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On Subclasses of Analytic Functions Associated with Miller-Ross-Type Poisson Distribution Series

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Abstract. The aim of this article is to obtain some necessary and sufficient conditions for functions, whose coefficients are probabilities of the Miller-Ross-type Poisson distribution series, to belong to certain subclasses of analytic and univalent functions. Furthermore, we consider an integral operator related to the Miller-Ross type Poisson distribution series.

1. Introduction

Let $\mathcal{A}$ stand for the standard class of analytic functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \quad z \in \mathbb{U} = \left\{z \in \mathbb{C} : |z| < 1\right\}, \tag{1.1}$$

and let $\mathcal{S}$ be the class of functions in $\mathcal{A}$ which are univalent in $\mathbb{U}$ (see [8]).

Let us define $\mathcal{T}$ as the subclass of functions $f \in \mathcal{A}$ of the form given by (see [23])

$$f(z) = z - \sum_{k=2}^{\infty} a_k z^k, \quad (a_k \geq 0). \tag{1.2}$$

Class $\mathcal{T}$ is given by Silverman [23]. Many studies have been done on class $\mathcal{T}$ and its subclasses in the literature. Altıntaş [3] defined the following class as being the subclass of $\mathcal{T}$.

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Definition 1.1 ([3])**.** A function $f \in \mathcal{T}$ is said to be in the class $T(\alpha,\lambda)$, if the following condition is satisfied:

$$Re\left(f'(z)+\lambda z f''(z)\right) > \alpha, \tag{1.3}$$

$$(\lambda \geq 0, 0 \leq \alpha < 1, z \in U).$$

Furthermore, Altıntaş and Owa [4] introduced the following class as a subclass of $\mathcal{T}$.

Definition 1.2 ([4])**.** A function $f \in \mathcal{T}$ is said to be in the class $C(\alpha,\beta)$, if the following condition is satisfied:

$$Re\left(\frac{f'(z)+zf''(z)}{f'(z)+\beta z f''(z)}\right) > \alpha, \tag{1.4}$$

$$(0 \leq \alpha < 1, 0 \leq \beta < 1, z \in U).$$

Kenneth S. Miller and Bertram Ross proposed a special function, which is called the Miller-Ross function and is defined as follows:

$$\mathbb{E}_{\nu,c}(z) = z^{\nu} e^{cz} \gamma^{*}(\nu, cz),$$

where γ^{*} is the incomplete gamma function (p.314, [13]). Using the properties of the incomplete gamma functions the Miller-Ross function can be written as follows:

$$\mathbb{E}_{\nu,c}(z) = z^{\nu} \sum_{n=0}^{\infty} \frac{(cz)^n}{\Gamma(n+\nu+1)}, \quad z, c, \nu \in \mathbb{C}. \tag{1.5}$$

In recent years a large literature has evolved on the use of distribution series such as Poisson, Pascal, Borel , etc. in geometric function theory. Many researchers have examined some important features in the field of geometric function theory, such as coefficient estimates, inclusion relations, and conditions of being in some known classes. They used different probability distributions, see for example [2, 5, 7, 10, 16–19, 21, 27–29].

We now recall that a discrete random variable X whose probability mass function is given by

$$P\left[X = i\right] = \frac{e^{-m} m^{i}}{i!}, \quad i = 0, 1, 2, \ldots, \; m > 0,$$

is said to have a Poisson distribution with parameter m.

Porwal and Dixit [20] have recently introduced Mittag-Leffler-type Poisson distributions and derived their moment-generating functions. Bajpai [6] introduced Mittag–Leffler-type Poisson distribution series. After that Murugusundaramoorthy and El-Deeb [15] studied the Mittag-Leffler type Borel distribution. Lately, Srivastava et al. [12] introduced the Miller-Ross-type Poisson distribution which is a two-parameter Poisson distribution and obtained moments, moment generating function.

Motivated by results on connections between various subclasses of analytic univalent functions using special functions and distribution series [1, 9, 11, 14, 20, 22, 24–26] we obtain some necessary and sufficient conditions for the Miller–Ross-type Poisson distribution series to be in the classes $T(\alpha,\lambda)$ and $C(\alpha,\beta)$. First, we recall the definition of the Miller–Ross-type distribution.

The probability mass function of the Miller–Ross-type Poisson distribution is given by

$$P_{\nu,c}(m;k)=\frac{m^{\nu}(cm)^{k}}{\mathbb{E}_{\nu,c}(m)\Gamma(k+\nu+1)},\quad k=0,1,2,\dots, \tag{1.6}$$

where $\nu>-1$, $c>0$ and $\mathbb{E}_{\nu,c}(z)$ is Miller-Ross function given in (1.5).

We introduce a power series whose coefficients are Miller–Ross-type Poisson distribution:

$$\mathbb{F}^{m}_{\nu,c}(z)=z+\sum_{n=2}^{\infty}\frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)\mathbb{E}_{\nu,c}(m)}z^{n}.$$

Now, we introduce the series

$$\begin{aligned}\mathbb{K}^{m}_{\nu,c}(z)&=2z-\mathbb{F}^{m}_{\nu,c}(z)\\&=z-\sum_{n=2}^{\infty}\frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)\mathbb{E}_{\nu,c}(m)}z^{n}.\end{aligned} \tag{1.7}$$

To establish our main results, we will use the following Lemmas:

Lemma 1.3 ([3])**.** *The function $f(z)$ defined by* (1.2) *belongs to the class $T(\alpha,\lambda)$ if and only if*

$$\sum_{n=2}^{\infty}n\left(1-\lambda+n\lambda\right)a_{n}\le 1-\alpha. \tag{1.8}$$

Lemma 1.4 ([4])**.** *The function $f(z)$ defined by* (1.2) *belongs to the class $C(\alpha,\beta)$ if and only if*

$$\sum_{n=2}^{\infty}n\left(n-\beta\alpha n-\alpha+\beta\alpha\right)a_{n}\le 1-\alpha. \tag{1.9}$$

2. Main Results

Theorem 2.1. *If $\nu>-1$ and $c>0$, then $\mathbb{K}^{m}_{\nu,c}(z)\in T(\alpha,\lambda)$ if and only if*

$$\frac{c}{\mathbb{E}_{\nu,c}(m)}\Big\{m^{2}\lambda\mathbb{E}_{\nu-1,c}(m)+m\left[2\lambda(1-\nu)+1\right]\mathbb{E}_{\nu,c}(m)+(1-\lambda\nu)(1-\nu)\mathbb{E}_{\nu+1,c}(m)\Big\}\le 1-\alpha. \tag{2.1}$$

Proof. Since

$$\mathbb{K}_{\nu,c}^{m}(z) = z - \sum_{n=2}^{\infty} \frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)\mathbb{E}_{\nu,c}(m)} z^{n},$$

and by virtue of Lemma 1.3, it suffices to show that

$$\sum_{n=2}^{\infty} n\,(1-\lambda+n\lambda)\, \frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)} \frac{1}{\mathbb{E}_{\nu,c}(m)} \leq 1-\alpha. \tag{2.2}$$

It follows from (2.2) that

$$\begin{aligned}
&\sum_{n=2}^{\infty} n\big(1-\lambda+n\lambda\big) \frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)} \frac{1}{\mathbb{E}_{\nu,c}(m)} \\
&= \frac{1}{\mathbb{E}_{\nu,c}(m)} \Bigg\{ \sum_{n=2}^{\infty} \lambda\big[(\nu+n-1)(\nu+n-2)\big] \frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)} \\
&\quad + \sum_{n=2}^{\infty} \lambda\big[(\nu+n-1)(3-2\nu)+(1-\nu)^{2}\big] \frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)} \\
&\quad + \sum_{n=2}^{\infty} (1-\lambda)\big[(\nu+n-1)+(1-\nu)\big] \frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)} \Bigg\} \\
&= \frac{1}{\mathbb{E}_{\nu,c}(m)} \Bigg\{ \lambda \sum_{n=2}^{\infty} \frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu-2)} + \big[2\lambda(1-\nu)+1\big] \sum_{n=2}^{\infty} \frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu-1)} \\
&\quad + \big[(1-\lambda\nu)(1-\nu)\big] \sum_{n=2}^{\infty} \frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)} \Bigg\} \\
&= \frac{cm}{\mathbb{E}_{\nu,c}(m)} \Bigg\{ \lambda \sum_{n=0}^{\infty} \frac{m^{\nu}(cm)^{n}}{\Gamma(n+\nu)} + \big[2\lambda(1-\nu)+1\big] \sum_{n=0}^{\infty} \frac{m^{\nu}(cm)^{n}}{\Gamma(n+\nu+1)} \\
&\quad + \big[(1-\lambda\nu)(1-\nu)\big] \sum_{n=0}^{\infty} \frac{m^{\nu}(cm)^{n}}{\Gamma(n+\nu+2)} \Bigg\} \\
&= \frac{c}{\mathbb{E}_{\nu,c}(m)} \Bigg\{ m^{2}\lambda \mathbb{E}_{\nu-1,c}(m) + m\big[2\lambda(1-\nu)+1\big]\mathbb{E}_{\nu,c}(m) \\
&\quad + (1-\lambda\nu)(1-\nu)\mathbb{E}_{\nu+1,c}(m) \Bigg\} \\
&\leq 1-\alpha
\end{aligned}$$

by the given hypothesis. This completes the proof of Theorem 2.1. □

The result given in Theorem 2.1 generalizes and improves the results given by Altınkaya and Yalçın [2]. Taking $\nu = 0$ and $\lambda = 0$ in Theorem 2.1, we get the following corollary.

Corollary 2.2. *If $m > 0$, then $\mathbb{K}^m_{0,c}(z) \in T(\alpha, 0)$ if and only if,*

$$\frac{c}{\mathbb{E}_{0,c}(m)}\left[m\mathbb{E}_{0,c}(m) + \mathbb{E}_{1,c}(m)\right] = mc + 1 - e^{-cm} \\ \leq 1 - \alpha.$$

Example 2.3. Taking $c = 1$ and $\nu = 0$ in (1.7), we get

$$\mathbb{K}^m_{0,1}(z) = z - \sum_{n=2}^{\infty} \frac{e^{-m}(m)^{n-1}}{(n-1)!} z^n.$$

In view of Lemma 1.3, we obtain

$$\sum_{n=2}^{\infty} na_n = \sum_{n=2}^{\infty} n\frac{e^{-m}(m)^{n-1}}{(n-1)!} \\ = m + 1 - e^{-m},$$

thus $\mathbb{K}^m_{0,1}(z) \in T(\alpha, 0)$ if and only if $m + 1 - e^{-m} \leq 1 - \alpha$.

This necessary and sufficient condition can also be easily obtained by taking $c = 1$ in Corollary 2.2.

Theorem 2.4. *If $\nu > -1$ and $c > 0$, then $\mathbb{K}^m_{\nu,c}(z) \in C(\alpha, \beta)$ if and only if*

(2.3)
$$\frac{c}{\mathbb{E}_{\nu,c}(m)}\left\{m^2(1-\beta\alpha)\mathbb{E}_{\nu-1,c}(m) + m\left[2(1-\nu)(1-\beta\nu) + (1-\alpha)\right]\mathbb{E}_{\nu,c}(m) \right. \\ \left. +(1-\nu)\left[(1-\nu) - \alpha(1-\beta\nu)\right]\mathbb{E}_{\nu+1,c}(m)\right\} \leq 1 - \alpha.$$

Proof. Since

$$\mathbb{K}^m_{\nu,c}(z) = z - \sum_{n=2}^{\infty} \frac{m^\nu (cm)^{n-1}}{\Gamma(n+\nu)\mathbb{E}_{\nu,c}(m)} z^n.$$

and by virtue of Lemma 1.4, it suffices to show that

$$\sum_{n=2}^{\infty} n\left(n - \beta\alpha n - \alpha + \beta\alpha\right)\frac{m^\nu (cm)^{n-1}}{\Gamma(n+\nu)}\frac{1}{\mathbb{E}_{\nu,c}(m)} \leq 1 - \alpha. \tag{2.4}$$

It follows from (2.4) that

$$\sum_{n=2}^{\infty} n\left(n - \beta\alpha n - \alpha + \beta\alpha\right)\frac{m^\nu (cm)^{n-1}}{\Gamma(n+\nu)}\frac{1}{\mathbb{E}_{\nu,c}(m)} \\ = \frac{1}{\mathbb{E}_{\nu,c}(m)}\left\{\sum_{n=2}^{\infty}(1-\beta\alpha)\left[(\nu+n-1)(\nu+n-2)\right]\frac{m^\nu (cm)^{n-1}}{\Gamma(n+\nu)}\right.$$

$$+\sum_{n=2}^{\infty}(1-\beta\alpha)\big[(\nu+n-1)(3-2\nu)+(1-\nu)^2\big]\frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)}$$
$$\left.-\sum_{n=2}^{\infty}\alpha(1-\beta)\big[(\nu+n-1)+(1-\nu)\big]\frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)}\right\}$$
$$=\frac{1}{\mathbb{E}_{\nu,c}(m)}\left\{(1-\beta\alpha)\sum_{n=2}^{\infty}\frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu-2)}\right.$$
$$+\big[2(1-\nu)(1-\beta\alpha)+1-\alpha\big]\sum_{n=2}^{\infty}\frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu-1)}$$
$$\left.+(1-\nu)\big[(1-\nu)-\alpha(1-\beta\nu)\big]\sum_{n=2}^{\infty}\frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)}\right\}$$
$$=\frac{cm}{\mathbb{E}_{\nu,c}(m)}\left\{(1-\beta\alpha)\sum_{n=0}^{\infty}\frac{m^{\nu}(cm)^{n}}{\Gamma(n+\nu)}\right.$$
$$+\big[2(1-\nu)(1-\beta\alpha)+1-\alpha\big]\sum_{n=0}^{\infty}\frac{m^{\nu}(cm)^{n}}{\Gamma(n+\nu+1)}$$
$$\left.+(1-\nu)\big[(1-\nu)-\alpha(1-\beta\nu)\big]\sum_{n=0}^{\infty}\frac{m^{\nu}(cm)^{n}}{\Gamma(n+\nu+2)}\right\}$$
$$=\frac{c}{\mathbb{E}_{\nu,c}(m)}\left\{m^2(1-\beta\alpha)\mathbb{E}_{\nu-1,c}(m)\right.$$
$$+m\big[2(1-\nu)(1-\beta\alpha)+1-\alpha\big]\mathbb{E}_{\nu,c}(m)$$
$$\left.+(1-\nu)\big[(1-\nu)-\alpha(1-\beta\nu)\big]\mathbb{E}_{\nu+1,c}(m)\right\}$$
$$\leq 1-\alpha,$$

by the given hypothesis. This completes the proof of Theorem 2.4. □

3. Integral Operators

In this section, we give conditions for the integral operators defined as follows:

$$\mathbb{G}^{m}_{\nu,c}(z)=\int_0^z\frac{\mathbb{K}^{m}_{\nu,c}(t)}{t}dt. \tag{3.1}$$

Theorem 3.1. *If $\nu>-1$ and $c>0$, then $\mathbb{G}^{m}_{\nu,c}(z)\in T(\alpha,\lambda)$ if and only if*

$$\frac{c}{\mathbb{E}_{\nu,c}(m)}\left\{(1-\lambda\nu)\mathbb{E}_{\nu+1,c}(m)+m\lambda\mathbb{E}_{\nu,c}(m)\right\}\leq 1-\alpha. \tag{3.2}$$

Proof. From (3.1), we can write

$$\mathbb{G}^m_{\nu,c}(z) = z - \sum_{n=2}^{\infty} \frac{m^\nu (cm)^{n-1}}{\Gamma(n+\nu)\mathbb{E}_{\nu,c}(m)} \frac{z^n}{n}.$$

According to Lemma 1.3, it is enough to show that

$$\sum_{n=2}^{\infty} n\,(1-\lambda+n\lambda)\,\frac{m^\nu (cm)^{n-1}}{\Gamma(n+\nu)} \frac{1}{\mathbb{E}_{\nu,c}(m)} \frac{z^n}{n} \leq 1-\alpha. \tag{3.3}$$

It follows from (3.3) that

$$\begin{aligned}
&\sum_{n=2}^{\infty} n\big(1-\lambda+n\lambda\big)\frac{m^\nu (cm)^{n-1}}{\Gamma(n+\nu)\mathbb{E}_{\nu,c}(m)} \frac{z^n}{n} \\
&\quad= \sum_{n=2}^{\infty} \big(1-\lambda+n\lambda\big)\frac{m^\nu (cm)^{n-1}}{\Gamma(n+\nu)\mathbb{E}_{\nu,c}(m)} z^n \\
&\quad= \frac{1}{\mathbb{E}_{\nu,c}(m)}\left\{ \sum_{n=2}^{\infty} \lambda\big[(\nu+n-1)+(1-\nu)\big]\frac{m^\nu (cm)^{n-1}}{\Gamma(n+\nu)} \right.\\
&\qquad \left. + \sum_{n=2}^{\infty}(1-\lambda)\frac{m^\nu (cm)^{n-1}}{\Gamma(n+\nu)}\right\} \\
&\quad= \frac{1}{\mathbb{E}_{\nu,c}(m)}\left\{ \sum_{n=2}^{\infty}(1-\lambda\nu)\frac{m^\nu (cm)^{n-1}}{\Gamma(n+\nu)} + \lambda\sum_{n=2}^{\infty}\frac{m^\nu (cm)^{n-1}}{\Gamma(n+\nu-1)}\right\} \\
&\quad= \frac{cm}{\mathbb{E}_{\nu,c}(m)}\left\{ \sum_{n=0}^{\infty}(1-\lambda\nu)\frac{m^\nu (cm)^{n}}{\Gamma(n+\nu+2)} + \lambda\sum_{n=0}^{\infty}\frac{m^\nu (cm)^{n}}{\Gamma(n+\nu+1)}\right\} \\
&\quad= \frac{c}{\mathbb{E}_{\nu,c}(m)}\left\{(1-\lambda\nu)\mathbb{E}_{\nu+1,c}(m) + m\lambda\mathbb{E}_{\nu,c}(m)\right\}.
\end{aligned}$$

The last expression is bounded above by $1-\alpha$ if and only if (3.2) is satisfied. This completes the proof. □

Theorem 3.2. *If $\nu > -1$ and $c > 0$, then $\mathbb{G}^m_{\nu,c}(z) \in C(\alpha,\beta)$ if and only if*

$$\frac{c}{\mathbb{E}_{\nu,c}(m)}\Big\{\big[(1-\nu)-\alpha(1-\beta\nu)\big]\mathbb{E}_{\nu+1,c}(m) + m(1-\beta\nu)\mathbb{E}_{\nu,c}(m)\Big\} \leq 1-\alpha. \tag{3.4}$$

Proof. From (3.1), we can write

$$\mathbb{G}^m_{\nu,c}(z) = z - \sum_{n=2}^{\infty} \frac{m^\nu (cm)^{n-1}}{\Gamma(n+\nu)\mathbb{E}_{\nu,c}(m)} \frac{z^n}{n}.$$

According to Lemma 1.4, it is enough to show that

$$\sum_{n=2}^{\infty} n\left(n-\beta\alpha n-\alpha+\beta\alpha\right)\frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)}\frac{1}{\mathbb{E}_{\nu,c}(m)}\frac{z^n}{n}\leq 1-\alpha. \tag{3.5}$$

It follows from (3.5) that

$$\begin{aligned}
&\sum_{n=2}^{\infty} n\big(n-\beta\alpha n-\alpha+\beta\alpha\big)\frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)\mathbb{E}_{\nu,c}(m)}\frac{z^n}{n}\\
&\quad=\sum_{n=2}^{\infty}\big(n-\beta\alpha n-\alpha+\beta\alpha\big)\frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)\mathbb{E}_{\nu,c}(m)}z^n\\
&\quad=\frac{1}{\mathbb{E}_{\nu,c}(m)}\Bigg\{\sum_{n=2}^{\infty}(1-\beta\nu)\big[(\nu+n-1)\\
&\qquad+(1-\nu)\big]\frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)}+\sum_{n=2}^{\infty}\alpha(1-\beta)\frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)}\Bigg\}\\
&\quad=\frac{1}{\mathbb{E}_{\nu,c}(m)}\Bigg\{\sum_{n=2}^{\infty}\big[(1-\nu)-\alpha(1-\beta\nu)\big]\frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)}\\
&\qquad+(1-\beta\nu)\sum_{n=2}^{\infty}\frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu-1)}\Bigg\}\\
&\quad=\frac{cm}{\mathbb{E}_{\nu,c}(m)}\Bigg\{\sum_{n=0}^{\infty}\big[(1-\nu)-\alpha(1-\beta\nu)\big]\frac{m^{\nu}(cm)^{n}}{\Gamma(n+\nu+2)}\\
&\qquad+(1-\beta\nu)\sum_{n=0}^{\infty}\frac{m^{\nu}(cm)^{n}}{\Gamma(n+\nu+1)}\Bigg\}\\
&\quad=\frac{c}{\mathbb{E}_{\nu,c}(m)}\Bigg\{\big[(1-\nu)-\alpha(1-\beta\nu)\big]\mathbb{E}_{\nu+1,c}(m)+m(1-\beta\nu)\mathbb{E}_{\nu,c}(m)\Bigg\}.
\end{aligned}$$

The last expression is bounded above by $1-\alpha$ if and only if (3.4) is satisfied. This completes the proof. □

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