Which is contradiction since $d(x_{2n+1}, x_{2n+2}) > 0$. Thus

$$f(d(x_{2n+1}, x_{2n+2})) \le \psi(f(d(x_{2n}, x_{2n+1}))).$$

Similarly,

$$f(d(x_{2n}, x_{2n+1})) \le \psi(f(d(x_{2n-1}, x_{2n})));$$

We have

$$f(d(x_n, x_{n+1})) \le \psi(f(d(x_{n-1}, x_n)));$$

For all $n \in \mathbb{N}_0$. By induction, we get

$$\begin{aligned} f(d(x_n, x_{n+1})) &\le \psi(f(d(x_{n-1}, x_n))) \\ &\vdots \\ &\le \psi^n(f(d(x_0, x_1))); \end{aligned}$$

for all $n \in \mathbb{N}_0$. Fix $\epsilon > 0$ and let $n(\epsilon) \in \mathbb{N}_0$ such that

$$\sum_{n=n(\epsilon)}^{\infty} \psi^n(f(d(x_0, x_1))) < \epsilon.$$

Let $n, m \in \mathbb{N}_0$ with $m > n > n(\epsilon)$. Using the triangle inequality, we obtain

$$\begin{aligned} f(d(x_n, x_m)) &\le \sum_{k=n}^{m-1} f(d(x_k, x_{k+1})) \\ &\le \sum_{k=n}^{m-1} \psi^k(f(d(x_0, x_1))) \\ &\le \sum_{n=n(\epsilon)}^{\infty} \psi^n(f(d(x_0, x_1))) \\ &< \epsilon. \end{aligned}$$

Thus we proved that $\{x_n\}$ is a Cauchy sequence in the metric space (X, d). Since (X, d) is complete metric space, there exists $x^* \in X$ such that $x_n \to x^*$ as $n \to +\infty$. From the order closed of T, it follows that $x_{2n+1} \in Tx_{2n} \to Tx^*$ as $n \to +\infty$, then $x^* \in Tx^*$. Similarly if S is order closed, we have $x^* \in Sx^*$.

Example 3.3. Let $X = [0, \infty)$, $d(x, y) = |x - y|$ and define $T, S : X \to 2^X$ by $Tx = [0, \frac{1}{3}x]$ and $Sx = [0, \frac{1}{2}x]$ for all $x \in X$ and $\alpha : X \times X \to [0, \infty)$ by $\alpha(x, y) = 1$ whenever $x, y \in [0, 1]$ And $\alpha(x, y) = 0$ whenever $x \notin [0, 1]$ or $y \notin [0, 1]$. Now, we show that T, S are α-admissible. If $\alpha(x, y) \ge 1$, then $x, y \in [0, 1]$ and so Tx and Sy are subsets of $[0, 1]$. Thus $a, b \in [0, 1]$ for all $a \in Tx$ and $b \in Sy$. Hence, $\alpha(a, b) = 1$ for all $a \in Tx$ and $b \in Sy$. This implies that $\alpha^*(Tx, Sy) = \inf\{\alpha(a, b) : a \in$