corollary 3.5 ([3, Theorem 3.1])**.** *Let Ω be a nonempty, bounded, closed and convex subset of a Banach space E and $T : \Omega \to \Omega$ be a continuous operator. If T is a N_μ-contraction with respect to $\xi \in Z$, then T has at least one fixed point in Ω.*

Proof. In Theorem 3.4, take $\alpha(x, y) = 1$. □

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