Nearly k-th Partial Ternary Cubic *-Derivations On Non-Archimedean l-Fuzzy C^*-Ternary Algebras

Mohammad Ali Abolfathi

Sahand Communications in Mathematical Analysis

Print ISSN: 2322-5807
Online ISSN: 2423-3900
Volume: 19
Number: 3
Pages: 13-33

Sahand Commun. Math. Anal.
DOI: 10.22130/scma.2021.543926.1026

Sahand Communications in Mathematical Analysis (SCMA) Vol. 19 No. 3 (2022), 13-33
http://scma.maragheh.ac.ir
DOI: 10.22130/scma.2021.543926.1026

Nearly $k-th$ Partial Ternary Cubic $*$-Derivations On Non-Archimedean ℓ-Fuzzy C^*-Ternary Algebras

Mohammad Ali Abolfathi

Abstract. In this paper, we investigate approximations of the $k-th$ partial ternary cubic derivations on non-Archimedean ℓ-fuzzy Banach ternary algebras and non-Archimedean ℓ-fuzzy C^*-ternary algebras. First, we study non-Archimedean and ℓ-fuzzy spaces, and then prove the stability of partial ternary cubic $*$-derivations on non-Archimedean ℓ-fuzzy C^*-ternary algebras. We therefore provide a link among different disciplines: fuzzy set theory, lattice theory, non-Archimedean spaces, and mathematical analysis.

1. Introduction

A classical equation in the theory of functional equations is the following: "when is it true that a function which approximately satisfies a functional equation must be close to an exact solution of the equation?". If the problem accepts a solution, we say that the equation is stable. The first problem concerning group homomorphisms was raised by Ulam [43] in 1940. In the next year, Hyers [21] gave the first affirmative answer to the question of Ulam in context of Banach spaces. Subsequently, the result of Hyers was generalized by Aoki [3] for additive mappings and Rassias [35] proved a generalization of the Hayers' theorem for linear mappings by considering an unbounded Cauchy difference. Furthermore, in 1994, Găvrua[12] provided a further generalization of Rassias' theorem in which he replaced the bound $\varepsilon(\|x\|^p + \|y\|^p)$ by a general control function $\varphi(x, y)$. Recently, several stability results have been obtained for various equations and mappings with more general

2020 *Mathematics Subject Classification.* 39B52, 46L05, 47B48.

Key words and phrases. Partial ternary derivation, Cubic derivation, Non-Archimedean ℓ-fuzzy algebra, C^*-ternary algebra, Hyers-Ulam-Rasias stability.
Received: 29 November 2021, Accepted: 04 April 2022.

domains and ranges by a number of authors [9, 20, 23, 31, 32]. We also refer the readers to books [7, 22, 36].

In 1897, Hensel [18] discovered the p-adic numbers as a number theoretical analogue of power series in complex analysis. The most important examples of non-Archimedean spaces are p-adic numbers. A key property of p-adic numbers is that they do not satisfy the Archimedean axiom: for all $x, y > 0$, there exists an integer n such that $x < ny$. During the last three decades, the theory of non-Archimedean spaces has gained the interest of physicists for their research, in particular the problems that coming from quantum physics, p-adic strings and superstrings [28]. Although many results in the classical normed space theory have a non-Archimedean counterpart, their proofs are essentially different and require an entirely new kind of intuition. One may note that for $|n| \leq 1$ in each valuation field, every triangle is isosceles and there many be no unit vector in a non-Archimedean normed space. These facts show that the non-Archimedean framework is of special interest. It turned out that non-Archimedean spaces have many nice applications [15, 37, 42].

Let $\mathbb{K}$ be a field. A non-Archimedean absolute value on $\mathbb{K}$ is a function (valuation) $|.| : \mathbb{K} \to \mathbb{R}$ such that, for any $a, b \in \mathbb{K}$, $|a| \geq 0$ and equality holds if and only if $a = 0$, $|ab| = |a|\,|b|$, $|a + b| \leq \max\{|a|, |b|\}$ (the strict triangle inequality). Note that $|1| = |-1| = 1$ and $|n| \leq 1$ for each integer n. A trivial example of a non-Archimedean valuation is the functional $|.|$ taking everything except for 0 into 1 and $|0| = 0$. We always assume, in addition, that $|.|$ is non-trivial, i.e., there exists an $a_0 \in \mathbb{K}$ such that $|a_0\| \notin \{0, 1\}$.

Let $\mathcal{X}$ be a linear space over a scaler field $\mathbb{K}$ with a non-Archimedean nontrivial valuation $|.|$. A $\| \, . \, \| : \mathcal{X} \to \mathbb{R}$ is a non-Archimedean norm (valuation) if it satisfies the following conditions: $\|x\| = 0$ if and only if $x = 0$, $\|rx\| = |r|\,\|x\|$, $\|x + y\| \leq \max\{\|x\|\,\|y\|\}$ (the strict triangle inequality (ultrametric) for all $x, y \in \mathcal{X}$. Then $(\mathcal{X}, \| \, . \, \|)$ is called non-Archimedean normed space. From the fact that

$$\|x_n - x_m\| \leq \max\{\|x_{i+1} - x_i\| : m \leq i \leq n-1\}, \quad (n > m).$$

holds, a sequence $\{x_n\}$ is a Cauchy if and only if $\{x_{n+1} - x_n\}$ converges to zero in a non-Archimedean normed space. By a complete non-Archimedean space, we mean one in which every Cauchy sequence is convergent.

Fix a prime number p. For any nonzero rational number x, there exists a unique integer n_x such that $x = \frac{a}{b} p^{n_x}$, where a and b are integers not divisible by p. Then $|x|_p := p^{-n_x}$ defines a non-Archimedean norm on $\mathbb{Q}$. The completion of $\mathbb{Q}$ with respect to the metric $d(x, y) = |x - y|_p$ is denoted by $\mathbb{Q}_p$, and it is called the p-adic number field. In fact $\mathbb{Q}_p$ is the set of all formal series $x=\sum\limits_{k\geq n}^{\infty}a^k p_k$, where $|a_k|\leq p-1$ are integers. The addition and multiplication between any two elements of $\mathbb{Q}_p$ are defined naturally. The norm $\left|\sum\limits_{k\geq n}^{\infty}a^k p_k\right|_p=p^{-n_x}$ is a non-Archimedean norm on $\mathbb{Q}_p$ and it makes $\mathbb{Q}_p$ a locally compact field [15, 37]. Note that if $p\geq 3$, then $|2^n|_p=1$ for each integer n.

On the other hand, the theory of fuzzy sets was introduced firstly by Zadeh in 1965 [45]. Fuzzy set theory is a powerful hand set for modeling uncertainty and vagueness in various problems arising in the field of science and engineering. After the pioneering work of Zadeh, there has been a great effort to obtain fuzzy analogues of classical theories. Among other fields, a progressive development is made in the field of fuzzy topology [2, 6, 13, 24, 26, 29, 44]. Goguen in [14] generalized the notion of a fuzzy subset of $\mathcal{X}$ to that of an ℓ-fuzzy subset, namely a function from $\mathcal{X}$ to a lattice L. One of the problems in ℓ-fuzzy topology is to obtain an appropriate concept of ℓ-fuzzy metric spaces and ℓ-fuzzy normed spaces. Saadati and Park [39], introduced and studied a notion of intuitionistic fuzzy metric(normed) spaces and then Deschrijver et al. and Saadati generalized the concept of intuitionistic fuzzy metric(normed) spaces and introduced and studied a notion of ℓ-fuzzy metric spaces and ℓ-fuzzy normed spaces [8, 38]. In 2009, Mirmostafaee and Moslehian [30], proved the stability of Cauchy functional equation in non-Archimedean fuzzy spaces in the spirit of Hyers-Ulam-Rassias-Găvrua. In 2010, Shakeri, Saadati and Park [41] investigated the classical quadratic functional equation and proved the generalized Hyers -Ulam stability in the context of non-Archimedean ℓ-fuzzy normed spaces, (see also[1, 10]).

A triangular norm (shortly, t-norm) is a binary operation $\mathcal{T}:[0,1]\times[0,1]\to[0,1]$ which is commutative, associative, monotone and has 1 as the unit element. Basic examples are the Lukasiewicz t-norm $\mathcal{T}_{\mathcal{L}}$, $\mathcal{T}_{\mathcal{L}}(x,y)=\max\{x+y-1,0\}$ for all $x,y\in[0,1]$ and the t-norms $\mathcal{T}_{\mathcal{M}}(x,y)=\min\{x,y\}$, $\mathcal{T}_{\mathcal{M}}(x,y)=xy$ and

$$\mathcal{T}_{\mathcal{D}}(x,y)=\begin{cases}\min\{x,y\}, & \text{if } \max\{x,y\}=1,\\ 0, & \text{otherwise.}\end{cases}$$

A t-norm $\mathcal{T}$ is said to be of Had*žić*-type (we denote by $\mathcal{T}\in\mathcal{H}$) if the family $(x^n_{\mathcal{T}})_{n\in\mathbb{N}}$ is equicontinuous at $x=1$, where is defined by

$$x^1_{\mathcal{T}}=x, \qquad x^n_{\mathcal{T}}=\mathcal{T}\left(x^{n-1}_{\mathcal{T}},x\right),$$

for all $x\in[0,1]$ and $n\geq 2$, [16].

A t-norm $\mathcal{T}$ can be extended (by associativity) in a unique way to an n-ary operation taking, for all $(x_1,\ldots,x_n)\in[0,1]^n$, the value

$\mathcal{T}(x_1,\ldots,x_n)$ defined by

$$\mathcal{T}_{i=1}^{0}x_i = 1, \qquad \mathcal{T}_{i=1}^{n}x_i = \mathcal{T}\left(\mathcal{T}_{i=1}^{n-1}x_i, x_n\right) = \mathcal{T}(x_1,\ldots,x_n).$$

The t-norm $\mathcal{T}$ can also be extended to a countable operation taking, for any sequence $\{x_n\}_{n\in\mathbb{N}}$ in $[0,1]$, the value

$$\mathcal{T}_{i=1}^{\infty}x_i = \lim_{n\to\infty}\mathcal{T}_{i=1}^{n}x_i.$$

Proposition 1.1 ([17]). (1) *For $\mathcal{T} \geq \mathcal{T}_{\mathcal{L}}$ the following implication holds:*

$$\lim_{n\to\infty}\mathcal{T}_{i=1}^{\infty}x_{n+i} = 1 \quad \Leftrightarrow \quad \sum_{n=1}^{\infty}(1-x_n) < \infty.$$

(2) *If $\mathcal{T}$ is of Hadžić-type, then*

$$\lim_{n\to\infty}\mathcal{T}_{i=1}^{\infty}x_{n+i} = 1,$$

for every sequence $\{x_n\}_{n\in\mathbb{N}}$ in $[0,1]$ such that $\lim_{n\to\infty}x_n = 1$.

Let $\ell = (L, \leq_L)$ be a complete lattice and let U be a nonempty set called the universe. An ℓ-fuzzy set in U is defined as a mapping $A : U \to L$. For each u in U, $A(u)$ represents the degree (in L) to which u is an element of A.

A t-norm on $([0,1], \leq)$ can be straightforwardly extended to any lattice $\ell = (L, \leq_L)$. Let $\ell = (L, \leq_L)$ be a lattice. A t-norm on ℓ is a mapping $\mathcal{T} : L \times L \to L$ satisfying the following conditions:

(i) $\mathcal{T}(x, 1_\ell) = x$ (boundary condition) $(x \in L)$;
(ii) $\mathcal{T}(x,y) = \mathcal{T}(y,x)$ (commutativity) $(x, y \in L)$;
(iii) $\mathcal{T}(x, \mathcal{T}(y,z)) = \mathcal{T}(\mathcal{T}(x,y), z)$ (associativity) $(x,y,z \in L)$;
(iv) $If x_1 \leq_L y_1$ and $x_2 \leq_L y_2$then$\mathcal{T}(x_1,x_2) \leq_L \mathcal{T}(y_1,y_2)$ (monotonicity) $(x_1,x_2,y_1,y_2 \in L)$.

A t-norm T on ℓ is said to be continuous if, for any $x, y \in L$ and any sequences $\{x_n\}$ and $\{y_n\}$ which converge to x and y respectively,

$$\lim_{n\to\infty}\mathcal{T}(x_n, y_n) = \mathcal{T}(x,y).$$

A t-norm $\mathcal{T}$ can also be defined recursively as an $(n+1)$-ary operation by $\mathcal{T}^1 = \mathcal{T}$ and

$$\mathcal{T}^n(x_1,\ldots,x_{n+1}) = \mathcal{T}\left(\mathcal{T}^{n-1}(x_1,\ldots,x_n), x_{n+1}\right),$$

for all $n \geq 2$ and $x_i \in L$.

A negator on ℓ is any decreasing mapping $\mathcal{N} : L \to L$ satisfying $\mathcal{N}(0_\ell) = 1_\ell$ and $\mathcal{N}(1_\ell) = 0_\ell$. If $\mathcal{N}(\mathcal{N}(x)) = x$, for all $x \in L$, then $\mathcal{N}$ is called a involutive negator. The negator $\mathcal{N}_s$ on $([0,1], \leq)$ defined as $\mathcal{N}_s(x) = 1 - x$ for all $x \in [0,1]$ is called the standard negator on $([0,1], \leq)$. In this paper, the involutive negator $\mathcal{N}$ is fixed.

Definition 1.2. A non-Archimedean ℓ-fuzzy normed space is a triple $(\mathcal{V}, \mathcal{P}, \mathcal{T})$, where $\mathcal{V}$ is a vector space, $\mathcal{T}$ is a continuous t-norm on L and $\mathcal{P}$ is an ℓ-fuzzy set on $\mathcal{V}\times]0,+\infty[$ satisfying the following conditions: for all $x, y \in V$ and $t, s \in]0,+\infty[$,

(i) $0_\ell <_L \mathcal{P}(x,t)$;
(ii) $\mathcal{P}(x,t) = 1_\ell$ for all $t > 0$ if and only if $x = 0$;
(iii) $\mathcal{P}(\alpha x, t) = \mathcal{P}\left(x, \frac{t}{|\alpha|}\right)$ for each $\alpha \neq 0$;
(iv) $\mathcal{T}(\mathcal{P}(x,t), \mathcal{P}(y,s)) \leq_L \mathcal{P}(x+y, \max\{t,s\})$;
(v) $\mathcal{P}(x,.) :]o,+\infty[\to L$ is continuous.
(vi) $\lim\limits_{t\to 0} \mathcal{P}(x,t) = 0_\ell$ and $\lim\limits_{t\to\infty} \mathcal{P}(x,t) = 1_\ell$.

In this case, $\mathcal{P}$ is called an non-Archimedean ℓ-fuzzy norm. Let $(\mathcal{A}, \|.\|)$ be a non-Archimedean normed linear space and

$$\mathcal{P}(x,t) = \begin{cases} 0, & t \leq \|x\|, \\ 1, & t > \|x\|. \end{cases}$$

Then, the triple $(\mathcal{A}, \mathcal{P}, \min)$ is a non-Archimedean ℓ-fuzzy normed space in which $L = [0,1]$.

A sequence $\{x_n\}_{n\in\mathbb{N}}$ in a non-Archimedean ℓ-fuzzy normed space $(\mathcal{V}, \mathcal{P}, \mathcal{T})$ is called a Cauchy sequence if, for each $\varepsilon \in L \setminus \{0_\ell\}$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that, for all $n, m \geq n_0$, $\mathcal{P}(x_n - x_m, t) >_L N(\varepsilon)$, where N is a negator on ℓ. A sequence $\{x_n\}_{n\in\mathbb{N}}$ is said to be convergent to $x \in \mathcal{V}$ in the non-Archimedean ℓ-fuzzy normed space $(\mathcal{V}, \mathcal{P}, \mathcal{T})$ which is denoted by $x_n \to x$ if $\mathcal{P}(x_n - x, t) \to 1_\ell$ where $n \to \infty$ for all $t > 0$. A non-Archimedean ℓ-fuzzy normed space $(\mathcal{V}, \mathcal{P}, \mathcal{T})$ is said be complete if and only if every Cauchy sequence in $\mathcal{V}$ is convergent.

Definition 1.3. A non-Archimedean ℓ-fuzzy normed algebra $(\mathcal{A}, \mathcal{P}, \mathcal{T}, \mathcal{T}')$ is a non-Archimedean ℓ-fuzzy normed space $(\mathcal{A}, \mathcal{P}, \mathcal{T})$ with algebraic structure if

$$\mathcal{P}(xy, ts) \geq_L \mathcal{T}'(\mathcal{P}(x,t), \mathcal{P}(y,s)),$$

for all $x, y \in \mathcal{A}$ and $t, s > 0$, in which $\mathcal{T}'$ is a continuous t-norm.

Definition 1.4. Let $(\mathcal{A}, \mathcal{P}, \mathcal{T}, \mathcal{T}')$ be a non-Archimedean ℓ-fuzzy Banach algebra. An involution on $\mathcal{A}$ is a mapping $x \to x^*$ from $\mathcal{A}$ into $\mathcal{A}$ satisfying the following conditions:

(i) $x^{**} = x$ for all $x \in \mathcal{A}$,
(ii) $(\alpha x + \beta y)^* = \overline{\alpha} x^* + \overline{\beta} y^*$ for all $x, y \in \mathcal{A}$ and $\alpha, \beta \in \mathbb{C}$,
(iii) $(xy)^* = y^* x^*$ for all $x, y \in \mathcal{A}$.

If, in addition, $\mathcal{P}(x^* x, ts) = \mathcal{T}'(\mathcal{P}(x,t), \mathcal{P}(x,s))$ for all $x \in \mathcal{A}$ and $t, s > 0$, then $\mathcal{A}$ is an non-Archimedean ℓ-fuzzy C^*-algebra.

Ternary algebraic operations have propounded originally in nineteenth century by several mathematicians such as Cayley [5] who introduced the notion of cubic matrix which in turn was generalized by Kapranov, Gelfand and Zelevinskii in 1990 [25]. Their structures appeared more or less naturally in various domains of mathematical physics and data processing. The application of ternary algebra in supersymmetry is presented in [27] and in Yang-Baxter equation in [33]. Cubic analogue of Laplace and d'alembert equations have been considered for the first time by Himbert in [19, 27].

Let $\mathcal{A}$ be a linear space over a complex field equipped with a mapping $[\quad]: \mathcal{A}\times\mathcal{A}\times\mathcal{A}\to\mathcal{A}$ (ternary product) with $(x,y,z)\to[xyz]$ that is linear in variables x,y,z and satisfies the associative identity, i.e., $[[xyz]\,vw]=[x\,[yzv]\,w]=[xy\,[zvw]]$ for all $x,y,z,v,w\in\mathcal{A}$. The pair $(\mathcal{A},[\quad])$ is called a ternary algebra. The ternary algebra $(\mathcal{A},[\quad])$ is called unital if it has an identity element, i.e. an element $e\in\mathcal{A}$ such that $[eex]=[xee]=x$ for every $x\in\mathcal{A}$. A $*$-ternary algebra is a ternary algebra together with a mapping $x\to x^*$ from $\mathcal{A}$ into $\mathcal{A}$ which satisfies $(x^*)^*=x$, $(\alpha x+\beta y)^*=\overline{\alpha}x^*+\overline{\beta}y^*$ and $[xyz]^*=[z^*y^*x^*]$ for all $x,y,z\in\mathcal{A}$ and $\alpha,\beta\in\mathbb{C}$. In the case that $\mathcal{A}$ is unital and e is its unit, we assume that $e^*=e$.

If $\mathcal{A}$ is a ternary algebra and there exists a norm $\|.\|$ on $\mathcal{A}$ which satisfies $\|[xyz]\|\le\|x\|\,\|y\|\,\|z\|$ for all $x,y,z\in\mathcal{A}$, then $\mathcal{A}$ is called a normed ternary algebra. If $\mathcal{A}$ is a unital ternary algebra with unit element e then $\|e\|=1$. By a Banach ternary algebra, we mean a normed ternary algebra with a complete norm $\|.\|$. If $\mathcal{A}$ is a ternary algebra, $x\in\mathcal{A}$ is called central if $[xyz]=[zxy]=[yzx]$ for all $y,z\in\mathcal{A}$. The set of central elements of $\mathcal{A}$ is called the center of $\mathcal{A}$ and is shown by $Z(\mathcal{A})$. If $\mathcal{A}$ is $*$-normed ternary algebra and $Z(\mathcal{A})=0$, then we have $\|x^*\|=\|x\|$.

By a non-Archimedean Banach ternary algebra, we mean a complete non-Archimedean vector spaces $\mathcal{A}$ equipped with a ternary product $(x,y,z)\to[xyz]$ of $\mathcal{A}^3$ into $\mathcal{A}$ which is $\mathbb{K}$-Linear in each variables and associative in the sense that $[xy\,[zvw]]=[x\,[yzv]\,w]=[[xyz]\,vw]$ and satisfies $\|[xyz]\|\le\|x\|\,\|y\|\,\|z\|$ for $x,y,z,v,w\in\mathcal{A}$. A non-Archimedean C^*ternary algebra is a non-Archimedean Banach $*$-ternary algebra $\mathcal{A}$ if $\|[x^*yx]\|=\|x\|^2\,\|y\|$ for all $x\in\mathcal{A}$ and $y\in Z(\mathcal{A})$.

Eshaghi and et. al. [11] introduced the concept of partial ternary derivation and proved the Hyers-Ulam-Rassias stability of partial ternary derivation in Banach ternary algebras. Recently, Arsalan and Inceboz [4] established the Hyers-Ulam-Rassias stability of the partial ternary derivation in Banach ternary algebras.

Definition 1.5. Let $\mathcal{A}$ be a ternary algebra and $(\mathcal{A},\mathcal{P},\mathcal{T})$ be a non-Archimedean ℓ-fuzzy normed space. Then

(i) $(\mathcal{A},\mathcal{P},\mathcal{T},\mathcal{T}')$ is called the non-Archimedean ℓ-fuzzy ternary normed algebra if

$$\mathcal{P}\left([xyz]\,,stu\right)\geq_L \mathcal{T}'\big(\mathcal{T}'\left(\mathcal{P}\left(x,s\right),\mathcal{P}\left(y,t\right)\right),\mathcal{P}\left(z,u\right)\big),$$

for all $x,y,z\in\mathcal{A}$ and all positive real numbers s,t and u.

(ii) A complete ternary non-Archimedean ℓ-fuzzy normed algebra is called a ternary non-Archimedean ℓ-fuzzy Banach algebra.

Let $\mathcal{A}_1,\ldots,\mathcal{A}_n$ be normed ternary algebras over the complex field $\mathbb{C}$ and let $\mathcal{B}$ be the Banach ternary algebra over $\mathbb{C}$. The mapping $\mathcal{D}_k$ is called $k-th$ a partial ternary cubic $*$-derivation if

$$\begin{aligned}2\mathcal{D}_k(x_1,x_2,x_3,\ldots,x_k+y_k,\ldots,x_n)&+2\mathcal{D}_k\left(x_1,x_2,x_3,\ldots,x_k-y_k,\ldots,x_n\right)\\ =\mathcal{D}_k&\left(x_1,x_2,x_3,\ldots,2x_k+y_k,\ldots,x_n\right)\\ &+\mathcal{D}_k\left(x_1,x_2,x_3,\ldots,2x_k-y_k,...,x_n\right)\\ &-12\mathcal{D}_k\left(x_1,x_2,x_3,\ldots,x_k,\ldots,x_n\right),\end{aligned}$$

and also there exists a mapping $\pi_k:\mathcal{A}_k\to\mathcal{B}$ such that

$$\begin{aligned}\mathcal{D}_k\left(x_1,\ldots,[a_kb_kc_k]\,,\ldots,x_n\right)&=\left[\pi_k\left(a_k\right)\pi_k\left(b_k\right)\mathcal{D}_k\left(x_1,\ldots,c_k,\ldots,x_n\right)\right]\\ &\quad+\left[\pi_k(a_k)\mathcal{D}_k\left(x_1,\ldots,b_k,\ldots,x_n\right)\pi_k(c_k)\right]\\ &\quad+\left[\mathcal{D}_k\left(x_1,\ldots,a_k,\ldots,x_n\right)\pi_k(b_k)\pi_k(c_k)\right],\end{aligned}$$

and

$$\mathcal{D}_k\left(x_1,\ldots,a_k^*,\ldots,x_n\right)=\left(\mathcal{D}_k\left(x_1,\ldots,a_k,\ldots,x_n\right)\right)^*,$$

for all $a_k,b_k,c_k\in\mathcal{A}_k, x_i\in\mathcal{A}_i(i\neq k)$.

In 2002, Jun and Kim [23] introduced the following functional equation

$$f\left(2x+y\right)+f\left(2x-y\right)=2\left(f\left(x+y\right)+f\left(x-y\right)\right)+12f\left(x\right),$$

and established the general solution and the Hyers-Ulam stability for it (see also [34]). This functional equation is called cubic functional equation and every solution of cubic equation is said to be a cubic function. Obviously, the function $f(x)=x^3$ satisfies this functional equation.

In this paper, we prove the Hyers-Ulam-Rassias stability of $k-th$ partial ternary cubic derivations on non-Archimedean ℓ-fuzzy Banach ternary algebras and non-Archimedean ℓ-fuzzy C^*-ternary algebras.

2. Stability of Partial Ternary Cubic Derivation on Non-Archimedean ℓ-fuzzy Banach Ternary Algebras

Let $\mathbb{K}$ be a non-Archimedean field, $\mathcal{X}$ be a vector space over $\mathbb{K}$ and $(\mathcal{X},\mathcal{P},\mathcal{T})$ be a non-Archimedean ℓ-fuzzy Banach space over $\mathbb{K}$. Let Ψ_i

be an ℓ-fuzzy set on $\mathcal{X} \times \mathcal{X} \times \mathcal{X} \times [0, \infty)$ such that $\Psi_i(x, y, z, .)$ is non-decreasing, i.e.,

$$\Psi_i\left(cx, cx, cx, t\right) \geq_L \Psi_i\left(x, x, x, \frac{t}{|c|}\right),$$

and

$$\lim_{t \to \infty} \Psi_i\left(x, y, z, t\right) = 1_\ell,$$

for all $i = 1, 2, , 3, \dots, n$, $x, y, z \in \mathcal{X}, t > 0$ and $c \neq 0$.

Theorem 2.1. *Let $\mathcal{G}_k : \mathcal{A}_1 \times \ldots \times \mathcal{A}_n \to \mathcal{B}$ be a mapping with $G_k\left(x_1, \dots, 0_k, \dots, x_n\right) = 0_{\mathcal{B}}$. Assume that there exists an ℓ-fuzzy set Ψ_k on $\mathcal{A}_1 \times \mathcal{A}_2 \times \mathcal{A}_3 \times [0, \infty)$ such that for some $\alpha \in (0, \infty)$ and some integer $\lambda \geq 2$ with $\left|2^\lambda\right| < \alpha$ which $|2| \neq 0$, we have*

$$\Psi_k\left(2^{-\lambda}x_k, 2^{-\lambda}y_k, 2^{-\lambda}z_k, t\right) \geq_L \Psi_k\left(x_k, y_k, z_k, \alpha t\right), \tag{2.1}$$

and

$$\lim_{l \to \infty} \mathcal{T}_{j=l}^{\infty} M\left(x_k, \frac{\alpha^j}{|2|^{\lambda j}} t\right) = 1_\ell, \tag{2.2}$$

for all $x_k, y_k, z_k \in \mathcal{A}_k$ and $t > 0$. Also assume that there exists a cubic mapping $\pi_k : \mathcal{A}_k \to \mathcal{B}$ satisfying

$$\begin{aligned} \mathcal{P}\Big(&\mathcal{G}_k\left(x_1, \dots, 2a_k + b_k, \dots, x_n\right) + \mathcal{G}_k\left(x_1, \dots, 2a_k - b_k, \dots, x_n\right) \\ &- 2\mathcal{G}_k\left(x_1, \dots, a_k + b_k, \dots, x_n\right) - 2\mathcal{G}_k\left(x_1, \dots, a_k - b_k, \dots, x_n\right) \\ &- 12\mathcal{G}_k\left(x_1, \dots, a_k, \dots, x_n\right), t\Big) \\ \geq_L\ & \Psi_k\left(a_k, b_k, 0_k, t\right), \end{aligned} \tag{2.3}$$

and

$$\begin{aligned} \mathcal{P}\Big(&\mathcal{G}_k\left(x_1, \dots, [a_k b_k c_k], \dots, x_n\right) - \left[\pi_k(a_k)\pi_k(b_k)\mathcal{G}_k\left(x_1, \dots, c_k, \dots, x_n\right)\right] \\ &- \left[\pi_k(a_k)\mathcal{G}_k\left(x_1, \dots, b_k, \dots, x_n\right)\pi_k(c_k)\right] \\ &+ \left[\mathcal{G}_k\left(x_1, \dots, a_k, \dots, x_n\right)\pi_k(b_k)\pi_k(c_k)\right], t\Big) \\ \geq_L\ & \Psi_k\left(a_k, b_k, c_k, t\right), \end{aligned} \tag{2.4}$$

for all $a_k, b_k, c_k \in \mathcal{A}_k, x_i \in \mathcal{A}_i (i \neq k)$ and $t > 0$. Then there exists a unique k-th partial cubic derivation $\mathcal{D}_k : \mathcal{A}_1 \times \cdots \times \mathcal{A}_n \to \mathcal{B}$ such that

$$\mathcal{P}\left(\mathcal{G}_k\left(x_1, \dots, x_k, \dots, x_n\right) - \mathcal{D}_k\left(x_1, \dots, x_k, \dots, x_n\right), t\right) \tag{2.5}$$

$$\geq_L \mathcal{T}_{j=1}^{\infty} M\left(x_k, \frac{\alpha^{j+1}}{|2|^{\lambda j}} t\right),$$

for all $x_i \in \mathcal{A}_i$ and $t > 0$ where

$M(x_k, t) := \mathcal{T}\left(\Psi_k(x_k, 0_k, 0_k, t), \Psi_k(2x_k, 0_k, 0_k, t), \ldots, \Psi_k\left(2^{\lambda-1}x_k, 0_k, 0_k, t\right)\right),$

for all $x_k \in \mathcal{A}_k$ and $t > 0$.

Proof. One can use induction on j to show that

(2.6)

$$\begin{aligned}
&\mathcal{P}\left(\mathcal{G}_k\left(x_1, \ldots, 2^j x_k, \ldots, x_n\right) - 2^{3j}\mathcal{G}_k(x_1, \ldots, x_k, \ldots, x_n), t\right) \\
&\quad \geq_L M_j(x_k, t) \\
&\quad = \mathcal{T}\left(\Psi_k(x_k, 0_k, 0_k, t), \Psi_k(2x_k, 0_k, 0_k, t), \ldots, \Psi_k\left(2^{j-1}x_k, 0_k, 0_k, t\right)\right),
\end{aligned}$$

for all $x_i \in \mathcal{A}_i$, $t > 0$. Replacing $a_k = x_k$ and $b_k = 0_k$ in (2.3), we have

$$\begin{aligned}
&\mathcal{P}(2\mathcal{G}_k(x_1, \ldots, 2x_k, \ldots, x_n) - 16\mathcal{G}_k(x_1, \ldots, x_k, \ldots, x_n), t) \\
&\quad \geq_L \Psi_k(x_k, 0_k, 0_k, t),
\end{aligned}$$

for all $x_i \in \mathcal{A}_i$ and $t > 0$. Hence

$$\begin{aligned}
&\mathcal{P}(\mathcal{G}_k(x_1, \ldots, 2x_k, \ldots, x_n) - 8\mathcal{G}_k(x_1, \ldots, x_k, \ldots, x_n), t) \\
&\quad \geq_L \Psi_k(x_k, 0_k, 0_k, 2t) \\
&\quad \geq_L \Psi_k(x_k, 0_k, 0_k, t),
\end{aligned}$$

for all $x_i \in \mathcal{A}_i$ and $t > 0$. This proves (2.6) for $j = 1$. Let (2.6) holds for some $j > 1$. Substituting a_k by $2^j x_k$ and b_k by 0_k in (2.3), we get

$$\begin{aligned}
&\mathcal{P}\left(\mathcal{G}_k\left(x_1, \ldots, 2^{j+1}x_k, \ldots, x_n\right) - 8\mathcal{G}_k\left(x_1, \ldots, 2^j x_k, \ldots, x_n\right), t\right) \\
&\quad \geq_L \Psi_k\left(2^j x_k, 0_k, 0_k, t\right),
\end{aligned}$$

for all $x_i \in \mathcal{A}_i$ and $t > 0$. Since $|8| \leq 1$, it follows that

$$\begin{aligned}
&\mathcal{P}\left(\mathcal{G}_k\left(x_1, \ldots, 2^{j+1}x_k, \ldots, x_n\right) - 2^{3(j+1)}\mathcal{G}_k(x_1, \ldots, x_k, \ldots, x_n), t\right) \\
&\quad \geq_L \mathcal{T}\Big(\mathcal{P}\left(\mathcal{G}_k\left(x_1, \ldots, 2^{j+1}x_k, \ldots, x_n\right) - 2^3\mathcal{G}_k\left(x_1, \ldots, 2^j x_k, \ldots, x_n\right), t\right) \\
&\qquad , 2^3\mathcal{P}\left(\mathcal{G}_k\left(x_1, \ldots, 2^{j+1}x_k, \ldots, x_n\right) - 2^{3(j+1)}\mathcal{G}_k(x_1, \ldots, x_k, \ldots, x_n), t\right)\Big) \\
&\quad = \mathcal{T}\Bigg(\mathcal{P}\left(\mathcal{G}_k\left(x_1, \ldots, 2^{j+1}x_k, \ldots, x_n\right) - 2^3\mathcal{G}_k\left(x_1, \ldots, 2^j x_k, \ldots, x_n\right), t\right) \\
&\qquad , \mathcal{P}\left(\mathcal{G}_k\left(x_1, \ldots, 2^j x_k, \ldots, x_n\right) - 2^{3j}\mathcal{G}_k(x_1, \ldots, x_k, \ldots, x_n), \frac{t}{|8|}\right)\Bigg) \\
&\quad \geq_L \mathcal{T}\Big(\mathcal{P}\left(\mathcal{G}_k\left(x_1, \ldots, 2^{j+1}x_k, \ldots, x_n\right) - 2^3\mathcal{G}_k\left(x_1, \ldots, 2^j x_k, \ldots, x_n\right), t\right)
\end{aligned}$$

$$, \mathcal{P}\left(\mathcal{G}_k\left(x_1,\ldots,2^j x_k,\ldots,x_n\right)-2^{3j}\mathcal{G}_k\left(x_1,\ldots,x_k,\ldots,x_n\right),t\right)\Big)$$
$$\geq_L \mathcal{T}\left(\Psi_k\left(2^j x_k,0_k,0_k,t\right),M_j\left(x_k,t\right)\right)$$
$$= M_{j+1}\left(x_k,t\right),$$

for all $x_i \in \mathcal{A}_i$ and $t > 0$. Therefore (2.6) holds for all $j \in \mathbb{N}$. In particular, we have

$$\mathcal{P}\left(\mathcal{G}_k\left(x_1,\ldots,2^\lambda x_k,\ldots,x_n\right)-2^{3\lambda}\mathcal{G}_k\left(x_1,\ldots,x_k,\ldots,x_n\right),t\right) \geq_L M\left(x_k,t\right), \tag{2.7}$$

for all $x_i \in \mathcal{A}_i$ and $t > 0$. Replacing x_k by $2^{-\lambda(l+1)}x_k$ in (2.7) and using (2.1), we obtain

$$\mathcal{P}\left(\mathcal{G}_k\left(x_1,\ldots,\frac{x_k}{2^{\lambda\lambda}},\ldots,x_n\right)-2^{3\lambda}\mathcal{G}_k\left(x_1,\ldots,\frac{x_k}{2^{\lambda(l+1)}},\ldots,x_n\right),t\right) \geq_L M\left(x_k,\alpha^{l+1}t\right), \tag{2.8}$$

for all $x_i \in \mathcal{A}_i$, $t > 0$ and $l \geq 0$. The above relation implies that

$$\mathcal{P}\left(\left(2^{3\lambda}\right)^l\mathcal{G}_k\left(x_1,\ldots,\frac{x_k}{2^{\lambda l}},\ldots,x_n\right)-\left(2^{3\lambda}\right)^{l+1}\mathcal{G}_k\left(x_1,\ldots,\frac{x_k}{2^{\lambda(l+1)}},\ldots,x_n\right),t\right)$$
$$\geq_L M\left(x_k,\frac{\alpha^{l+1}}{|(2^{3\lambda})^l|}t\right)$$
$$\geq_L M\left(x_k,\frac{\alpha^{l+1}}{|(2^{\lambda})^l|}t\right),$$

for all $x_i \in \mathcal{A}_i$, $t > 0$ and $l \geq 0$. Therefore

$$\mathcal{P}\left(\left(2^{3\lambda}\right)^l\mathcal{G}_k\left(x_1,...,\frac{x_k}{2^{\lambda l}},\ldots,x_n\right)-\left(2^{3\lambda}\right)^{l+p}\mathcal{G}_k\left(x_1,\ldots,\frac{x_k}{2^{\lambda(l+p)}},\ldots,x_n\right),t\right)$$
$$\geq_L \mathcal{T}_{j=l}^{l+p}\Bigg(\left(2^{3\lambda}\right)^j\mathcal{G}_k\left(x_1,\ldots,\frac{x_k}{2^{\lambda j}},\ldots,x_n\right)$$
$$-\left(2^{3\lambda}\right)^{j+p}\mathcal{G}_k\left(x_1,\ldots,\frac{x_k}{2^{\lambda(j+p)}},\ldots,x_n\right),t\Bigg)$$
$$\geq_L \mathcal{T}_{j=l}^{l+p}M\left(x_k,\frac{\alpha^{j+1}}{\left|\left(2^\lambda\right)^j\right|}t\right),$$

for all $x_i \in \mathcal{A}_i$, $t > 0$ and $l \geq 0$. Since $\lim_{l\to\infty}\mathcal{T}_{j=l}^{l+p}M\big(x_k,\frac{\alpha^{j+1}}{|(2^\lambda)^j|}t\big) = 1_\ell$, for all $x_i \in \mathcal{A}_i$ and $t > 0$, then the sequence

$$\left\{(2^{3\lambda})^l\mathcal{G}_k\left(x_1,\ldots,\frac{x_k}{2^{\lambda l}},\ldots,x_n\right)\right\},$$

is Cauchy in the non-Archimedean ℓ-fuzzy Banach space $(\mathcal{B},\mathcal{P},\mathcal{T})$. Hence, we can define a mapping $\mathcal{D}_k:\mathcal{A}_1\times\ldots\times\mathcal{A}_n\to\mathcal{B}$ such that

(2.9)

$$\lim_{l\to\infty}\mathcal{P}\left(\left(2^{3\lambda}\right)^l\mathcal{G}_k\left(x_1,\ldots,\frac{x_k}{2^{\lambda l}},\ldots,x_n\right)-\mathcal{D}_k\left(x_1,\ldots,x_k,\ldots,x_n\right),t\right)=1_\ell,$$

for all $x_i\in\mathcal{A}_i$ and $t>0$. For each $l\geq 1$,$x_i\in\mathcal{A}_i$ and $t>0$, we get

$$\begin{aligned}
&\mathcal{P}\left(\mathcal{G}_k\left(x_1,\ldots,a_k,\ldots,x_n\right)-2^{3\lambda l}\mathcal{G}_k\left(x_1,\ldots,\frac{a_k}{2^{\lambda l}},\ldots,x_n\right),t\right)\\
&\quad=\mathcal{P}\Bigg(\sum_{j=0}^{l-1}2^{3\lambda j}\mathcal{G}_k\left(x_1,\ldots,\frac{a_k}{2^{\lambda l}},\ldots,x_n\right)\\
&\qquad-2^{3\lambda(j+1)}\mathcal{G}_k\left(x_1,\ldots,\frac{a_k}{2^{\lambda(j+1)}},\ldots,x_n\right),t\Bigg)\\
&\quad\geq_L\mathcal{T}_{j=0}^{l-1}\Bigg(\mathcal{P}\Bigg(2^{3\lambda j}\mathcal{G}_k\left(x_1,\ldots,\frac{a_k}{2^{\lambda l}},\ldots,x_n\right)\\
&\qquad-2^{3\lambda(j+1)}\mathcal{G}_k\left(x_1,\ldots,\frac{a_k}{2^{\lambda(j+1)}},\ldots,x_n\right),t\Bigg)\Bigg)\\
&\quad\geq_L\mathcal{T}_{j=0}^{l-1}M\left(x_k,\frac{\alpha^{j+1}}{|2^\lambda|^j}t\right),
\end{aligned}$$

and so

(2.10)

$$\begin{aligned}
&\mathcal{P}\Big(\mathcal{G}_k\left(x_1,\ldots,a_k,\ldots,x_n\right)-\mathcal{D}_k\left(x_1,\ldots,a_k,\ldots,x_n\right),t\Big)\\
&\quad\geq_L\mathcal{T}\Big(P\Big(\mathcal{G}_k\left(x_1,\ldots,a_k,\ldots,x_n\right)-2^{3\lambda l}\mathcal{G}_k\left(x_1,\ldots,\frac{a_k}{2^{\lambda l}},\ldots,x_n\right),t\Big)\\
&\qquad,\mathcal{P}\left(2^{3\lambda l}\mathcal{G}_k\left(x_1,\ldots,\frac{a_k}{2^{\lambda l}},\ldots,x_n\right)-\mathcal{D}_k\left(x_1,\ldots,a_k,\ldots,x_n\right),t\right)\Big)\\
&\quad\geq_L\mathcal{T}\Big(\mathcal{T}_{j=0}^{l-1}M\left(x_k,\frac{\alpha^{j+1}}{|2^\lambda|^j}t\right)\\
&\qquad,\mathcal{P}\left(2^{3\lambda l}\mathcal{G}_k\left(x_1,\ldots,\frac{a_k}{2^{\lambda l}},\ldots,x_n\right)-\mathcal{D}_k\left(x_1,\ldots,a_k,\ldots,x_n\right),t\right)\Big).
\end{aligned}$$

By taking limit as $l\to\infty$ in (2.10), we obtain

$$\begin{aligned}
&\mathcal{P}\left(\mathcal{G}_k\left(x_1,\ldots,x_k,\ldots,x_n\right)-\mathcal{D}_k\left(x_1,\ldots,x_k,\ldots,x_n\right),t\right)\\
&\quad\geq_L\mathcal{T}_{j=1}^{\infty}M\left(x_k,\frac{\alpha^{j+1}}{|2|^{\lambda j}}t\right),
\end{aligned}$$

for all $x_i \in \mathcal{A}_i$ and $t > 0$. Now, replacing a_k, b_k, c_k with $2^{-\lambda l}a_k, 2^{-\lambda l}b_k$, $2^{-\lambda l}c_k$, respectively, in (2.4), we obtain

$$\begin{aligned}
&\mathcal{P}\Bigg(\mathcal{G}_k\left(x_1,\ldots,\frac{[a_kb_kc_k]}{2^{3\lambda l}},\ldots,x_n\right)-\left[\frac{\pi_k(a_k)}{2^{3\lambda l}}\frac{\pi_k b_k}{2^{3\lambda l}}\mathcal{G}_k\left(x_1,\ldots,\frac{c_k}{2^{\lambda l}},\ldots,x_n\right)\right]\\
&\quad-\left[\frac{\pi_k(a_k)}{2^{3\lambda l}}\mathcal{D}_k\left(x_1,\ldots,\frac{b_k}{2^{\lambda l}},\ldots,x_n\right)\frac{\pi_k(c_k)}{2^{3\lambda l}}\right]\\
&\quad-\left[\mathcal{D}_k\left(x_1,\ldots,\frac{a_k}{2^{\lambda l}},\ldots,x_n\right)\frac{\pi_k(b_k)}{2^{3\lambda l}}\frac{\pi_k(c_k)}{2^{3\lambda l}}\right],t\Bigg)\\
&\quad\geq_L \Psi_k\left(\frac{a_k}{2^{\lambda l}},\frac{b_k}{2^{\lambda l}},\frac{c_k}{2^{\lambda l}},t\right),
\end{aligned}$$

for all $a_k, b_k, c_k \in \mathcal{A}_k, x_i \in \mathcal{A}_i (i \neq k)$ and $t > 0$. Hence

$$\begin{aligned}
&\mathcal{P}\Bigg(2^{9\lambda l}\mathcal{G}_k\left(x_1,\ldots,\frac{[a_kb_kc_k]}{2^{3\lambda l}},\ldots,x_n\right)\\
&\qquad-2^{9\lambda l}\left[\frac{\pi_k\left(a_k\right)}{2^{3\lambda l}}\frac{\pi_k\left(b_k\right)}{2^{3\lambda l}}\mathcal{G}_k\left(x_1,\ldots,\frac{c_k}{2^{\lambda l}},\ldots,x_n\right)\right]\\
&\qquad-2^{9\lambda l}\left[\frac{\pi_k\left(a_k\right)}{2^{3\lambda l}}\mathcal{D}_k\left(x_1,\ldots,\frac{b_k}{2^{\lambda l}},\ldots,x_n\right)\frac{\pi_k\left(c_k\right)}{2^{3\lambda l}}\right]\\
&\qquad-2^{9\lambda m}\left[\mathcal{D}_k\left(x_1,\ldots,\frac{a_k}{2^{\lambda l}},\ldots,x_n\right)\frac{\pi_k\left(b_k\right)}{2^{3\lambda l}}\frac{\pi_k\left(c_k\right)}{2^{3\lambda l}}\right],t\Bigg)\\
&\quad\geq_L \Psi_k\left(\frac{a_k}{2^{\lambda l}},\frac{b_k}{2^{\lambda l}},\frac{c_k}{2^{\lambda l}},\frac{t}{|2|^{9\lambda l}}\right)\\
&\quad\geq_L \Psi_k\left(a_k,b_k,c_k,\frac{\alpha^l}{|2|^{\lambda l}}t\right),
\end{aligned}$$

for all $a_k, b_k, c_k \in \mathcal{A}_k, x_i \in \mathcal{A}_i (i \neq k)$ and $t > 0$.

By $\lim_{l\to\infty} \Psi_k(a_k, b_k, c_k, \frac{\alpha^l}{|2|^{\lambda l}}t) = 1_\ell$, we get

$$\begin{aligned}
\mathcal{D}_k\left(x_1,\ldots,\left[a_kb_kc_k\right],\ldots,x_n\right)&=\left[\pi_k\left(a_k\right)\pi_k\left(b_k\right)\mathcal{D}_k\left(x_1,\ldots,c_k,\ldots,x_n\right)\right]\\
&\quad+\left[\pi_k\left(a_k\right)\mathcal{D}_k\left(x_1,\ldots,b_k,\ldots,x_n\right)\pi_k\left(c_k\right)\right]\\
&\quad+\left[\mathcal{D}_k\left(x_1,\ldots,a_k,\ldots,x_n\right)\pi_k\left(b_k\right)\pi_k\left(c_k\right)\right],
\end{aligned}$$

for all $a_k, b_k, c_k \in \mathcal{A}_k, x_i \in \mathcal{A}_i (i \neq k)$. As $\mathcal{T}$ is continuous, form a well known result in ℓ-fuzzy (probabilistic) normed spaces [40], it follows that

$$\lim_{l\to\infty}\mathcal{P}\Bigg(8^{\lambda l}\mathcal{G}_k\left(x_1,\ldots,2^{-\lambda l}(2a_k+b_k),\ldots,x_n\right)$$

$$
\begin{aligned}
&+\left(8^{\lambda l}\mathcal{G}_k\left(x_1,\ldots,2^{-\lambda l}(2a_k-b_k),\ldots,x_n\right)\right)\\
&-2\left(8^{\lambda l}\mathcal{G}_k\left(x_1,...,2^{-\lambda l}(a_k+b_k),\ldots,x_n\right)\right)\\
&-2\left(8^{\lambda l}\mathcal{G}_k\left(x_1,\ldots,2^{-\lambda l}(a_k-b_k),\ldots,x_n\right)\right)\\
&-12\left(8^{\lambda l}\mathcal{G}_k\left(x_1,\ldots,2^{-\lambda l}a_k,\ldots,x_n\right)\right),t\Bigg)\\
=\ &\mathcal{P}\Big(\mathcal{D}_k\left(x_1,\ldots,(2a_k+b_k),\ldots,x_n\right)\\
&+\mathcal{D}_k\left(x_1,\ldots,(2a_k-b_k),\ldots,x_n\right)\\
&-2\mathcal{D}_k\left(x_1,\ldots,(a_k+b_k),\ldots,x_n\right)\\
&-2\mathcal{D}_k\left(x_1,\ldots,(a_k-b_k),\ldots,x_n\right)\\
&-12\mathcal{D}_k\left(x_1,\ldots,a_k,\ldots,x_n\right),t\Big),
\end{aligned}
$$

for all $a_k, b_k \in \mathcal{A}_k, x_i \in \mathcal{A}_i (i \neq k, i = 1, 2, ..., n)$ and $t > 0$. Replacing a_k, b_k by $2^{-\lambda l}a_k, 2^{-\lambda l}b_k$ in (2.3) and by (2.1), we get

$$
\begin{aligned}
&\mathcal{P}\Big(8^{\lambda l}\mathcal{D}_k\left(x_1,\ldots,2^{-\lambda l}(2a_k+b_k),\ldots,x_n\right)\\
&\quad+\left(8^{\lambda l}\mathcal{D}_k\left(x_1,\ldots,2^{-\lambda l}(2a_k-b_k),\ldots,x_n\right)\right)\Big)\\
&\quad-2\left(8^{\lambda l}\mathcal{D}_k\left(x_1,\ldots,2^{-\lambda l}(a_k+b_k),\ldots,x_n\right)\right)\\
&\quad-2\left(8^{\lambda l}\mathcal{D}_k\left(x_1,\ldots,2^{-\lambda l}(a_k-b_k),...,x_n\right)\right)\\
&\quad-12\left(8^{\lambda l}\mathcal{D}_k\left(x_1,\ldots,2^{-\lambda l}a_k,\ldots,x_n\right)\right),t\Big)\\
&\geq_L \Psi_k\left(2^{-\lambda l}a_k,2^{-\lambda l}b_k,0_k,t\right)\\
&\geq_L \Psi_k\left(a_k,b_k,0_k,\frac{\alpha^l}{|2^\lambda|^l}t\right),
\end{aligned}
$$

for all $a_k, b_k \in \mathcal{A}_k, x_i \in \mathcal{A}_i(i \neq k, i = 1, 2, \ldots, n)$ and $t > 0$. Since $\lim_{l\to\infty}\Psi_k\left(a_k,b_k,0_k,\frac{\alpha^l}{|2^\lambda|^l}t\right) = 1_\ell$, we infer that $\mathcal{D}$ is a cubic mapping with respect to the $k-th$ variable.

For the uniqueness of $\mathcal{D}$, let $\mathcal{D}'_k : \mathcal{A}_1 \times \ldots \times \mathcal{A}_n \to \mathcal{B}$ be another $k-th$ partial ternary cubic derivation such that

$$
\begin{aligned}
\text{(2.11)}\qquad &\mathcal{P}\left(\mathcal{G}_k\left(x_1,\ldots,x_k,\ldots,x_n\right)-\mathcal{D}'_k\left(x_1,\ldots,x_k,\ldots,x_n\right),t\right)\\
&\geq_L \mathcal{T}_{j=1}^{\infty}M\left(x_k,\frac{\alpha^{j+1}}{|2|^{\lambda j}}t\right),
\end{aligned}
$$

for all $x_i \in \mathcal{A}_i$ and $t > 0$. Then for each $l = 1, 2, \ldots, x_i \in \mathcal{A}_i$ and $t > 0$, we have

$$\begin{aligned}
&\mathcal{P}\left(\mathcal{D}_k\left(x_1, \ldots, x_k, \ldots, x_n\right) - \mathcal{D}'_k\left(x_1, \ldots, x_k, \ldots, x_n\right), t\right) \\
&\quad \geq_L \mathcal{T}\Bigg(\mathcal{P}\left(\mathcal{D}_k\left(x_1, \ldots, x_k, \ldots, x_n\right) - 2^{3\lambda l}\mathcal{G}_k\left(x_1, \ldots, \frac{x_k}{2^{\lambda l}}, \ldots, x_n\right), t\right) \\
&\qquad , \mathcal{P}\left(2^{3\lambda l}\mathcal{G}_k\left(x_1, \ldots, \frac{x_k}{2^{\lambda l}}, \ldots, x_n\right) - \mathcal{D}'_k\left(x_1, \ldots, x_k, \ldots, x_n\right), t\right)\Bigg),
\end{aligned}$$

for all $x_i \in \mathcal{A}_i$ and $t > 0$. From (2.9), we conclude that $\mathcal{D}_k = \mathcal{D}'_k$. This completes the proof. □

Corollary 2.2. *Let $(\mathcal{X}, \mathcal{P}, \mathcal{T})$ be a non-Archimedean ℓ-fuzzy Banach space over $\mathbb{K}$ under a t-norm Hadžić-type ($\mathcal{T} \in \mathcal{H}$). Let $\mathcal{G}_k : \mathcal{A}_1 \times \ldots \times \mathcal{A}_n \to \mathcal{B}$ be a mapping with $\mathcal{G}_k\left(x_1, \ldots, 0_k, \ldots, x_n\right) = 0_{\mathcal{B}}$. Assume that there exists an ℓ-fuzzy set Ψ_k on $\mathcal{A}_1 \times \mathcal{A}_2 \times \mathcal{A}_3 \times [0, \infty)$ satisfying (2.1) and (2.2) for some $\alpha \in (0, \infty)$ and some integer $\lambda \geq 2$ with $\left|2^\lambda\right| < \alpha$ which $|2| \neq 0$. Also assume that there exists a cubic mapping $\pi_k : \mathcal{A}_k \to \mathcal{B}$ satisfying (2.3) and (2.4). Then there exists a unique k-th partial cubic derivation $\mathcal{D}_k : \mathcal{A}_1 \times \ldots \times \mathcal{A}_n \to \mathcal{B}$ such that*

$$\begin{aligned}
&\mathcal{P}\left(\mathcal{G}_k\left(x_1, \ldots, x_k, \ldots, x_n\right) - \mathcal{D}_k\left(x_1, \ldots, x_k, \ldots, x_n\right), t\right) \\
&\quad \geq_\ell \mathcal{T}_{j=1}^{\infty} M\left(x_k, \frac{\alpha^{j+1}}{|2|^{\lambda j}} t\right),
\end{aligned}$$

for all $x_i \in \mathcal{A}_i$ and $t > 0$ where

$$M(x_k, t := \mathcal{T}\left(\Psi_k\left(x_k, 0_k, 0_k, t\right), \Psi_k\left(2x_k, 0_k, 0_k, t\right), \ldots, \Psi_k\left(2^{\lambda-1}x_k, 0_k, 0_k, t\right)\right),$$

for all $x_k \in \mathcal{A}_k$ and $t > 0$.

Proof. Since

$$\lim_{n\to\infty} M\left(x, \frac{\alpha^{j+1}}{|2|^{\lambda j}} t\right) = 1_\ell,$$

for all $x_k \in \mathcal{A}_k$, $t > 0$ and $\mathcal{T}$ is of Had*žić*-type, it follows from Proposition 1.1 that

$$\lim_{n\to\infty} \mathcal{T}_{j=n}^{\infty} M\left(x, \frac{\alpha^{j+1}}{|2|^{\lambda j}} t\right) = 1_\ell,$$

for all $x_k \in \mathcal{A}_k$ and $t > 0$. Now, we get the conclusion by applying Theorem 2.1. □

Similarly, we can obtain the following theorem.

Theorem 2.3. *Let $\mathcal{G}_k : \mathcal{A}_1 \times ... \times \mathcal{A}_n \to \mathcal{B}$ be a mapping with*

$$\mathcal{G}_k\left(x_1, \dots, 0_k, \dots, x_n\right) = 0_{\mathcal{B}}.$$

Assume that there exists an ℓ-fuzzy set Ψ_k on $\mathcal{A}_1 \times \mathcal{A}_2 \times \mathcal{A}_3 \times [0,\infty)$ such that for some $\alpha \in (0,\infty)$ and for some integer $\lambda \geq 2$ with $\frac{1}{|2|^{6\lambda}} < \alpha$ which $|2| \neq 0$, satisfies

$$\Psi\left(2^{\lambda}x_k, 2^{\lambda}y_k, 2^{\lambda}z_k, t\right) \geq_{\ell} \Psi_k\left(x_k, y_k, z_k, \frac{\alpha}{|2|^{3\lambda}}t\right), \tag{2.12}$$

and

$$\lim_{n\to\infty} \mathcal{T}_{j=n}^{\infty} M\left(x, \alpha^j t\right) = 1_{\ell}, \tag{2.13}$$

for all $x_k, y_k, z_k \in \mathcal{A}_k$ and $t > 0$. Also assume that there exists a cubic mapping $\pi_k : \mathcal{A}_k \to \mathcal{B}$ satisfying (2.3) and (2.4) for all $a_k, b_k, c_k \in \mathcal{A}_k, x_i \in \mathcal{A}_i (i \neq k)$ and $t > 0$. Then there exists a unique k-th partial cubic derivation $\mathcal{D}_k : \mathcal{A}_1 \times \cdots \times \mathcal{A}_n \to \mathcal{B}$ such that

$$\begin{aligned} &\mathcal{P}\left(\mathcal{G}_k\left(x_1, \dots, x_k, \dots, x_n\right) - \mathcal{D}_k\left(x_1, \dots, x_k, \dots, x_n\right), t\right) \\ &\qquad \geq_L \mathcal{T}_{j=1}^{\infty} M\left(x_k, \alpha^{j+1}t\right), \end{aligned} \tag{2.14}$$

for all $x_i \in \mathcal{A}_i$ and $t > 0$, where

$$M(x_k, t) := \mathcal{T}\Big(\Psi_k\left(\frac{x_k}{2}, 0_k, 0_k, t\right), \Psi_k\left(\frac{x_k}{4}, 0_k, 0_k, t\right), \dots, \Psi_k\left(\frac{x_k}{2^{\lambda}}, 0_k, 0_k, t\right)\Big),$$

for all $x_k \in \mathcal{A}_k$ and $t > 0$.

Proof. Replacing x_k by $\frac{x_k}{2}$ in (2.7), we obtain

$$\begin{aligned} &\mathcal{P}\left(\frac{1}{2^{3\lambda}}\mathcal{G}_k\left(x_1, \dots, x_k, \dots, x_n\right) - 2^{3\lambda}\mathcal{G}_k\left(x_1, \dots, \frac{x_k}{2^{\lambda}}, \dots, x_n\right), t\right) \\ &\quad \geq_L \mathcal{T}\left(\Psi_k\left(\frac{x_k}{2}, 0_k, 0_k, |2|^{3\lambda}t\right), \Psi_k\left(\frac{x_k}{4}, 0_k, 0_k, |2|^{3\lambda}t\right)\right. \\ &\qquad \left., \dots, \Psi_k\left(\frac{x_k}{2^{\lambda}}, 0_k, 0_k, |2|^{3\lambda}t\right)\right) \\ &\quad = M\left(x_k, |2|^{3\lambda}t\right). \end{aligned} \tag{2.15}$$

Replacing x_k by $2^{\lambda(l+1)}x_k$ in (2.15) and using (2.12), we have

$$\begin{aligned} &\mathcal{P}\left(\frac{1}{2^{3\lambda}}\mathcal{G}_k\left(x_1, \dots, 2^{\lambda(l+1)}x_k, \dots, x_n\right) - \mathcal{G}_k\left(x_1, \dots, 2^{\lambda l}x_k, \dots, x_n\right), t\right) \\ &\quad \geq_L \mathcal{T}\left(\Psi_k\left(\frac{x_k}{2}, 0_k, 0_k, |2|^{3\lambda}t\right), \Psi_k(\frac{x_k}{4}, 0_k, 0_k, |2|^{3\lambda}t)\right. \\ &\qquad \left., \dots, \Psi_k\left(\frac{x_k}{2^{\lambda}}, 0_k, 0_k, |2|^{3\lambda}t\right)\right) \end{aligned}$$

$$= M\left(x_k, \frac{\alpha^{l+1}}{|2|^{3\lambda l}}t\right),$$

for all $x_i \in \mathcal{A}_i$, $t > 0$ and $l \geq 0$. Then, we have

$$\mathcal{P}\Big(\frac{1}{2^{3\lambda(l+1)}}\mathcal{G}_k\left(x_1, \dots, 2^{\lambda(l+1)}x_k, \dots, x_n\right) - \frac{1}{2^{3\lambda l}}\mathcal{G}_k\left(x_1, \dots, 2^{\lambda l}x_k, \dots, x_n\right), t\Big)$$
$$\geq_L M\left(x_k, \alpha^{l+1}t\right),$$

for all $x_i \in \mathcal{A}_i$, $t > 0$ and $l \geq 0$. Hence

$$\mathcal{P}\left(\frac{1}{2^{3\lambda(l+1)}}\mathcal{G}_k\left(x_1, \dots, 2^{\lambda(l+1)}x_k, \dots, x_n\right) - \frac{1}{2^{3\lambda l}}\mathcal{G}_k\left(x_1, \dots, 2^{\lambda l}x_k, \dots, x_n\right), t\right)$$
$$\geq_L \mathcal{T}_{j=l}^{l+p}\mathcal{P}\left(\frac{1}{2^{3\lambda(p+j)}}\mathcal{G}_k\left(x_1, \dots, 2^{\lambda(p+j)}x_k, \dots, x_n\right)\right.$$
$$\left.-\frac{1}{2^{3\lambda j}}\mathcal{G}_k\left(x_1, \dots, 2^{\lambda j}x_k, \dots, x_n\right), t\right)$$
$$\geq_L \mathcal{T}_{j=l}^{l+p}M\left(x_k, \alpha^{j+1}t\right).$$

By (2.13), the sequence $\left\{\frac{1}{2^{3\lambda l}}\mathcal{G}_k\left(x_1, \dots, 2^{\lambda l}x_k, \dots, x_n\right)\right\}_{l\in\mathbb{N}}$ is Cauchy in $\mathcal{B}$ and by the completeness of $\mathcal{B}$, this sequence is convergent. Hence, we can define the mapping $\mathcal{D}_k : \mathcal{A}_1 \times \cdots \times \mathcal{A}_n \to \mathcal{B}$ by

$$\lim_{l\to\infty}\mathcal{P}\left(\frac{1}{2^{3\lambda l}}\mathcal{G}_k\left(x_1, \dots, 2^{\lambda l}x_k, \dots, x_n\right) - \mathcal{D}_k\left(x_1, \dots, x_k, \dots, x_n\right), t\right) = 1_\ell,$$

for all $x_i \in \mathcal{A}_i$ and $t > 0$. The rest of the proof is similar to the proof of Theorem 2.1. □

3. Stability of Partial Ternary Cubic $*$-Derivation on Non-Archimedean ℓ-Fuzzy C^*-Ternary Algebras

A complex non-Archimedean ℓ-fuzzy $*$-Banach algebra $(\mathcal{B}, \mathcal{P}, \mathcal{T}, \mathcal{T}')$, which has a ternary product $(x, y, z) \mapsto [x, y, z]$ of $\mathcal{B}^3$ into $\mathcal{B}$ is a non-Archimedean ℓ-fuzzy C^*-ternary algebra if the product is linear on each variable and

(i) $[x, y, [z, u, v]] = [a, [u, z, y], v] = [[x, y, z], u, v]$;
(ii) $\|[x, y, z]\| \leq \|x\|\,\|y\|\,\|z\|$;
(iii) $\|[x, x, x]\| = \|x\|^3$,

for all $x, y, z, u, v \in \mathcal{B}$.

If $(\mathcal{B}, \mathcal{P}, \mathcal{T}, \mathcal{T}')$ has the element e so that $x = [x, e, e] = [e, e, x]$ for all $x \in \mathcal{B}$, then e is called the unite element of the non-Archimedean ℓ-fuzzy C^*-ternary algebra. If for $x \in \mathcal{B}$, we have $[e, x, e] = x^*$, then $*$ is an involution on the C^*-ternary algebra.

In this section, assume that $\mathcal{A}_1, \mathcal{A}_2, ..., \mathcal{A}_n$ are non-Archimedean ℓ-fuzzy $*$-normed ternary algebras over $\mathbb{C}$, and $\mathcal{B}$ is a non-Archimedean ℓ-fuzzy Banach C^*-ternary algebra.

Theorem 3.1. *Let $\mathcal{G}_k : \mathcal{A}_1 \times \cdots \times \mathcal{A}_n \to \mathcal{B}$ be a mapping with*

$$\mathcal{G}_k(x_1, \ldots, 0_k, \ldots, x_n) = 0_{\mathcal{B}}.$$

Suppose that there exists an ℓ-fuzzy set Ψ_k on $\mathcal{A}_k \times \mathcal{A}_k \times \mathcal{A}_k \times [0, \infty)$ and a cubic mapping $\pi_k : \mathcal{A}_k \to \mathcal{B}$ such that (2.1)-(2.4) hold. Also assume that

$$\begin{aligned}&\mathcal{P}\left(\mathcal{G}_k\left(x_1, \ldots, a_k^*, \ldots, x_n\right) - \mathcal{G}_k\left(x_1, \ldots, a_k, \ldots, x_n\right)^*, t\right)\\ &\qquad \geq_L \Psi_k\left(a_k, 0_k, 0_k, t\right),\end{aligned}$$

for all $a_k \in \mathcal{A}_k, x_i \in \mathcal{A}_i (i \neq k)$ and $t > 0$. Then there exists a unique k-th partial cubic $*$-derivation $\mathcal{D}_k : \mathcal{A}_1 \times \ldots \times \mathcal{A}_n \to \mathcal{B}$ such that

$$\begin{aligned}&\mathcal{P}\left(\mathcal{G}_k\left(x_1, \ldots, x_k, \ldots, x_n\right) - \mathcal{D}_k\left(x_1, \ldots, x_k, \ldots, x_n\right), t\right)\\ &\qquad \geq_L \mathcal{T}_{j=1}^{\infty} M\left(x_k, \frac{\alpha^{j+1}}{|2|^{\lambda j}} t\right),\end{aligned}$$

for all $x_i \in \mathcal{A}_i$ and $t > 0$ where

$$M(x_k, t) := \mathcal{T}\left(\Psi_k\left(x_k, 0_k, 0_k, t\right), \Psi_k\left(2x_k, 0_k, 0_k, t\right), \ldots, \Psi_k\left(2^{\lambda-1} x_k, 0_k, 0_k, t\right)\right)$$

for all $x_k \in \mathcal{A}_k$ and $t > 0$.

Proof. By a similar argument to that used the proof of theorem 2.1, there exists a unique $k-th$ partial ternary cubic derivation $\mathcal{D}_k : \mathcal{A}_1 \times \ldots \times \mathcal{A}_n \to \mathcal{B}$ which satisfy (2.5), and

$$\begin{aligned}&\lim_{l\to\infty} \mathcal{P}\left(\left(2^{3\lambda}\right)^l \mathcal{G}_k\left(x_1, \ldots, \frac{x_k}{2^{\lambda m}}, \ldots, x_n\right) - \mathcal{D}_k\left(x_1, \ldots, x_k, \ldots, x_n\right), t\right)\\ &\qquad = 1_\ell,\end{aligned}$$

for all $x_i \in \mathcal{A}_i$ and $t > 0$. So, we have

$$\begin{aligned}&\mathcal{P}\left(\mathcal{D}_k\left(x_1, \ldots, a_k^*, \ldots, x_n\right) - \mathcal{D}_k\left(x_1, \ldots, a_k, \ldots, x_n\right)^*, t\right)\\ &\quad = \lim_{l\to\infty} \mathcal{P}\left(\left(2^{3\lambda}\right)^l \mathcal{G}_k\left(x_1, \ldots, \frac{x_k^*}{2^{\lambda m}}, \ldots, x_n\right)\right.\\ &\qquad \left. - \left(2^{3\lambda}\right)^l \mathcal{G}_k\left(x_1, \ldots, \frac{x_k}{2^{\lambda l}}, \ldots, x_n\right)^*, t\right)\\ &\quad = \lim_{l\to\infty} \mathcal{P}\left(\left(2^{3\lambda}\right)^l \mathcal{G}_k\left(x_1, \ldots, \left(\frac{x_k}{2^{\lambda l}}\right)^*, \ldots, x_n\right)\right.\\ &\qquad \left. - \left(2^{3\lambda}\right)^l \mathcal{G}_k\left(x_1, \ldots, \frac{x_k}{2^{\lambda l}}, \ldots, x_n\right)^*, t\right)\\ &\quad \geq_L \lim_{l\to\infty} \Psi_k\left(a_k, 0_k, 0_k, \frac{\alpha^l}{|2^\lambda|^l} t\right)\\ &\quad = 1_\ell,\end{aligned}$$

for all $x_k \in \mathcal{A}_k, x_i \in \mathcal{A}_i (i \neq k)$ and $t > 0$. □

Corollary 3.2. *Let $(\mathcal{X}, \mathcal{P}, \mathcal{T})$ be a non-Archimedean ℓ-fuzzy Banach space over $\mathbb{K}$ under a t-norm Hadžić-type ($\mathcal{T} \in \mathcal{H}$). Let $\mathcal{G}_k : \mathcal{A}_1 \times \cdots \times \mathcal{A}_n \to \mathcal{B}$ be a mapping with $G_k(x_1, \ldots, 0_k, \ldots, x_n) = 0_{\mathcal{B}}$. Suppose that there exists an ℓ-fuzzy set Ψ_k on $\mathcal{A}_k \times \mathcal{A}_k \times \mathcal{A}_k \times [0, \infty)$ and a cubic mapping $\pi_k : \mathcal{A}_k \to \mathcal{B}$ such that (2.1)-(2.4) hold. Also assume that*

$$\mathcal{P}\left(\mathcal{G}_k\left(x_1, x_2, x_3, \ldots, a_k^*, \ldots, x_n\right) - \mathcal{G}_k\left(x_1, \ldots, a_k, \ldots, x_n\right)^*, t\right) \geq_\ell \Psi_k\left(a_k, 0_k, 0_k, t\right)$$

for all $a_k \in \mathcal{A}_k, x_i \in \mathcal{A}_i (i \neq k)$ and $t > 0$. Then there exists a unique k-th partial cubic $*$-derivatio $\mathcal{D}_k : \mathcal{A}_1 \times \ldots \times \mathcal{A}_n \to \mathcal{B}$ such that

$$\mathcal{P}\left(\mathcal{G}_k\left(x_1, \ldots, x_k, \ldots, x_n\right) - \mathcal{D}_k\left(x_1, \ldots, x_k, \ldots, x_n\right), t\right) \geq_\ell \mathcal{T}_{j=1}^{\infty} M\left(x_k, \frac{\alpha^{j+1}}{|2|^{aj}} t\right)$$

for all $x_i \in \mathcal{A}_i$ and $t > 0$ where

$$M\left(x_k, t\right) := \mathcal{T}\left(\Psi_k\left(x_k, 0_k, 0_k, t\right), \Psi_k\left(2x_k, 0_k, 0_k, t\right), \ldots, \Psi_k\left(2^{\lambda-1} x_k, 0_k, 0_k, t\right)\right)$$

for all $x_k \in \mathcal{A}_k$ and $t > 0$.

Proof. we get the conclusion by applying Proposition 1.1 and Theorem 3.1. □

References

1. M.A. Abolfathi, A. Ebadian and R. Aghalary, *Stability of mixed additive-quadratic Jensen type functional equation in non-Archimedean ℓ-fuzzy normed spaces*, Ann. Univ. Ferrara Sez. VII Sci. Mat., 60(2) (2014), pp. 307-319.
2. M. Amini and R. Saadati, *Topics in fuzzy metric space*, J. Fuzzy. Math., 4 (2003), pp. 765-768.
3. T. Aoki, *On the stability of linear trasformation in Banach spaces*, J. Math. Soc. Japan, 2 (1950), pp. 64-66.
4. B. Arsalan and H. Inceboz, *Nearly k-th Partial Ternary Quadratic $*$-Derivations*, Kyungpook Math. J., 55 (2015), pp. 893-907.
5. A. Cayley, *On the 34 concomitants of the ternary cubic*, Am. J. Math., 4 (1981), pp. 1-15.
6. S.C. Cheng and J.N. Mordeson, *Fuzzy linear operator and fuzzy normed linear spaces*, Bull. Calcutta Math. Soc., 86 (1994), pp. 429-436.

7. P. Czerwik, *Functional Equations and Inequalities in Several Variable*, World Scientific Publishing Company, New Jersey, Hong Kong, Singapore and London, 2002.
8. G. Deschrijver, D. O'Regan, R. Saadati and S.M. Vaezpour, *ℓ-fuzzy Euclidean normed spaces and compactness*, Chaos Solitions Fractals, 42 (2009), pp. 40-45.
9. A. Ebadian, R. Aghalary and M.A. Abolfathi, *On approximate dectic mappings in non-Archimedean spaces: a fixed point approah*, Int. J. Nonlinear Anal. Appl., 5(2) (2014), pp. 111-–122.
10. A. Ebadian, N. Ghobadipour, B. Savadkouhi and M. Eshaghi Gordji, *of a mixed type cubic and quartic functional equation in non-Archimedean ℓ-fuzzy normed spaces*, Thai J. Math., 9 (2011), pp. 243-259.
11. M. Eshaghi, M.B. Savadkouhi, M. Bidkham, C. park and J.R. Lee, *Nearly partial derivations on Banach ternary algebras*, J. Math. Stat., 6 (4) (2010) pp. 454-461.
12. P. Găvruta, *A generalization of the Hyers-Ulam-Rassias stability of the approximately additive mappings*, J. Math. Anal. Appl., 184 (1994), pp. 431-436.
13. A. George and p. Veeramani, *On some result in fuzzy metric space*, Fuzzy Sets Syst., 64 (1994), pp. 395-399.
14. J.A. Goguen, *L-fuzzy sets*, J. Math. Anal. Appl., 18 (1967), pp. 145-174.
15. F.Q. Gouvêa, *p-Adic Numbers. An Introduction*, Springer-Verlag, Berlin, 1997.
16. O. Hadžić and E. Pap, *Fixed point Theory in Probabilistic Metric Spaces*, Kluwer Academic, Dordrecht, 2001.
17. O. Hadžić, E. Pap and M. Budincević, *Countable extension of triangular norms and their applications to the fixed point theory in probabilistic metric spaces*, Kybernetica, 38 (2002), pp. 363-381.
18. K. Hensel, *Über eine neue Begundung der Theorie der algebraischen Zahlen*, Jahresbericht der Deutschen Mathematiker-Vereinigung, 6 (1897), pp. 83-88.
19. A. Himbert, *Comptes Rendus del'Acad. Sci.*, Paris, (1985).
20. N.E. Hoseinzadeh, A. Bodaghi and M.R. Mardanbeigi, *Almost Multi-Cubic Mappings and a Fixed point Application*, Sahand Commun. Math. Anal., 17 (3) (2020), pp. 131-142.
21. D.H. Hyers, *On the Stability of the linear functional equation*, Proc. Nat. Acad. Sci. U.S.A., 27 (1941), pp. 222-224.
22. D.H. Hyers, G. Isac and Th.M. Rssias, *Stability of Functional Equation in Several Variables*, Birkhäuse, Basel, 1998.

23. K. Jun and H. Kim, *The gegeralized Hyers-Ulam-Rassias stability of cubic functional equation*, J. Math. Anal. Appl., 274 (2002), pp. 867-878.
24. O. Kaleva and S. Seikkala, *On fuzzy metric spaces*, Fuzzy Set Syst., 12 (3) (1984), pp. 1-7.
25. M. Kapranov, IM. Gelfand and A. Zelevinskii, *Discriminінants, Reesultants and Multidimensional Determinants(Modern Bikhäuser Classics)*, Berlin, (1994).
26. A.K. Katsaras, *Fuzzy topological vector spaces*, Fuzzy Set Syst., 12 (1984), pp. 143-154.
27. R. Kerner, *The cubic chessboard: Geometry and physics*, Class. Quantum Grav., 14 (1997), pp. A203-A225 .
28. A. Khrennikov, *Non-Archimedean Analysis: Quantu Paradoxes, Dynamical Systems and Biological Models*, Math. Appl., vol.427, Kluwer Academic publisher Dordrecht, 1997.
29. S.V. Krishna and K.K.M. Sarma, *Separation of fuzzy normed linear spaces*, Fuzzy Set Syst., 63 (1994) 207-217.
30. A.K. Mirmostafaee and M.S. Moslehian, *Stability of additive mapping in non Archimedean fuzzy normed spaces*, Fuzzy Set Syst., 160 (2009), 1643-1652.
31. A. Najati, B. Noori and M.B. Moghimi, *On Approximation of Some Mixed Functional Equations*, Sahand Commun. Math. Anal., 18 (1) (2021), pp. 35-46.
32. M. Nazarianpoor and G. Sadeghi, *On the stability of the Pexiderized cubic functionalequation in multi-normed spaces*, Sahand Commun. Math. Anal., 9 (1) (2018), pp. 45-83.
33. S. Okabo, *Triple products and Yang-Baxter equation I, II. Octonionic and quaternionic triple systems*, J. Math. Phys., 34(7) (1993), 3273-3291 and 3291-3315.
34. K.H. Park and Y.S. Jung, *Stability of a cubic functional equation on groups*, Bull. Korean Math. Soc., 41 (2004) 347–357.
35. Th.M. Rassias, *On the stability of the linear mapping in Banach spaces*, Proc. Amer. Math. Soc., 72 (1978) 297-300.
36. Th.M. Rassias, *Functional Equation, Inequalities and Applications*, KLuwer Academic publishers Co., Dordrecht, Boston, London, 2003.
37. A.M. Robert, *A Course in p-Adic Analysis,(Graduate Texts in Mathematics)*, Vol.198, Springer-Verlag, New York, 2000.
38. R. Saadati, *On the ℓ-fuzzy topological spaces*, Chaos Solitions Fractals, 37 (2008), pp. 1419-1426.
39. R. Saadati and J. Park, *On the intuitionistic fuzzy topological spaces*, Chaos Solitions Fractals, 27 (2006), pp. 331-344.

40. B. Schweizer and A. Sklar, *Probabilistic Metric Spaces*, Elsevier, North Holand, New York, 1983.
41. S. Shakeri, R. Saadati and C. Park, *Stability of the functional equations in non- Archimedean ℓ-fuzzy normed spaces*, Int. J. Nonlinear Anal. Appl., 1(2) (2010), pp. 72-83.
42. N. Shilkret, *Non-Archimedian Banach algebras*, Ph.D. thesis, Polytechnic University, 1968.
43. S.M. Ulam, *Problem in Modern Mathematics, Chapter VI, Science Editions*, Wiley, New York, 1964.
44. J.Z. Xiao and X.H. Zhu, *Fuzzy normed spaces of operators and its completeness*, Fuzzy Set Syst., 133 (2003) pp. 389-399.
45. L.A. Zadeh, *Fuzzy sets*, Inf. Control, 8 (1965) pp. 338-353.

Department of Mathematics, Faculty of Science, Urmia University, P.O.Box 165, Urmia, Iran.

Email address: m.abolfathi@urmia.ac.ir