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ON RARELY GENERALIZED REGULAR FUZZY CONTINUOUS FUNCTIONS IN FUZZY TOPOLOGICAL SPACES

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Abstract. In this paper, we introduce the concept of rarely generalized regular fuzzy continuous functions in the sense of A.P. Sostak's and Ramadan is introduced. Some interesting properties and characterizations of them are investigated. Also, some applications to fuzzy compact spaces are established.

1. Introduction and preliminaries

The concept of fuzzy set was introduced by Zadeh [18] in his classical paper. Fuzzy sets have applications in many fields such as information [12] and control [13]. Chang [2] introduced the notion of a fuzzy topology. Later Lowen [9] redefined what is now known as stratified fuzzy topology. Šostak [14] introduced the notion of fuzzy topology as an extension of Chang an Lowen's fuzzy topology. Later on he has developed the theory of fuzzy topological spaces in [15] and [16]. Popa [10] introduced the notion of rarely continuity as a generalization of weak continuity [7] which has been further investigated by Long and Herrington [8] and Jafari [4] and [5]. Recently Vadivel et al. [17] introduced the concept of generalized regular fuzzy closed sets in fuzzy topological spaces in the sense of Šostak's.

In this paper, we introduce the concept of rarely generalized regular fuzzy continuous functions in the sense of A. P. Sostak's [14] and Ramadan [11] is introduced. Some intresting properties and characterizations of them are investigated. Also, some applications to fuzzy compact spaces are established.

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Throughout this paper, let X be a nonempty set, $I = [0,\ 1]$ and $I_0 = (0,\ 1]$. For $\lambda \in I^X$, $\overline{\lambda}(x) = \lambda$ for all $x \in X$. For $x \in X$ and $t \in I_0$, a fuzzy point x_t is defined by

$$x_t(y) = \begin{cases} t & \text{if } y = x \\ 0 & \text{if } y \neq x. \end{cases}$$

Let $Pt(X)$ be the family of all fuzzy points in X. A fuzzy point $x_t \in \lambda$ iff $t < \lambda(x)$. All other notations and definitions are standard, for all in the fuzzy set theory.

Definition 1.1 ([14])**.** A function $\tau : I^X \to I$ is called a fuzzy topology on X if it satisfies the following conditions:

(O1) $\tau(\overline{0}) = \tau(\overline{1}) = 1$,
(O2) $\tau(\bigvee_{i\in\Gamma} \mu_i) \geq \bigwedge_{i\in\Gamma} \tau(\mu_i)$, for any $\{\mu_i\}_{i\in\Gamma} \subset I^X$,
(O3) $\tau(\mu_1 \wedge \mu_2) \geq \tau(\mu_1) \wedge \tau(\mu_2)$, for any $\mu_1,\ \mu_2 \in I^X$.

The pair $(X,\ \tau)$ is called a fuzzy topological space (for short, fts). A fuzzy set λ is called an r-fuzzy open (r-fo, for short) if $\tau(\lambda) \geq r$. A fuzzy set λ is called an r-fuzzy closed (r-fc, for short) set iff $\overline{1} - \lambda$ is an r-fo set.

Theorem 1.2 ([3])**.** *Let (X, τ) be a fts. Then for each $\lambda \in I^X$ and $r \in I_0$, we define an operator $C_\tau : I^X \times I_0 \to I^X$ as follows: $C_\tau(\lambda,\ r) = \bigwedge\{\mu \in I^X : \lambda \leq \mu,\ \tau(\overline{1} - \mu) \geq r\}$. For $\lambda,\ \mu \in I^X$ and $r, s \in I_0$, the operator C_τ satisfies the following statements:*

(C1) $C_\tau(\overline{0}, r) = \overline{0}$,
(C2) $\lambda \leq C_\tau(\lambda,\ r)$,
(C3) $C_\tau(\lambda,\ r) \vee C_\tau(\mu,\ r) = C_\tau(\lambda \vee \mu,\ r)$,
(C4) $C_\tau(\lambda,\ r) \leq C_\tau(\lambda,\ s)$ *if* $r \leq s$,
(C5) $C_\tau(C_\tau(\lambda,\ r),\ r) = C_\tau(\lambda,\ r)$.

Theorem 1.3 ([3])**.** *Let (X, τ) be a fts. Then for each $\lambda \in I^X$ and $r \in I_0$, we define an operator $I_\tau : I^X \times I_0 \to I^X$ as follows: $I_\tau(\lambda,\ r) = \bigvee\{\mu \in I^X : \mu \leq \lambda,\ \tau(\mu) \geq r\}$. For $\lambda,\ \mu \in I^X$ and $r, s \in I_0$, the operator I_τ satisfies the following statements:*

(I1) $I_\tau(\overline{1}, r) = \overline{1}$,
(I2) $I_\tau(\lambda,\ r) \leq \lambda$,
(I3) $I_\tau(\lambda,\ r) \wedge I_\tau(\mu,\ r) = I_\tau(\lambda \wedge \mu,\ r)$,
(I4) $I_\tau(\lambda,\ r) \leq I_\tau(\lambda,\ s)$ *if* $s \leq r$,
(I5) $I_\tau(I_\tau(\lambda,\ r),\ r) = I_\tau(\lambda,\ r)$,
(I6) $I_\tau(\overline{1} - \lambda, r) = \overline{1} - C_\tau(\lambda, r)$ *and* $C_\tau(\overline{1} - \lambda, r) = \overline{1} - I_\tau(\lambda,\ r)$.

Definition 1.4 ([6])**.** Let $(X,\ \tau)$ be a fts, $\lambda \in I^X$ and $r \in I_0$. Then

(1) a fuzzy set λ is called r-fuzzy regular open (for short, r-fro) if $\lambda = I_\tau(C_\tau(\lambda, r), r)$.
(2) a fuzzy set λ is called r-fuzzy regular closed (for short, r-frc) if $\lambda = C_\tau(I_\tau(\lambda, r), r)$.

Definition 1.5 ([17])**.** Let $f : (X,\ \tau) \to (Y,\ \sigma)$ be a function and $r \in I_0$. Then f is called:

(1) fuzzy regular continuous (for short, fr-continuous) if $f^{-1}(\lambda)$ is a r-fro set in I^X for each $\lambda \in I^Y$ with $\sigma(\lambda) \geq r$.
(2) fuzzy regular open (for short, fr-open) if $f(\lambda)$ is a r-fro set in I^Y for each $\lambda \in I^X$ with $\tau(\lambda) \geq r$.
(3) fuzzy regular closed (for short, fr-closed) if $f(\lambda)$ is a r-frc set in I^Y for each $\lambda \in I^X$ with $\tau(\overline{1} - \lambda) \geq r$.

Definition 1.6 ([17])**.** Let $(X,\ \tau)$ be a fts. Let $\lambda, \mu \in I^X$ and $r \in I_0$.

(1) The r-fuzzy regular closure of λ, denoted by $RC_\tau(\lambda, r)$, and is defined by $RC_\tau(\lambda, r) = \bigwedge\{\mu \in I^X | \mu \geq \lambda, \mu \text{ is } r\text{-frc }\}$.
(2) The r-fuzzy regular interiror of λ, denoted by $RI_\tau(\lambda, r)$, and is defined by $RI_\tau(\lambda, r) = \bigvee\{\mu \in I^X | \mu \leq \lambda, \mu \text{ is } r\text{-fro }\}$.

Definition 1.7 ([17])**.** Let $(X,\ \tau)$ be a fts. Let $\lambda,\ \mu \in I^X$ and $r \in I_0$,

(1) Fuzzy set λ is called r-generalized regular fuzzy closed (for short, r-grfc) set if $RC_\tau(\lambda, r) \leq \mu$, whenever $\lambda \leq \mu$ and $\tau(\mu) \geq r$.
(2) Fuzzy set λ is called r-generalized regular fuzzy open (for short, r-grfo) set if $\overline{1} - \lambda$ is r-grfc.

Definition 1.8 ([17])**.** Let $(X,\ \tau)$ and $(Y,\ \eta)$ be a fts's. Let $f : (X,\ \tau) \to (Y,\ \eta)$ be a function.

(1) f is called generalized regular fuzzy continuous (for short, grf-continuous) iff $f^{-1}(\mu)$ is r-grfc for each $\mu \in I^Y,\ r \in I_0$ with $\eta(\overline{1} - \mu) \geq r$.
(2) f is called generalized regular fuzzy open (for short, grf-open) iff $f(\lambda)$ is r-grfo for each $\lambda \in I^X,\ r \in I_0$ with $\tau(\lambda) \geq r$.
(3) f is called generalized regular fuzzy closed (for short, grf-closed) iff $f(\lambda)$ is r-grfc for each $\lambda \in I^X,\ r \in I_0$ with $\tau(\overline{1} - \lambda) \geq r$.

Definition 1.9 ([1])**.** Let (X, τ) be a fts and $r \in I_0$. For $\lambda \in I^X$, λ is called a r-fuzzy rare set if $I_\tau(\lambda, r) = \overline{0}$.

Definition 1.10 ([1])**.** Let $(X,\ \tau)$ and $(Y,\ \eta)$ be two fts's. Let $f : (X,\ \tau) \to (Y,\ \eta)$ be a function. Then f is called

(1) weakly continuous if for each $\mu \in I^Y$, where $\sigma(\mu) \geq r,\ r \in I_0$,
$$f^{-1}(\mu) \leq I_\tau(f^{-1}(C_\sigma(\mu, r)), r).$$

(2) rarely continuous if for each $\mu \in I^Y$, where $\sigma(\mu) \geq r,\ r \in I_0$, there exists a r-fuzzy rare set $\lambda \in I^Y$ with $\mu + C_\sigma(\lambda, r) \geq 1$ and $\rho \in I^X$, where $\tau(\rho) \geq r$ such that $f(\rho) \leq \mu \vee \lambda$.

Proposition 1.11 ([1]). *Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be any two fts's, $r \in I_0$ and $f : (X,\ \tau) \to (Y,\ \sigma)$ be fuzzy open and one-to-one, then f preserves r-fuzzy rare sets.*

2. Rarely generalized regular fuzzy continuous functions

Definition 2.1. Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be two fts's, and $f : (X,\ \tau) \to (Y,\ \eta)$ be a function. Then f is called

(1) rarely fuzzy regular continuous (for short, rarely fr-continuous) if for each $\mu \in I^Y$, where $\sigma(\mu) \geq r,\ r \in I_0$, there exists a r-fuzzy rare set $\lambda \in I^Y$ with $\mu + C_\sigma(\lambda, r) \geq 1$ and a r-fro set $\rho \in I^X$ such that $f(\rho) \leq \mu \vee \lambda$.

(2) rarely generalized regular fuzzy continuous (for short, rarely grf-continuous) if for each $\mu \in I^Y$, where $\sigma(\mu) \geq r,\ r \in I_0$, there exists a r-fuzzy rare set $\lambda \in I^Y$ with $\mu + C_\sigma(\lambda, r) \geq 1$ and a r-grfo set $\rho \in I^X$, such that $f(\rho) \leq \mu \vee \lambda$.

Remark 2.2. (1) Every weakly continuous (resp. fr-continuous) function is rarely continuous [1] (resp. grf-continuous [17]) but converse is not true.

(2) Every rarely fr-continuous (resp. grf-continuous) function is rarely grf-continuous but converse is not true.

(3) Every rarely fr-continuous function is rarely continuous but converse is not true.

From the above definitions and remarks it is not difficult to conclude that the following diagram of implications is true.

$$\begin{tikzcd}
\text{weakly continuous} \arrow[r] & \text{rarely continuous} & \text{rarely fr-continuous} \arrow[l] \arrow[d] \\
\text{fr-continuous} \arrow[r] & \text{grf-continuous} \arrow[r] & \text{rarely grf-continuous}
\end{tikzcd}$$

Example 2.3. Let $X = \{a, b\} = Y$. Define $\lambda_1 \in I^X,\ \lambda_2 \in I^Y$ as follows: $\lambda_1(a) = 0.8,\ \lambda_1(b) = 0.5,\ \lambda_2(a) = 0.8,\ \lambda_2(b) = 0.6$. Define the fuzzy topologies $\tau,\ \sigma : I^X \to I$ as follows:

$$\tau(\lambda) = \begin{cases} 1 & \text{if } \lambda = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{10} & \text{if } \lambda = \lambda_1, \\ 0 & \text{otherwise}, \end{cases} \qquad \sigma(\lambda) = \begin{cases} 1 & \text{if } \lambda = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{10} & \text{if } \lambda = \lambda_2, \\ 0 & \text{otherwise}. \end{cases}$$

Let $r = 1/10$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = a,\ f(b) = a$ and $\gamma \in I^Y$ be a 1/10-fuzzy rare set defined by $\gamma(a) = 0.6,\ \gamma(b) = 0.8$ and a r-grfo set $\lambda \in I^X$ is defined by $\lambda(a) = 0.2,\ \lambda(b) = 0.3,\ f(\lambda) = (0.8,\ 0.8) \leq \lambda_2 \vee \gamma = (0.8,\ 0.8)$. Then f is rarely grf-continuous but not rarely fr-continuous, because $\lambda \in I^X$ is not a r-fro set.

Example 2.4. In Example 2.3 f is rarely continuous but not rarely fr-continuous.

Example 2.5. Let $X = \{a, b\} = Y$. Define $\lambda_1 \in I^X,\ \lambda_2 \in I^Y$ as follows: $\lambda_1(a) = 0.7,\ \lambda_1(b) = 0.9$. Define the fuzzy topologies $\tau,\ \sigma : I^X \to I$ as follows:

$$\tau(\lambda) = \begin{cases} 1 & \text{if } \lambda = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{10} & \text{if } \lambda = \lambda_1, \\ 0 & \text{otherwise,} \end{cases} \qquad \sigma(\lambda) = \begin{cases} 1 & \text{if } \lambda = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{10} & \text{if } \lambda = \lambda_1, \\ 0 & \text{otherwise.} \end{cases}$$

Let $r = 1/10$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = a,\ f(b) = b$ and $\gamma \in I^Y$ be a 1/10-fuzzy rare set defined by $\gamma(a) = 0.7,\ \gamma(b) = 0.7$ and a r-grfo set $\lambda \in I^X$ is defined by $\lambda(a) = 0.2,\ \lambda(b) = 0.1,\ f(\lambda) = (0.7,\ 0.9) \leq \lambda_2 \vee \gamma = (0.7,\ 0.9)$. Then f is rarely grf-continuous but not grf-continuous, because $f^{-1}(\overline{1} - \lambda_1)$ is not a r-grfo set in I^X for each $(\overline{1} - \lambda_1) \in I^Y$ and $\sigma(\overline{1} - \lambda_1) \geq r$.

Definition 2.6. Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be two fts's, and $f : (X,\ \tau) \to (Y,\ \eta)$ be a function. Then f is called weakly generalized regular fuzzy continuous (for short, weakly grf-continuous) if for each r-grfo set $\mu \in I^Y$, $r \in I_0$, $f^{-1}(\mu) \leq I_\tau(f^{-1}(C_\sigma(\mu, r)), r)$.

Definition 2.7 ([17])**.** A fts (X, τ) is said to be a fuzzy $FRT_{1/2}$-space if every r-grfo set $\lambda \in I^X,\ r \in I_0$ is a r-fro set.

Proposition 2.8. *Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be any two fuzzy topological spaces. If $f : (X,\ \tau) \to (Y,\ \sigma)$ is both grf-open and grf-irresolute and $(X,\ \tau)$ is a $FRT_{1/2}$ space, then it is weakly grf-continuous.*

Proof. Let $\lambda \in I^X$ be such that $\tau(\lambda) \geq r$. Since f is grf-open, $f(\lambda) \in I^Y$ is a r-grfo. Also, since f is grf-irresolute, $f^{-1}(f(\lambda)) \in I^X$ is a r-grfo set. Since $(X,\ \tau)$ is $FRT_{1/2}$ space, every r-grfo set is a r-fro set and also every r-fro set is a r-fo set, now, $\tau(f^{-1}(f(\lambda))) \geq r$. Consider

$$f^{-1}(f(\lambda)) \leq f^{-1}(C_\sigma(f(\lambda), r))$$

from which

$$I_\tau\left(f^{-1}(f(\lambda)), r\right) \leq I_\tau\left(f^{-1}(C_\sigma(f(\lambda), r)), r\right).$$

Since

$$\tau\left(f^{-1}(f(\lambda))\right) \geq r, \qquad f^{-1}\left(f(\lambda)\right) \leq I_\tau\left(f^{-1}(C_\sigma(f(\lambda), r)), r\right),$$

thus f is weakly grf-continuous. $\square$

Definition 2.9. Let (X, τ) be a fts. A grf-open cover of (X, τ) is the collection $\{\lambda_i \in I^X,\ \lambda_i \text{ is } r\text{-grfo},\ i \in J\}$ such that $\bigvee_{i\in J} \lambda_i = \overline{1}$.

Definition 2.10. A fts (X, τ) is said to be a grf-compact space if every grf-open cover of (X, τ) has a finite sub cover.

Definition 2.11. A fts (X, τ) is said to be rarely grf-almost compact if for every grf-open cover $\{\lambda_i \in I^X,\ \lambda_i \text{ is } r\text{-grfo},\ i \in J\}$ of (X, τ), there exists a finite subset J_0 of J such that $\bigvee_{i\in J} \lambda_i \vee \rho_i = \overline{1}$ where $\rho_i \in I^X$ are r-fuzzy rare sets.

Proposition 2.12. *Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be any two fts's, $r \in I_0$ and $f : (X,\ \tau) \to (Y,\ \sigma)$ be rarely grf-continuous. If $(X,\ \tau)$ is grf-compact then $(Y,\ \sigma)$ is rarely grf-almost compact.*

Proof. Let $\{\lambda_i \in I^Y,\ i \in J\}$ be the grf-open cover of (Y, σ). Then $\overline{1} = \vee_{i\in J}\lambda_i$. Since f is rarely grf-continuous, there exists a r-fuzzy rare set $\rho_i \in I^Y$ such that $\lambda_i + C_\sigma(\rho_i, r) \geq \overline{1}$ and a r-grfo set $\mu_i \in I^X$ such that $f(\mu_i) \leq \lambda_i \vee \rho_i$. Since (X, τ) is grf-compact, every grf-open cover of (X, τ) has a finite sub cover. Thus $\overline{1} \leq \vee_{i\in J_0}\mu_i$. Hence $\overline{1} = f(\overline{1}) = f(\vee_{i\in J_0}\mu_i) = \vee_{i\in J_0} f(\mu_i) \leq \vee_{i\in J_0}\lambda_i \vee \rho_i$. Therefore (Y, σ) is rarely grf-almost compact. $\square$

Proposition 2.13. *Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be any two fts's, $r \in I_0$ and $f : (X,\ \tau) \to (Y,\ \sigma)$ be rarely continuous. If $(X,\ \tau)$ is grf-compact then $(Y,\ \sigma)$ is rarely grf-almost compact.*

Proof. Since every rarely continuous function is rarely grf-continuous, then proof follows immediatly from the Proposition 2.12. $\square$

Proposition 2.14. *Let $(X,\ \tau)$, $(Y,\ \sigma)$ and $(Z,\ \eta)$ be any fts's and $r \in I_0$. If $f : (X,\ \tau) \to (Y,\ \sigma)$ be rarely grf-continuous, grf-open and $g : (Y,\ \sigma) \to (Z,\ \eta)$ be fuzzy open and one-to-one, then $g \circ f : (X,\ \tau) \to (Z,\ \eta)$ is rarely grf-continuous.*

Proof. Let $\lambda \in I^X$ with $\tau(\lambda) \geq r$. Since f is grf-open, $f(\lambda) \in I^Y$ with $\sigma(f(\lambda)) \geq r$. Since f is rarely grf-continuous and there exists a r-fuzzy rare set $\rho \in I^Y$ with $f(\lambda) + C_\sigma(\rho, r) \geq \overline{1}$ and a r-grfo set $\mu \in I^X$ such that $f(\mu) \leq f(\lambda) \vee \rho$. By the proposition 1.11, $g(\rho) \in I^Z$ is also a r-fuzzy rare set. Since $\rho \in I^Y$ is such that $\rho < \gamma$ for all $\gamma \in I^Y$ with $\sigma(\gamma) \geq r$, and g is injective, it follows that $(g \circ f)(\lambda) + C_\eta(g(\rho), r) \geq \overline{1}$. Then $(g \circ f)(\mu) = g(f(\mu)) \leq g(f(\lambda) \vee \rho) \leq g(f(\lambda)) \vee g(\rho) \leq (g \circ f)(\lambda) \vee g(\rho)$. Hence the result. $\square$

Proposition 2.15. *Let $(X,\ \tau)$, $(Y,\ \sigma)$ and $(Z,\ \eta)$ be any fts's and $r \in I_0$. If $f : (X,\ \tau) \to (Y,\ \sigma)$ be grf-open and onto and $g : (Y,\ \sigma) \to (Z,\ \eta)$*

be a function such that $g \circ f : (X,\ \tau) \to (Z,\ \eta)$ is rarely grf-continuous, then g is rarely grf-continuous.

Proof. Let $\lambda \in I^X$ and $\mu \in I^Y$ be such that $f(\lambda) = \mu$. Let $(g \circ f)(\lambda) = \gamma \in I^Z$ with $\eta(\gamma) \geq r$. Since $(g \circ f)$ is grf-continuous, there exists a rare set $\rho \in I^Z$ with $\gamma + C_\eta(\rho, r) \geq \overline{1}$ and a r-grfo set $\delta \in I^X$ such that $(g \circ f)(\delta) \leq \gamma \vee \rho$. Since f is grf-open, $f(\delta) \in I^Y$ is a r-grfo set. Thus there exists a r-fuzzy rare set $\rho \in I^Z$ with $\gamma + C_\eta(\rho, r) \geq \overline{1}$ and a r-grfo set $f(\delta) \in I^Y$ such that $g(f(\delta)) \leq \gamma \vee \rho$. Hence g is rarely grf-continuous. □

Proposition 2.16. *Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be any two fuzzy topological spaces and $r \in I_0$. If $f : (X,\ \tau) \to (Y,\ \sigma)$ be rarely grf-continuous and $(X,\ \tau)$ be a fuzzy $FRT_{1/2}$-space, then f is rarely fr-continuous.*

Proof. The proof is trivial. □

Definition 2.17. A fts (X, τ) is said to be a rarely grf-T_2-space if for each pair $\lambda,\ \mu \in I^X$ with $\lambda \neq \mu$ there exist r-grfo sets $\rho_1,\ \rho_2 \in I^X$ with $\rho_1 \neq \rho_1$ and a r-fuzzy rare set $\gamma \in I^X$ with $\rho_1 + C_\tau(\gamma, r) \geq \overline{1}$ and $\rho_2 + C_\tau(\gamma, r) \geq \overline{1}$ such that $\lambda \leq \rho_1 \vee \gamma$ and $\mu \leq \rho_2 \vee \gamma$.

Proposition 2.18. *Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be any two fuzzy topological spaces and $r \in I_0$. If $f : (X,\ \tau) \to (Y,\ \sigma)$ be grf-open and injective and $(X,\ \tau)$ be a rarely grf-T_2 space, then $(Y,\ \sigma)$ is also a rarely grf-T_2 space.*

Proof. Let $\lambda,\ \mu \in I^X$ with $\lambda \neq \mu$. Since f is injective, $f(\lambda) \neq f(\mu)$. Since $(X,\ \tau)$ is a rarely grf-T_2-space, there exist r-grfo sets $\rho_1,\ \rho_2 \in I^X$ with $\rho_1 \neq \rho_1$ and a r-fuzzy rare set $\gamma \in I^X$ with $\rho_1 + C_\tau(\gamma, r) \geq \overline{1}$ and $\rho_2 + C_\tau(\gamma, r) \geq \overline{1}$ such that $\lambda \leq \rho_1 \vee \gamma$ and $\mu \leq \rho_2 \vee \gamma$. Since f is grf-open, $f(\rho_1),\ f(\rho_2) \in I^Y$ are r-grfo sets with $f(\rho_1) \neq f(\rho_2)$. Since f is grf-open and one-to-one, $f(\gamma)$ is also a r-fuzzy rare set with $f(\rho_1) + C_\sigma(\gamma, r) \geq \overline{1}$ and $f(\rho_2) + C_\sigma(\gamma, r) \geq \overline{1}$ such that $f(\lambda) \leq f(\rho_1 \vee \gamma)$ and $f(\mu) \leq f(\rho_1 \vee \gamma)$. Thus $(Y,\ \sigma)$ is a grf-rarely T_2-space. □

3. Conclusion

Šostak's fuzzy topology has been recently of major interest among fuzzy topologies. In this paper, we have introduced rarely generalized regular fuzzy continuous functions in fuzzy topological spaces of Šostak's. We have also introduced grf-compact spaces, rarely grf-almost compact spaces and rarely-grf-T_2-spaces, some intresting properties and characterizations of them are investigated.

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