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CHARACTERIZATION OF FUZZY COMPLETE NORMED SPACE AND FUZZY B-COMPLETE SET

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Abstract. The present paper introduces the notion of the complete fuzzy norm on a linear space. And, some relations between the fuzzy completeness and ordinary completeness on a linear space is considered, moreover a new form of fuzzy compact spaces, namely b-compact spaces and b-closed spaces are introduced. Some characterizations of their properties are obtained.

1. Introduction

The notions of fuzzy vector spaces and fuzzy topological vector spaces were introduced in Katsaras and Liu [4]. These ideas were modified by Katsaras [2], and in [3] Katsaras defined the fuzzy norm on a vector space. In [5] Krishna and Sarma discussed the generation of a fuzzy vector topology from an ordinary vector topology on vector spaces. Also Krishna and Sarma [6] observed the convergence of sequence of fuzzy points. Rhie et al. [9] introduced the notion of fuzzy α-Cauchy sequence of fuzzy points and fuzzy completeness. Throughout the paper X and Y mean fuzzy topological spaces (fts). The notions Cl (A) will stand for the fuzzy closure of a fuzzy set A in a fts X. Support of a fuzzy set A in X will be denoted by S (A). The fuzzy sets in X taking on respectively the constant value 0 and 1 are denoted by 0_x and 1_x respectively. In this paper, a type of the convergence of sequences as an analogy of Bag and Samanta [1] in a fuzzy normed linear space was observed. and, the notion of a complete fuzzy norm, using the convergence of a sequence of a linear space was introduced. Some relations between the fuzzy completeness and the ordinary completeness on a linear space was introduced. Finally, fuzzy b-compact space was introduced.

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Definition 1.1. [4] For two fuzzy subsets μ_1 and μ_2 of X, the fuzzy subset $\mu_1 + \mu_2$ is defined by

$$(\mu_1 + \mu_2)(x) = \vee \{\mu_1(x_1) \wedge \mu_2(x_2) \,|x = x_1 + x_2\}.$$

And for a scalar t of K and a fuzzy subset μ of X, the fuzzy subset $t\mu$ is defined by

$$(t\mu)(x) = \begin{cases} \mu({}^{x}\!/_{t}) & \text{if} \quad t \neq 0 \\ 0 & \text{if} \quad t \neq 0 \quad \text{and} \quad x \neq 0 \\ \vee\{\mu(y)|\, y \in X\} & \text{if} \quad t \neq 0 \quad \text{and} \quad x = 0 \end{cases}$$

Definition 1.2. [2] $\mu \in I^x$ is said to be

1. Convex if $t\mu + (1-t)\,\mu \subseteq \mu$ for each $t \in [0,1]$,
2. Balanced if $t\mu \subseteq \mu$ for each $t \in K$ with $|t| \leq 1$,
3. Absorbing if $\vee \{t\mu(x) \,|t > 0\} = 1$ for all $x \in X$.

Definition 1.3. [2] Let (X, τ) be a topological space moreover

$$\omega(\tau) = \{f : (X, \tau) \to [0,1] \,|\, \text{f is lower semicontinuous}\}$$

then $\omega(\tau)$ is a fuzzy topology on X. This topology is called the fuzzy topology generated by τ on X. The fuzzy usual topology on K means the fuzzy topology generated by the usual topology of K.

Definition 1.4. [2] A fuzzy linear topology on a vector space Xover K is a fuzzy topology on X such that the two mappings

$$+ : X \times X \to X, \quad (x, y) \to x + y$$
$$\cdot : K \times K \to X, \quad (t, y) \to tx$$

are continuous when K has the fuzzy usual topology and $K \times X$ and $X \times X$ have the corresponding product fuzzy topologies. A linear space with a fuzzy linear topology is called a fuzzy topological linear space or a fuzzy topological vector space.

Definition 1.5. [2] Let x be a point in a fuzzy topological space X. A family F of neighborhood of x is called a base for the system of all neighborhoods of x if for each neighborhood μ of x and each $0 < \theta < \mu(x)$, there exists $\mu_1 \in F$ with $\mu_1 \leq \mu$ and $\mu_1(x) > \theta$.

Definition 1.6. [3] A fuzzy semi norm on X is a fuzzy set ρ in X which is convex, balanced and absorbing. If in addition $\wedge \{(t_\rho)(x) \,|t > 0\}$ for $x \neq 0$, then ρ is called a fuzzy norm.

Definition 1.7. [3] If ρ is a fuzzy semi norm on X, then the family $B_\rho = \{\theta \wedge (t_\rho) \,|0 < \theta \leq 1, t > 0\}$ is a base at zero for a fuzzy linear topology τ_ρ . The fuzzy topology τ_ρ is called the fuzzy topology induced by the fuzzy semi norm ρ. And a linear space equipped with a fuzzy semi norm is called a fuzzy semi normed linear space.

Definition 1.8. [5] Let ρ be a fuzzy semi norm on X. $P_\varepsilon : X \to R_+$ is defined by

$$P_\varepsilon (x) = \wedge \{t > 0 | t\rho (x) > \varepsilon\}$$

for each $\varepsilon \in (0,1)$.

Theorem 1.9. [5] *The P_ε is a semi norm on X for each $\varepsilon \in (0,1)$. Further P_ε is norm on X for each $\varepsilon \in (0,1)$ if and only if ρ is a fuzzy norm on X.*

Definition 1.10. [3] Two fuzzy semi norms ρ_1, ρ_2 on X are said to be equivalent iff $\tau_{\rho_1} = \tau_{\rho_2}$.

Theorem 1.11. [9] *Let $(X, \| \,.\, \|)$ be a normed linear space. If ρ is a lower semi continuous fuzzy norm on X and has the bounded support $\{x \in X | \rho (x) > 0\}$ is bounded, then ρ is equivalent to the fuzzy norm χ_B where B is the closed unit ball of X.*

2. Fuzzy convergence and fuzzy completeness

Definition 2.1. Let (X, ρ) be a fuzzy normed linear space. A sequence $\{x_n\} \subset X$ is said to be convergent to $x \in X$ if for every $t > 0$ and $\varepsilon \in (0,1)$, there exists a positive integer M such that $n \geq M$ implies

$$t_\rho (x_n - x) > 1 - \varepsilon.$$

Theorem 2.2. *Let (X, ρ) be a fuzzy normed linear space. A sequence $\{x_n\} \subset X$ converges to $x \in X$ iff for every $t > 0$ and $\varepsilon \in (0,1)$, there exists a positive integer M such that $n \geq M$ implies $P_{1-\varepsilon} (x_n - x)$.*

Proof. Let $t > 0$ and $\varepsilon \in (0,1)$ be given. Since $\{x_n\}$ converges to x, there exists a positive integer M such that

$$n \geq M \text{ implies } \frac{t}{2}\rho (x_n - x) > 1 - \varepsilon,$$

it follows that

$$n \geq M \text{ implies } P_{1-\varepsilon} (x_n - x) \leq \frac{t}{2} < t.$$

For the converse, let $t > 0$ and $\varepsilon > 0$ be given. Then there exists a positive integer M such that

$$n, m \geq M \text{ implies } P_{1-\varepsilon} (x_n - x_m) < t,$$

it follows that

$$\begin{aligned} n, m \geq M \text{ implies } & t_\rho' (x_n - x_m) \\ & > 1 - \varepsilon \text{ for some } t' \in (P_{1-\varepsilon} (x_n - x_m), t), \end{aligned}$$

so we have

$$\begin{aligned} n,m \geq M \text{ implies } & t_\rho (x_n - x_m) \\ & \geq t_\rho{}' (x_n - x_m) \\ & > 1 - \varepsilon. \end{aligned}$$

This completes the proof.

□

Definition 2.3. Let (X, ρ) be a fuzzy normed linear space. A sequence $\{x_n\} \subset X$ is a Cauchy sequence iff for every $t > 0$ and $\varepsilon \in (0, 1)$, there exists a positive integer M such that $n, m \geq M$ implies

$$t_\rho (x_n - x_m) < 1 - \varepsilon.$$

Theorem 2.4. *Let (X, ρ) be a fuzzy normed linear space. A sequence $\{x_n\} \subset X$ is a Cauchy sequence iff for every $t > 0$ and $\varepsilon \in (0, 1)$, there exists a positive integer M such that $n, m \geq M$ implies*

$$P_{1-\varepsilon} (x_n - x_m) < t.$$

Proof. The proof is similar to the proof of Theorem 2.1 □

The following theorem is easily verified with elementary skills from Theorem 2.1 and Theorem 2.2.

Definition 2.5. A fuzzy norm ρ on a linear space X is said to be fuzzy complete if every Cauchy sequence in X converges to a point in X.

Lemma 2.6. *Let $(X, \| \, . \, \|)$ be a normed linear space and B the closed unit ball of X. Then every Cauchy sequence in the fuzzy normed linear space (X, χ_B) is a Cauchy sequence with respect to the ordinary norm.*

Proof. Let $\{x_n\} \subset X$ be a Cauchy sequence and $\delta > 0$. Since $\{x_n\}$ is a Cauchy sequence, for this δ and for every $\varepsilon \in (0, 1)$, there exists a positive integer M such that

$$n, m \geq M \text{ implies } \frac{\delta}{2} \chi_B (x_n - x_m) > 1 - \varepsilon,$$

so we have

$$n, m \geq M \text{ implies } \chi_B \left(\frac{\delta}{2} (x_n - x_m) \right) > 1 - \varepsilon,$$

it follows that

$$n, m \geq M \text{ implies } \chi_B \left(\frac{\delta}{2} (x_n - x_m) \right) = 1,$$

hence

$$n, m \geq M \text{ implies } \|x_n - x_m\| \leq \frac{\delta}{2}.$$

Therefore $\{x_n\}$ is Cauchy sequence in $(X, \|\ .\ \|)$. This proves the lemma. □

Theorem 2.7. *Let $(X, \|\ .\ \|)$ be a Banach space. Then the fuzzy normed linear space (X, χ_B) is fuzzy complete where B is the closed unit ball of X.*

Proof. Let $\{x_n\}$ be a Cauchy sequence in (X, χ_B). Then it is a Cauchy sequence with respect to the ordinary norm $\|\ .\ \|$ by the above lemma. Since $(X, \|\ .\ \|)$ is complete, there exists an $x \in X$ such that

$$\|x_n - x_m\| \to 0.$$

Now, it is shown that $\{x_n\}$ converges to this x in (X, χ_B). Let $t > 0$ and $\varepsilon \in (0, 1)$. Then there exists a positive integer M such that

$$n \geq m \text{ implies } \|x_n - x\| < t,$$

it follows that

$$n \geq m \text{ implies } \left\|\frac{1}{t}(x_n - x)\right\| < t,$$

so we have

$$n \geq m \text{ implies } \chi_B\left(\frac{1}{t}(x_n - x)\right) = 1,$$

hence

$$n \geq m \text{ implies } \mathrm{t}\chi_B(x_n - x) > 1 - \varepsilon.$$

That is $\{x_n\}$ converges to x, therefor (X, χ_B) is fuzzy complete. This completes the proof. □

3. Fuzzy b-closed Spaces

A fts X is said to be fuzzy b-closed iff for every family λ of fuzzy b-open set such that $\underset{A\in\lambda}{\vee} A = 1_x$ there is a finite subfamily $\delta \subseteq \lambda$ such that $\left(\underset{A\in\delta}{\vee} bCl(A)\right)(x) = 1_x$, for every $x \in X$.

Definition 3.1. A fuzzy set U in a fts X is said to be fuzzy b-closed relative to X iff for every family λ of fuzzy b-open set such that

$$\underset{A\in\lambda}{\vee} A = 1_x$$

there is a finite subfamily $\delta \subseteq \lambda$ such that $\underset{A\in\delta}{\vee} bCl(A)(x) = U(x)$, for every $x \in S(U)$.

Remark 3.2. Every fuzzy b-compact space is fuzzy b-closed, but the converse is not true.

Theorem 3.3. *A fts X is fuzzy b-closed iff for every fuzzy filterbases Γ in X, $\left(\underset{G\in\Gamma}{\wedge} bCl\left(G\right)\right) \neq 0_x$.*

Proof. Let μ be a fuzzy b-open set cover of X and let for every finite family of μ, $\underset{A\in\partial}{\vee} bCl\left(A\right)\left(x\right) < 1_x$ for some $x \in X$. Then

$$\left(\underset{A\in\partial}{\wedge} \overline{bCl\left(G\right)}\right)\left(x\right) > 0_x$$

for some $x \in X$. Thus $\left\{\left(\overline{bCl\left(A\right)} : A \in \mu\right)\right\} = \Gamma$ forms a fuzzy b-open filterbases in X . Since μ is a fuzzy b-open set cover of X, then $\left(\underset{A\in\mu}{\wedge} A\right) = 0_x$, which implies $\left(\underset{A\in\mu}{\wedge} bCl\left(\overline{bCl\left(G\right)}\right)\right)\left(x\right) = 0_x$, which is a contradiction. Then every fuzzy b-open μ of X has a finite subfamily ∂ such that $\left(\underset{A\in\partial}{\vee} bCl\left(A\right)\left(x\right)\right) = 1_x$ for every $x \in X$. Hence X is a fuzzy b-closed.

Conversely, suppose there exists a fuzzy b-open filterbases Γ in X such that $\left(\underset{G\in\Gamma}{\wedge} bCl\left(G\right)\right) = 0_x$. That implies $\left(\underset{G\in\Gamma}{\vee} \left(\overline{bCl\left(G\right)}\right)\right)\left(x\right) = 1_x$ for $x \in X$ and hence $\mu = \left\{\overline{\left(bCl\left(G\right)\right)} : G \in \Gamma\right\}$ is a fuzzy b-open set cover of X . Since X is fuzzy b-closed, by definition μ has a finite subfamily ∂ such that $\left(\underset{G\in\partial}{\vee} bCl\left(\overline{bCl\left(G\right)}\right)\right)\left(x\right) = 1_x$ for every $x \in X$, and hence $\underset{\lambda\in\partial}{\wedge} \left(\overline{bCl\left(\overline{bCl\left(G\right)}\right)}\right) = 0_x$. Thus $\underset{G\in\partial}{\wedge} G = 0_x$ is a contradiction. Hence

$$\underset{G\in\Gamma}{\wedge} bCl\left(G\right) \neq 0_x.$$

□

Theorem 3.4. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a fuzzy b^*-continuous surjection. If X is fuzzy b-closed space, then Y is fuzzy b-closed space.*

Proof. Let $\{A_\lambda : \lambda \in \Lambda\}$ be a fuzzy b-open cover of Y. Since f is fuzzy b^*-continuous, $\left\{f^{-1}\left(A_\lambda\right) : \lambda \in \Lambda\right\}$ is fuzzy b-open cover of X .

By hypothesis, there exists a finite subset Δ of Γ such that

$$\underset{\lambda\in\Delta}{\vee} bCl\left(f^{-1}\left(A_\lambda\right)\right) = 1_x.$$

Since f is surjection and by theorem we have,

$$\begin{aligned}1_Y &= f(1_x)\\ &= f\left(\underset{\lambda\in\Delta}{\vee} bCl\left(f^{-1}(A_\lambda)\right)\right)\\ &\leq \underset{\lambda\in\Delta}{\vee} bCl\left(f\left(f^{-1}(A_\lambda) = \underset{\lambda\in\Delta}{\vee} bCl(A_\lambda)\right)\right).\end{aligned}$$

Hence Y is fuzzy b-closed space. □

4. Fuzzy b-Compact Spaces

Definition 4.1. A fts X is said to be fuzzy b-compact iff for every family μ of fuzzy b-open fuzzy sets such that $\underset{A\in\delta}{\vee} A = 1_x$ There is a finite subfamily $\delta \subseteq \mu$ such that $\underset{A\in\delta}{\vee} A = 1_x$ for every $x \in S(u)$.

Definition 4.2. A fuzzy set U in a fts X is said to be fuzzy b-compact relative to X iff for every family μ of fuzzy b-open sets such that $\underset{A\in\mu}{\vee} A \geq U(x)$ there is a finite subfamily $\delta \subseteq \mu$ such that $\underset{A\in\delta}{\vee} A \geq U(x)$ for every $x \in S(U)$.

Theorem 4.3. *A fts X is b-compact iff for every collection $\{A_\lambda : \lambda \in \Lambda\}$ of fuzzy b-closed sets of X having the finite intersection property,*

$$\underset{\lambda\in}{\wedge} A_\lambda \neq 0_x.$$

Proof. Let $\{A_\lambda : \lambda \in \Lambda\}$ be a collection of fuzzy b-closed sets with the finite intersection property. Suppose that $\underset{\lambda\in\Lambda}{\wedge} A_\lambda = 0_x$. Then $\underset{\lambda\in\Lambda}{\vee} \left(\overline{A_\lambda}\right) = 1_x$ since $\left\{\overline{A_\lambda} : \lambda \in \Lambda\right\}$ is a collection of fuzzy b-compactness of X, it follows that there exists a finite subset $\Delta \subseteq \Lambda$ such that $\underset{\lambda\in\Delta}{\vee} \overline{A_\lambda} = 1_x$. Then $\underset{\lambda\in\Delta}{\wedge} A_\lambda = 0_x$, which gives a contradiction. Therefore $\underset{\lambda\in\Lambda}{\wedge} A_\lambda \neq 0_x$.

Conversely, let $\{A_\lambda : \lambda \in \Lambda\}$ be a collection of fuzzy b-open sets covering X. Suppose that for every finite subset $\Delta \subseteq \Lambda$, we have $\underset{\lambda\in\Delta}{\vee} A_\lambda \neq 1_x$. Then $\underset{\lambda\in\Delta}{\wedge} \left(\overline{A_\lambda}\right) \neq 0_x$. Hence $\left\{\overline{A_\lambda} : \lambda \in \Lambda\right\}$ satisfies the finite intersection property. Then by definition we have $\underset{\lambda\in\Lambda}{\wedge} \left(\overline{A_\lambda}\right) \neq 0_x$ which implies $\underset{\lambda\in\Delta}{\vee} \left(\overline{A_\lambda}\right) \neq 1_x$ and this contradicts that $\{A_\lambda : \lambda \in \Lambda\}$ is a fuzzy b-cover of X. Thus X is fuzzy b-compact. □

The following characterization on b-compactness is in term of fuzzy filterbases.

Theorem 4.4. *A fts is fuzzy b-compact if and only if every filterbases* Γ *in* X, $\underset{G\in\Gamma}{\wedge} bClG \neq 0_x$.

Proof. Let μ be a fuzzy b-open cover which has no finite sub-cover in X. Then for every finite subcollection of $\{A_1, ..., A_n\}$ of μ, there exists $x \in X$ such that $A_\lambda(x) < 1$ for every $\lambda = 1, ..., n$. Then $\overline{A_\lambda} > 0$, so that $\overset{n}{\underset{\lambda=1}{\wedge}} \overline{A_\lambda}(x) \neq 0_x$. Thus $\left\{\overline{A_\lambda}(x) : A_\lambda \in \mu\right\}$ forms a filterbase in X. Since μ is fuzzy b-open set cover of X, then $\left(\underset{A_\lambda\in\mu}{\vee} A_\lambda\right)(x) = 1_x$ for every $x \in X$ and hence $\underset{A_\lambda\in\mu}{\wedge} bCl\overline{A_\lambda}(x) = 0_x$, which is a contradiction. Then every fuzzy b-open set cover of has a finite subcover and hence is fuzzy b-compact.

Conversely, suppose that there exists a filterbases Γ in X, such that

$$\underset{G\in\Gamma}{\wedge} bCl(G) = 0_x,$$

so that

$$\left(\underset{G\in\Gamma}{\vee} \overline{(bCl(G))}\right)(x) = 1_x,$$

for every and hence $\mu = \left\{\overline{bCl(G)} : G \in \Gamma\right\}$ is a fuzzy b-open cover of X. Since X is fuzzy b-compact, by definition Γ has a finite subcover such that $\left(\overset{n}{\underset{\lambda=1}{\vee}} \left(\overline{bCl(G_\lambda)}\right)(x)\right) = 1_x$ and hence $\left(\overset{n}{\underset{\lambda=1}{\vee}} \overline{(G_\lambda)}\right)(x) = 1_x$, so that $\overset{n}{\underset{\lambda=1}{\wedge}} (G_\lambda) = 0_x$, which is a contradiction. Therefore,

$$\underset{G\in\Gamma}{\wedge} bCl(G) \neq 0_x$$

for every filterbases Γ. □

Theorem 4.5. *A fuzzy set* U *in a fts* X *is fuzzy b-compact relative to* X *iff for every filterbase* Γ *such that every finite members of* Γ *is quasi coincident with* U, $\left(\underset{G\in\Gamma}{\wedge} bCl(G)\right) \wedge U \neq 0_x$.

Proof. Suppose U is not fuzzy b-compact relative to X , then there exists a fuzzy b-open set μ covering of U with no finite subcover υ. Then

$$\left(\underset{A_\lambda\in\upsilon}{\vee} A_\lambda(x)\right) < U(x)$$

for some $x \in S(U)$ so that

$$\left(\underset{A_\lambda\in\upsilon}{\wedge} \overline{A_\lambda}\right)(x) > \overline{U}(x) \geq 0_x$$

and hence $\Gamma = \left\{\overline{A_\lambda}(x) : A_\lambda \in \mu\right\}$ forms a filterbases and $\underset{A_\lambda \in \upsilon}{\wedge} \overline{A_\lambda} \text{ q } U$.

By hypothesis

$$\left(\underset{A_\lambda \in \upsilon}{\wedge} bCl\overline{A_\lambda}\right) \wedge U \neq 0_x$$

and hence $\left(\underset{A_\lambda \in \upsilon}{\wedge} \overline{A_\lambda}\right) \wedge U \neq 0_x$. Then for some $x \in S(U)$,

$$\left(\underset{A_\lambda \in \mu}{\wedge} \overline{A_\lambda}\right)(x) > 0_x,$$

that is $\left(\underset{A_\lambda \in \mu}{\vee} A_\lambda\right)(x) < 1_x$ which is a contradiction. Hence U is a fuzzy b-compact relative to X.

Conversely, suppose that there exists a filterbases Γ such that every finite members of Γ is quasi-coincident with U and

$$\left(\underset{G \in \Gamma}{\wedge} bCl(G)\right) \wedge U \neq 0_x.$$

Then for every $x \in S(U)$, $\left(\underset{G \in \Gamma}{\wedge} bCl(G)\right)(x) = 0_x$ and hence

$$\underset{G \in \Gamma}{\vee} \overline{bCl(G)}(x) = 1_x$$

for every $x \in S(U)$. Thus $\mu = \left\{\overline{bCl(G)} : G \in \Gamma\right\}$ is a fuzzy b-open set cover of U. Since is fuzzy b-compact relative to X, there exists a finite subcover, say $\left\{\left(\overline{bCl(G_\lambda)}\right) : \lambda = 1, ...n\right\}$ such that

$$\left(\overset{n}{\underset{\lambda=1}{\wedge}} \overline{bCl(G_\lambda)}\right)(x) \geq U(x)$$

for every $x \in S(U)$.

Hence $\left(\overset{n}{\underset{\lambda=1}{\wedge}} bCl(G_\lambda)\right)(x) \leq \overline{U}(x)$ for every $x \in S(U)$. So that $\overset{n}{\underset{\lambda=1}{\wedge}} bCl(G_\lambda) \ \overline{\text{q}}\ U$, which is a contradiction. Therefore, for every filterbases Γ, every finite member of Γ is quasi coincident with U. Hence

$$\left(\underset{G \in \Gamma}{\wedge} bCl(G)\right) \wedge u \neq 0_x.$$

□

The following theorem proves the hereditary property for fuzzy b-compact spaces.

Theorem 4.6. *Every fuzzy b-closed subset of a fuzzy b-compact spaces is fuzzy b-compact relative to X.*

Proof. Suppose Γ be a fuzzy filter bases, Δ be its finite subcollection in X and for a fuzzy b-closed set U let U q $\wedge \{G \in \Gamma\}$. Let $\Gamma^* = \{U\} \cup \Gamma$. For any finite subcollection Δ^* of Γ^*, if $U \notin \Delta^*$ then $\wedge\Delta^* \neq 0_x$. If $U \in \Delta^*$and U q $\wedge \{G : G \in \Delta^* - U\}$, then $\wedge\Delta^* \neq 0_x$. Hence Δ^* is a fuzzy filterbases in X. Since is fuzzy b-compact, then

$$\left(\underset{G\in\Gamma}{\wedge} bCl\left(G\right)\right) \neq 0_x,$$

such that

$$\left(\underset{G\in\Gamma}{\wedge} bCl\left(G\right)\right) \underset{G\in\Gamma}{\wedge} U = \left(\underset{G\in\Gamma}{\wedge} bCl\left(G\right)\right) \wedge bCl\left(U\right) \neq 0_x.$$

Hence, by Theorem 3.3 is fuzzy b-compact relative to X. □

In the following theorem it shows that image of a fuzzy b-compact space under a fuzzy b*-continuous mapping is fuzzy b-compact.

Theorem 4.7. *If a function $f : (X, \tau) \to (Y, \sigma)$ is fuzzy b*-continuous and U is fuzzy b-compact relative to X, then $f(U)$ is fuzzy b-compact.*

Proof. Let $\{A_\lambda : \lambda \in \Lambda\}$ be a fuzzy b-open cover of $S(f(u))$ in Y. For

$$x \in S\left(U\right), \quad f\left(x\right) \in f\left(S\left(U\right)\right).$$

Since f is fuzzy b*-continuous, $\left\{f^{-1}\left(A_\lambda\right) : \lambda \in \Lambda\right\}$ is fuzzy b-open cover of $S(U)$ in X. Since U is fuzzy b-compact relative to X, there is a finite subfamily $\left\{f^{-1}\left(A_\lambda\right) : \lambda = 1, ..., n\right\}$ such that

$$\begin{aligned} S\left(U\right) &\leq \overset{n}{\underset{\lambda=1}{\vee}} f^{-1}\left(A_\lambda\right) \\ &= f^{-1}\left(\overset{n}{\underset{\lambda=1}{\vee}} A_\lambda\right). \end{aligned}$$

Hence

$$\begin{aligned} S\left(f\left(U\right)\right) &= f\left(S\left(U\right)\right) \\ &\leq f\left(f^{-1}\left(\overset{n}{\underset{\lambda=1}{\vee}} A_\lambda\right)\right) \\ &\leq \overset{n}{\underset{\lambda=1}{\vee}} A_\lambda. \end{aligned}$$

Therefore $f(U)$ is λ fuzzy b-compact relative to Y. □

The pre image of fuzzy b-compact space under fuzzy b*-open bijective mapping is fuzzy b-compact.

Theorem 4.8. *If a function $f : (X, \tau) \to (Y, \sigma)$ is fuzzy b*-open bijective mapping and Y be fuzzy b-compact, then X fuzzy b-compact.*

Proof. Let $\{A_\lambda : \lambda \in \Lambda\}$ be a family of fuzzy b-open covering of X. Then let $\{f(A_\lambda) : \lambda \in \Lambda\}$ be a fuzzy b-open cover is a family of fuzzy b-open sets covering Y. Since Y is fuzzy b-compact, by definition there exists a finite family $\Delta \subseteq \Lambda$ such that $\{f(A_\lambda) : \lambda \in \Delta\}$ covers Y. Also, since f is bijective we have

$$\begin{aligned} 1_x &= f^{-1}(1_Y) \\ &= f^{-1} f\left(\bigvee_{\lambda\in\Delta} A_\lambda\right) \\ &= \bigvee_{\lambda\in\Delta} A_\lambda. \end{aligned}$$

Thus X is fuzzy b-compact. □

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