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On Deferred Statistical Convergence of Sequences in Neutrosophic Normed Spaces

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Abstract. In this paper, we introduce the notion of deferred statistical convergence in the neutrosophic normed spaces as an extension of statistical convergence, λ-statistical convergence, and lacunary statistical convergence. We investigate a few fundamental properties of the newly introduced notion. Finally, we introduce the concept of deferred statistical Cauchy sequence and show that deferred statistical Cauchy sequences are equivalent to deferred statistical convergent sequences in the neutrosophic normed spaces.

1. Introduction

In 1951, the idea of statistical convergence was introduced independently by Fast [9] and Steinhaus [32] with the aim of providing deeper insights into summability theory. Later on, it was further investigated from the sequence space point of view by Fridy [12], Salat [27], and many mathematicians across the globe. Following their work, several investigations and generalizations have been made by Altinok et. al. [2], Hazarika and Esi [13], Mursaleen [24], Savas and Gurdal [29], Tripathy and Sen [33], and many others [4, 7, 8, 28, 30]. Statistical convergence has become one of the most active areas of research due to its wide applicability in various branches of mathematics such as number theory, mathematical analysis, probability theory, etc.

In 1932, Agnew [1] generalized Cesàro mean to deferred Cesàro mean to obtain a more useful method having stronger features. Then, in 2016, Küçükaslan and Yilmaztürk [20] utilize this to introduce the notion of

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deferred statistical convergence mainly as a natural generalization of statistical convergence, lambda statistical convergence, and lacunary statistical convergence. They proved some fundamental properties and established a few implication relationships of this convergence with strong deferred Cesàro mean and statistical convergence. For more details related to deferred statistical convergence and its generalizations, [6] can be addressed where many more references can be found.

The concept of fuzzy sets was first introduced by Zadeh [35] in the year 1965 which was an extension of the classical set-theoretical concept. Nowadays it has wide applicability in different branches of science and engineering. The theory of fuzzy sets cannot always cope with the lack of knowledge of membership degrees. To overcome the drawbacks, in 1986 Atanassov [3] introduced intuitionistic fuzzy sets as an extension of fuzzy sets. Intuitionistic fuzzy sets have been widely used to solve various decision-making problems.

Many times, decision-makers face some hesitations besides going to direct approaches (i.e., yes or no) in decision making. In addition, we can obtain a tricomponent outcome in some real events like sports, the procedure for voting, etc. Considering all in 2005, Smarandache [31] introduced the notion of neutrosophic set as a generalization of both fuzzy set and intuitionistic fuzzy set. An element belonging to a neutrosophic set consists of a triplet namely truth-membership function (T), indeterminacy-membership function (F), and falsity-membership function (I). A neutrosophic set is determined as a set where every component of the universe has a degree of T, F, and I.

The notion of fuzzy normed space was introduced by Felbin [10] in the year 1992. Later on, in 2006 the concept of intuitionistic fuzzy normed spaces was introduced by Saadati and Park [26]. In 2008, statistical convergence in intuitionistic fuzzy normed spaces was introduced and investigated by Karakus et. al. [14]. Furthermore in 2018, Melliani et. al. [21] generalized it to deferred statistical convergence. For more details on statistical convergence and its related generalizations in intuitionistic fuzzy normed spaces, one may refer to [5, 18, 23, 25]. Recently, Kirisci and Simsek [19] introduced neutrosophic normed linear spaces and investigated the notion of statistical convergence. Following their work, several researchers investigated various notions of convergence of sequences in the neutrosophic normed spaces. For more details, one may refer to [15–17]. Research on the convergence of sequences in neutrosophic normed linear spaces has not yet gained much ground and it is still in its infant stage. The research carried out so far shows a strong analogy in the behavior of convergence of sequences in neutrosophic normed spaces.

2. Definitions and Preliminaries

Definition 2.1 ([11])**.** Let K be a subset of the positive integers $\mathbb{N}$ and K_n denotes the set $\{k \leq n : k \in K\}$. The natural density of K is denoted and defined by

$$\delta(K) = \lim_{n\to\infty} \frac{|K_n|}{n}.$$

Here, $|K_n|$ denotes the cardinality of the set K_n.

Definition 2.2 ([12])**.** A sequence (x_k) is said to be statistical convergent to l if for each $\varepsilon > 0$,

$$\delta(A(\varepsilon)) = 0,$$

where $A(\varepsilon) = \{k \in \mathbb{N} : |x_k - l| \geq \varepsilon\}$. In this case, l is called the statistical limit of the sequence (x_k) and symbolically it is expressed as $x_k \xrightarrow{st} l$.

Definition 2.3 ([34])**.** Let K be a subset of the positive integers $\mathbb{N}$ and $K_{p,q}(n)$ denotes the set $\{p(n)+1 \leq k \leq q(n) : k \in K\}$, where $p = (p(n))$ and $q = (q(n))$ are the sequence of non-negative integers satisfying

$$p(n) < q(n), \quad \forall n \in \mathbb{N} \text{ and } \lim_{n\to\infty} q(n) = \infty. \tag{2.1}$$

The deferred density of K is denoted and defined by

$$\delta_{p,q}(K) = \lim_{n\to\infty} \frac{1}{q(n)-p(n)} |K_{p,q}(n)|.$$

Definition 2.4 ([20])**.** Let (x_k) be a real-valued sequence. Then, (x_k) is said to be strong deferred convergent to $l \in \mathbb{R}$ if

$$\lim_{n\to\infty} \frac{1}{q(n)-p(n)} \sum_{k=p(n)+1}^{q(n)} |x_k - l| = 0,$$

holds and it is denoted by $x_k \to l(D[p,q])$.

Definition 2.5 ([20])**.** Let (x_k) be a real valued sequence. Then (x_k) is said to be deferred statistical convergent to $l \in \mathbb{R}$ if for every $\varepsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{q(n)-p(n)} |\{p(n)+1 \leq k \leq q(n) : |x_k - l| \geq \varepsilon\}| = 0.$$

Symbolically, $x_k \to l(DS[p,q])$.

Definition 2.6 ([22])**.** A binary operation $\circ : [0,1] \times [0,1] \to [0,1]$, is said to be a continuous t-norm if the following conditions are satisfied:

(i) $\circ$ is associative and commutative,
(ii) $\circ$ is continuous,
(iii) $s \circ 1 = s$, for all $s \in [0,1]$,

(iv) $s \circ t \leq u \circ v$ whenever $s \leq u$ and $t \leq v$, for all $s, t, u, v \in [0, 1]$.

Definition 2.7 ([22])**.** A binary operation $\bullet : [0, 1] \times [0, 1] \to [0, 1]$, is said to be a continuous t-conorm if the following conditions are satisfied:

(i) $\bullet$ is associative and commutative,
(ii) $\bullet$ is continuous,
(iii) $s \bullet 0 = s$, for all $s \in [0, 1]$,
(iv) $s \bullet t \leq u \bullet v$ whenever $s \leq u$ and $t \leq v$, for all $s, t, u, v \in [0, 1]$.

Definition 2.8 ([31])**.** Let X be the universe of discourse. Then, the set $A_{NS} \subseteq X$ defined by

$$A_{NS} = \{\langle u, \mathcal{R}_A(u), \mathcal{T}_A(u), \mathcal{W}_A(u)\rangle : u \in X\}$$

is called a neutrosophic set, where $\mathcal{R}_A(u), \mathcal{T}_A(u), \mathcal{W}_A(u) : X \to [0, 1]$ represent the degree of truth-membership, degree of indeterminacy-membership, and degree of false-membership respectively, with

$$0 \leq \mathcal{R}_A(u) + \mathcal{T}_A(u) + \mathcal{W}_A(u) \leq 3.$$

Definition 2.9 ([19])**.** Let F be a vector space and

$$\mathcal{N} = \{\langle u, \mathcal{R}(u), \mathcal{T}(u), \mathcal{W}(u)\rangle : u \in F\},$$

be a normed space (NS) such that $\mathcal{R}, \mathcal{T}, \mathcal{W} : F \times \mathbb{R}^+ \to [0, 1]$. Let $\circ$ and $\bullet$ be the continuous t-norm and continuous t-conorm, respectively. Then the four-tuple $V = (F, \mathcal{N}, \circ, \bullet)$ is called a neutrosophic normed space (NNS) if the following conditions are hold, for all $u, v \in F$ and $\lambda, \mu > 0$ and for each $\sigma \neq 0$:

(i) $0 \leq \mathcal{R}(u, \lambda) \leq 1$, $0 \leq \mathcal{T}(u, \lambda) \leq 1$, $0 \leq \mathcal{W}(u, \lambda) \leq 1$,
(ii) $\mathcal{R}(u, \lambda) + \mathcal{T}(u, \lambda) + \mathcal{W}(u, \lambda) \leq 3$,
(iii) $\mathcal{R}(u, \lambda) = 1$ (for $\lambda > 0$) if and only if $u = 0$,
(iv) $\mathcal{R}(\sigma u, \lambda) = \mathcal{R}\left(u, \frac{\lambda}{|\sigma|}\right)$,
(v) $\mathcal{R}(u, \lambda) \circ \mathcal{R}(v, \mu) \leq \mathcal{R}(u + v, \lambda + \mu)$,
(vi) $\mathcal{R}(u, .)$ is a continuous and non-decreasing function,
(vii) $\lim_{\lambda \to \infty} \mathcal{R}(u, \lambda) = 1$,
(viii) $\mathcal{T}(u, \lambda) = 0$ (for $\lambda > 0$) if and only if $u = 0$,
(ix) $\mathcal{T}(\sigma u, \lambda) = \mathcal{T}\left(u, \frac{\lambda}{|\sigma|}\right)$,
(x) $\mathcal{T}(u, \mu) \bullet \mathcal{T}(v, \lambda) \geq \mathcal{T}(u + v, \lambda + \mu)$,
(xi) $\mathcal{T}(u, .)$ is a continuous and non-increasing function,
(xii) $\lim_{\lambda \to \infty} \mathcal{T}(u, \lambda) = 0$,
(xiii) $\mathcal{W}(u, \lambda) = 0$ (for $\lambda > 0$) if and only if $u = 0$,
(xiv) $\mathcal{W}(\sigma u, \lambda) = \mathcal{W}\left(u, \frac{\lambda}{|\sigma|}\right)$,
(xv) $\mathcal{W}(u, \mu) \bullet \mathcal{W}(v, \lambda) \geq \mathcal{W}(u + v, \lambda + \mu)$,
(xvi) $\mathcal{W}(u, .)$ is a continuous non-increasing function,

(xvii) $\lim_{\lambda\to\infty} \mathcal{W}(u,\lambda) = 0$,

(xviii) If $\lambda \leq 0$, then $\mathcal{R}(u,\lambda) = 0, \mathcal{T}(u,\lambda) = 1$ and $\mathcal{W}(u,\lambda) = 1$.

Then, $\mathcal{N} = (\mathcal{R}, \mathcal{T}, \mathcal{W})$ is called neutrosophic norm (NN).

Example 2.10 ([19])**.** Suppose $(F, \|\cdot\|)$ be a NS. For $s, t \in [0,1]$, define the t-norm $\circ$ and the t-conorm $\bullet$ as $s \circ t = st$ and $s \bullet t = s + t - st$ respectively. For $\lambda > \|u\|$, let

$$\mathcal{R}(u,\lambda) = \frac{\lambda}{\lambda + \|u\|}, \qquad \mathcal{T}(u,\lambda) = \frac{\|u\|}{\lambda + \|u\|}, \qquad \mathcal{W}(u,\lambda) = \frac{\|u\|}{\lambda},$$

$\forall u (\neq 0) \in F$ and for $\lambda \leq 0$, let $\mathcal{R}(u,\lambda) = 0, \mathcal{T}(u,\lambda) = 1$ and $\mathcal{W}(u,\lambda) = 1$. Then, $(F, \mathcal{N}, \circ, \bullet)$ is a NNS.

Definition 2.11 ([19])**.** Let V be a NNS. A sequence $(x_k) \subset V$ is said to be statistical convergent to l with respect to the neutrosophic norm (NN), if for every $0 < \varepsilon < 1$,

$$\delta(K_\varepsilon) = 0,$$

where

$$K_\varepsilon = \{k \leq n : \mathcal{R}(x_k - l, \lambda) \leq 1 - \varepsilon, \mathcal{T}(x_k - l, \lambda) \geq \varepsilon \text{ and } \mathcal{W}(x_k - l, \lambda) \geq \varepsilon\}.$$

Symbolically it is denoted as $st - \mathcal{N} - \lim x_k = l$ or $x_k \to l(st - \mathcal{N})$.

Definition 2.12 ([19])**.** Let V be a NNS and $(x_k) \subset V$ be a sequence. Then, (x_k) is said to be statistical Cauchy if for any $0 < \varepsilon < 1$, there exists $N = N(\varepsilon) \in \mathbb{N}$ such that $\delta(KC_\varepsilon) = 0$, where

$$KC_\varepsilon = \{k \leq n : \mathcal{R}(x_k - x_N, \lambda) \leq 1 - \varepsilon, \mathcal{T}(x_k - x_N, \lambda) \geq \varepsilon \text{ and } \mathcal{W}(x_k - x_N, \lambda) \geq \varepsilon\}.$$

3. Main Results

Definition 3.1. Let V be a NNS. A sequence (x_k) is said to be deferred statistical convergent to l with respect to neutrosophic norm (NN), if for every $0 < \varepsilon < 1$,

$$\delta_{p,q}(K_\varepsilon) = 0,$$

where

$$K_\varepsilon = \{p(n) + 1 \leq k \leq q(n) : \mathcal{R}(x_k - l, \lambda) \leq 1 - \varepsilon,\ \mathcal{T}(x_k - l, \lambda) \geq \varepsilon \text{ and } \mathcal{W}(x_k - l, \lambda) \geq \varepsilon\}.$$

In this case, we write, $x_k \to l\,(\mathcal{N} - DS[p,q])$.

Remark 3.2. It is clear that,

(i) If $q(n)=n$ and $p(n)=0$, then Definition 3.1 coincides with the definition of statistical convergence in neutrosophic normed spaces given in [19].

(ii) If $q(n)=\lambda_n$ and $p(n)=0$ (where λ is a non-decreasing sequence of positive integers tending to ∞ such that $\lambda_{n+1}\leq\lambda_n+1$, $\lambda_1=0$), then Definition 3.1 turns to the definition of λ-statistical convergence sequence in NNS [15].

(iii) If $q(n)=k_n$ and $p(n)=k_{n-1}$(for any lacunary sequence of non-negative integer satisfying $k_n-k_{n-1}\to\infty$ as $n\to\infty$), then Definition 3.1 coincides with the definition of lacunary statistical convergence in NNS [16].

Example 3.3. Let $(F,\|\cdot\|)$ be a NS. For all $s,t\in[0,1]$, define the continuous t-norm $s\circ t=st$ and the continuous t-conorm $s\bullet t=\min\{s+t,1\}$. We take $\mathcal{R},\mathcal{T},\mathcal{W}$ in Example 2.10, for all $\lambda>0$. Then, V is a NNS. Define the sequence (x_k) as

$$x_k=\begin{cases} k^3, & \left[\sqrt[3]{q(n)}\right]-1<k\leq\left[\sqrt[3]{q(n)}\right],\ n=1,2,3,\dots \\ 0, & \text{otherwise,}\end{cases}$$

where $0<p(n)\leq\left[\sqrt[3]{q(n)}\right]-1$ and $q(n)$ is a monotonic increasing sequence. Then, $x_k\to 0\,(\mathcal{N}-DS[p,q])$.

Justification: For every $0<\varepsilon<1$, we have

$$K_\varepsilon^n=\{p(n)+1\leq k\leq q(n):\mathcal{R}(x_k-0,\lambda)\leq 1-\varepsilon,\mathcal{T}(x_k-0,\lambda)\geq\varepsilon,\\ \text{and }\mathcal{W}(x_k-0,\lambda)\geq\varepsilon\}.$$

This implies that,

$$\begin{aligned}K_\varepsilon^n&=\Bigg\{p(n)+1\leq k\leq q(n):\frac{\lambda}{\lambda+\|x_k\|}\leq 1-\varepsilon,\\ &\qquad\qquad\frac{\|x_k\|}{\lambda+\|x_k\|}\geq\varepsilon\text{ and }\frac{\|x_k\|}{\lambda}\geq\varepsilon\Bigg\}\\ &=\left\{p(n)+1\leq k\leq q(n):\|x_k\|\geq\frac{\lambda\varepsilon}{1-\varepsilon}\text{ and }\|x_k\|\geq\lambda\varepsilon\right\}\\ &\subseteq\left\{p(n)+1\leq k\leq q(n):x_k=k^3\right\}.\end{aligned}$$

Then, $\delta_{p,q}(K_\varepsilon)=\lim_{n\to\infty}\frac{|K_\varepsilon^n|}{q(n)-p(n)}=0$. Hence, $x_k\to 0\,(\mathcal{N}-DS[p,q])$.

Lemma 3.4. *Let V be a NNS. Then, for any $0<\varepsilon<1$, the following statements are equivalent:*

(i) $x_k\to l\,(\mathcal{N}-DS[p,q])$;

(ii)

$$\begin{aligned}\delta_{p,q}\left(\{k \in \mathbb{N} : \mathcal{R}(x_k - l, \lambda) \le 1 - \varepsilon\}\right) \\ &= \delta_{p,q}\left(\{k \in \mathbb{N} : \mathcal{T}(x_k - l, \lambda) \ge \varepsilon\}\right) \\ &= \delta_{p,q}\left(\{k \in \mathbb{N} : \mathcal{W}(x_k - l, \lambda) \ge \varepsilon\}\right) \\ &= 0;\end{aligned}$$

(iii) $\delta_{p,q}(\{k \in \mathbb{N} : \mathcal{R}(x_k - l, \lambda) > 1 - \varepsilon,\ \mathcal{T}(x_k - l, \lambda) < \varepsilon$ *and* $\mathcal{W}(x_k - l, \lambda) < \varepsilon\}) = 1$;

(iv)

$$\begin{aligned}\delta_{p,q}(\{k \in \mathbb{N} : \mathcal{R}(x_k - l, \lambda) > 1 - \varepsilon\}) \\ &= \delta_{p,q}(\{k \in \mathbb{N} : \mathcal{T}(x_k - l, \lambda) < \varepsilon\}) \\ &= \delta_{p,q}(\{k \in \mathbb{N} : \mathcal{W}(x_k - l, \lambda) < \varepsilon\}) \\ &= 1;\end{aligned}$$

(v) $\mathcal{R}(x_k - l, \lambda) \to 1\,(\mathcal{N} - DS[p,q])$, $\mathcal{T}(x_k - l, \lambda) \to 0\,(\mathcal{N} - DS[p,q])$ *and* $\mathcal{W}(x_k - l, \lambda) \to 0\,(\mathcal{N} - DS[p,q])$.

Theorem 3.5. *Let V be a NNS and suppose (x_k) be a sequence such that $x_k \to l\,(\mathcal{N} - DS[p,q])$. Then l is unique.*

Proof. Suppose that

$$x_k \to l_1\,(\mathcal{N} - DS[p,q])\,, \qquad x_k \to l_2\,(\mathcal{N} - DS[p,q])$$

for $l_1 \ne l_2$. Then, for a given $0 < \varepsilon < 1$, we can choose $\mu > 0$ such that $(1-\varepsilon) \circ (1-\varepsilon) > 1 - \mu$ and $\varepsilon \bullet \varepsilon < \mu$. Now, for any $\lambda > 0$ we define the following sets:

$$\begin{aligned}K_{\mathcal{R}_1}(\varepsilon,\lambda) &= \left\{p(n)+1 \le k \le q(n) : \mathcal{R}\left(x_k - l_1, \frac{\lambda}{2}\right) \le 1 - \varepsilon\right\},\\ K_{\mathcal{R}_2}(\varepsilon,\lambda) &= \left\{p(n)+1 \le k \le q(n) : \mathcal{R}\left(x_k - l_2, \frac{\lambda}{2}\right) \le 1 - \varepsilon\right\},\\ K_{\mathcal{T}_1}(\varepsilon,\lambda) &= \left\{p(n)+1 \le k \le q(n) : \mathcal{T}\left(x_k - l_1, \frac{\lambda}{2}\right) \ge \varepsilon\right\},\\ K_{\mathcal{T}_2}(\varepsilon,\lambda) &= \left\{p(n)+1 \le k \le q(n) : \mathcal{T}\left(x_k - l_2, \frac{\lambda}{2}\right) \ge \varepsilon\right\},\\ K_{\mathcal{W}_1}(\varepsilon,\lambda) &= \left\{p(n)+1 \le k \le q(n) : \mathcal{W}\left(x_k - l_1, \frac{\lambda}{2}\right) \ge \varepsilon\right\},\\ K_{\mathcal{W}_2}(\varepsilon,\lambda) &= \left\{p(n)+1 \le k \le q(n) : \mathcal{W}\left(x_k - l_2, \frac{\lambda}{2}\right) \ge \varepsilon\right\}.\end{aligned}$$

Since $x_k \to l_1\,(\mathcal{N} - DS[p,q])$, so by Lemma 3.4, we have for any $\lambda > 0$,

$$\delta_{p,q}(K_{\mathcal{R}_1}(\varepsilon,\lambda)) = \delta_{p,q}(K_{\mathcal{T}_1}(\varepsilon,\lambda))$$

$$= \delta_{p,q}(K_{\mathcal{W}_1}(\varepsilon,\lambda)) = 0.$$

Again since $x_k \to l_2\,(\mathcal{N}-DS[p,q])$, so by Lemma 3.4, we have for any $\lambda > 0$,

$$\begin{aligned}\delta_{p,q}(K_{\mathcal{R}_2}(\varepsilon,\lambda)) &= \delta_{p,q}(K_{\mathcal{T}_2}(\varepsilon,\lambda))\\ &= \delta_{p,q}(K_{\mathcal{W}_2}(\varepsilon,\lambda))\\ &= 0.\end{aligned}$$

Let

$$\begin{aligned}K(\varepsilon,\lambda) = {} & (K_{\mathcal{R}_1}(\varepsilon,\lambda)\cup K_{\mathcal{R}_2}(\varepsilon,\lambda))\cap(K_{\mathcal{T}_1}(\varepsilon,\lambda)\cup K_{\mathcal{T}_2}(\varepsilon,\lambda))\\ &\cap(K_{\mathcal{W}_1}(\varepsilon,\lambda)\cup K_{\mathcal{W}_2}(\varepsilon,\lambda)).\end{aligned}$$

Then, we have $\delta_{p,q}(K(\varepsilon,\lambda)) = 0$ and eventually $\delta_{p,q}(\mathbb{N}\setminus K(\varepsilon,\lambda)) = 1$ and therefore $\mathbb{N}\setminus K(\varepsilon,\lambda)$ is non-empty. Choose $p \in \mathbb{N}\setminus K(\varepsilon,\lambda)$. Then, there are three possibilities:

(i) $p \in (\mathbb{N}\setminus(K_{\mathcal{R}_1}(\varepsilon,\lambda))\cup(\mathbb{N}\setminus(K_{\mathcal{R}_2}(\varepsilon,\lambda))$,
(ii) $p \in (\mathbb{N}\setminus(K_{\mathcal{T}_1}(\varepsilon,\lambda))\cup(\mathbb{N}\setminus(K_{\mathcal{T}_2}(\varepsilon,\lambda))$, and
(iii) $p \in (\mathbb{N}\setminus(K_{\mathcal{W}_1}(\varepsilon,\lambda))\cup(\mathbb{N}\setminus(K_{\mathcal{W}_2}(\varepsilon,\lambda))$.

If we consider (i), then we have the following

$$\begin{aligned}\mathcal{R}(l_1-l_2,\lambda) &\geq \mathcal{R}\left(x_k-l_1,\frac{\lambda}{2}\right)\circ\mathcal{R}\left(x_k-l_2,\frac{\lambda}{2}\right)\\ &> (1-\varepsilon)\circ(1-\varepsilon)\\ &> 1-\mu.\end{aligned}\tag{3.1}$$

Now, since μ is arbitrary, from Equation (3.1), for any $\lambda > 0$, we obtain $\mathcal{R}(l_1-l_2,\lambda) = 1$ i.e., $l_1 = l_2$.

Again, if we consider (ii), then we have the following

$$\begin{aligned}\mathcal{T}(l_1-l_2,\lambda) &\leq \mathcal{T}\left(x_k-l_1,\frac{\lambda}{2}\right)\bullet\mathcal{T}\left(x_k-l_2,\frac{\lambda}{2}\right)\\ &< \varepsilon\bullet\varepsilon\\ &< \mu.\end{aligned}\tag{3.2}$$

Now, since μ is arbitrary, from Equation (3.2), for any $\lambda > 0$, we obtain $\mathcal{T}(l_1-l_2,\lambda) = 0$ i.e., $l_1 = l_2$.

Finally, if we consider (iii), then we have the following

$$\begin{aligned}\mathcal{W}(l_1-l_2,\lambda) &\leq \mathcal{W}\left(x_k-l_1,\frac{\lambda}{2}\right)\bullet\mathcal{W}\left(x_k-l_2,\frac{\lambda}{2}\right)\\ &< \varepsilon\bullet\varepsilon\\ &< \mu.\end{aligned}\tag{3.3}$$

Now, since μ is arbitrary, from Equation (3.3), for any $\lambda > 0$, we obtain $\mathcal{W}(l_1 - l_2, \lambda) = 0$ i.e., $l_1 = l_2$. Thus, in all cases we obtain $l_1 = l_2$ and this completes the proof. □

Theorem 3.6. *Let (x_k) and (y_k) be two sequences in the NNS V such that $x_k \to l_1\,(\mathcal{N} - DS[p,q])$ and $y_k \to l_2\,(\mathcal{N} - DS[p,q])$. Then,*

(i) $x_k + y_k \to l_1 + l_2\,(\mathcal{N} - DS[p,q])$ and
(ii) $\alpha x_k \to \alpha l_1\,(\mathcal{N} - DS[p,q])$, where $\alpha \in \mathbb{R}$.

Proof. (i) Suppose $x_k \to l_1\,(\mathcal{N} - DS[p,q])$ and $y_k \to l_2\,(\mathcal{N} - DS[p,q])$. Now by definition for any $0 < \varepsilon < 1$, $\delta_{p,q}(K_\varepsilon) = \delta_{p,q}(K'_\varepsilon) = 0$, where

$$K_\varepsilon = \left\{p(n) + 1 \le k \le q(n) : \mathcal{R}\left(x_k - l_1, \frac{\lambda}{2}\right) \le 1 - \varepsilon, \right.$$
$$\left. \mathcal{T}\left(x_k - l_1, \frac{\lambda}{2}\right) \ge \varepsilon \text{ and } \mathcal{W}\left(x_k - l_1, \frac{\lambda}{2}\right) \ge \varepsilon\right\},$$

and

$$K'_\varepsilon = \left\{p(n) + 1 \le k \le q(n) : \mathcal{R}\left(y_k - l_2, \frac{\lambda}{2}\right) \le 1 - \varepsilon, \right.$$
$$\left. \mathcal{T}\left(y_k - l_2, \frac{\lambda}{2}\right) \ge \varepsilon \text{ and } \mathcal{W}\left(y_k - l_2, \frac{\lambda}{2}\right) \ge \varepsilon\right\}.$$

Now as the inclusion

$$(\mathbb{N}\backslash K_\varepsilon) \cap (\mathbb{N} \setminus K'_\varepsilon) \subseteq \{k \in \mathbb{N} : \mathcal{R}(x_k + y_k - l_1 - l_2, \lambda) > 1 - \varepsilon,$$
$$\mathcal{T}(x_k + y_k - l_1 - l_2, \lambda) < \varepsilon \text{ and } \mathcal{W}(x_k + y_k - l_1 - l_2, \lambda) < \varepsilon\},$$

holds, so we must have

$$K''_\varepsilon = \{p(n) + 1 \le k \le q(n) : \mathcal{R}(x_k + y_k - l_1 - l_2, \lambda) \le 1 - \varepsilon,$$
$$\mathcal{T}(x_k + y_k - l_1 - l_2, \lambda) \ge \varepsilon \text{ and } \mathcal{W}(x_k + y_k - l_1 - l_2, \lambda) \ge \varepsilon\}$$
$$\subseteq K_\varepsilon \cup K'_\varepsilon,$$

and consequently, $\delta_{p,q}(K''_\varepsilon) = 0$ i.e.,

$$x_k + y_k \to l_1 + l_2\,(\mathcal{N} - DS[p,q])\,.$$

(ii) The proof is easy, so is omitted.

□

Theorem 3.7. *Let (x_k) and (y_k) be two sequences in the NNS V such that*

$$\delta_{p,q}(\{k \in \mathbb{N} : x_k \ne y_k\}) = 0.$$

If $y_k \to l(\mathcal{N})$, then $x_k \to l\,(\mathcal{N} - DS[p,q])$.

Proof. Suppose $\delta_{p,q}(\{k \in \mathbb{N} : x_k \neq y_k\}) = 0$ holds and $y_k \to l(\mathcal{N})$. Then, by definition for every $0 < \varepsilon < 1$, the set $K_\varepsilon = \{k \leq n : \mathcal{R}(y_k - l, \lambda) \leq 1 - \varepsilon,\ \mathcal{T}(y_k - l, \lambda) \geq \varepsilon$ and $\mathcal{W}(y_k - l, \lambda) \geq \varepsilon\}$ contains atmost a finite number of elements and consequently, $\delta_{p,q}(K_\varepsilon) = 0$. Now, since the inclusion

$$K'_\varepsilon = \{p(n) + 1 \leq k \leq q(n) : \mathcal{R}(x_k - l, \lambda) \leq 1 - \varepsilon,\ \mathcal{T}(x_k - l, \lambda) \geq \varepsilon \text{ and } \mathcal{W}(x_k - l, \lambda) \geq \varepsilon\} \subseteq K_\varepsilon \cap \{k \in \mathbb{N} : x_k \neq y_k\},$$

holds, so we must have, $\delta_{p,q}(K'_\varepsilon) = 0$. Hence, $x_k \to l\,(\mathcal{N} - DS[p,q])$. □

Theorem 3.8. *Let $p = (p(n))$, and $q = (q(n))$ be sequences of non-negative integers satisfying* (2.1)*. If the sequence $\left(\frac{p(n)}{q(n)-p(n)}\right)$ is bounded then $x_k \to l\,(st - \mathcal{N})$ implies $x_k \to l\,(\mathcal{N} - DS[p,q])$.*

Proof. Since, $\lim_{n\to\infty} q(n) = \infty$ and $x_k \to l\,(st - \mathcal{N})$ holds, so proceeding as Theorem 2.2.1 in [20], we obtain

$$\lim_{n\to\infty} \frac{1}{q(n)} \left|\{1 \leq k \leq q(n) : \mathcal{R}(x_k - l, \lambda) \leq 1 - \varepsilon, \mathcal{T}(x_k - l, \lambda) \geq \varepsilon \text{ and } \mathcal{W}(x_k - l, \lambda) \geq \varepsilon\}\right| = 0.$$

Now as the following inclusion

$$\{p(n) + 1 \leq k \leq q(n) : \mathcal{R}(x_k - l, \lambda) \leq 1 - \varepsilon,\ \mathcal{T}(x_k - l, \lambda) \geq \varepsilon \text{ and } \mathcal{W}(x_k - l, \lambda) \geq \varepsilon\}$$
$$\subseteq \{1 \leq k \leq q(n) : \mathcal{R}(x_k - l, \lambda) \leq 1 - \varepsilon,\ \mathcal{T}(x_k - l, \lambda) \geq \varepsilon \text{ and } \mathcal{W}(x_k - l, \lambda) \geq \varepsilon\},$$

holds, so we must have

$$|\{p(n) + 1 \leq k \leq q(n) : \mathcal{R}(x_k - l, \lambda) \leq 1 - \varepsilon,\ \mathcal{T}(x_k - l, \lambda) \geq \varepsilon \text{ and } \mathcal{W}(x_k - l, \lambda) \geq \varepsilon\}|$$
$$\leq |\{1 \leq k \leq q(n) : \mathcal{R}(x_k - l, \lambda) \leq 1 - \varepsilon,\ \mathcal{T}(x_k - l, \lambda) \geq \varepsilon \text{ and } \mathcal{W}(x_k - l, \lambda) \geq \varepsilon\}|.$$

Consequently, the following inequation holds

(3.4)
$$\frac{1}{q(n) - p(n)} |\{p(n) + 1 \leq k \leq q(n) : \mathcal{R}(x_k - l, \lambda) \leq 1 - \varepsilon, \mathcal{T}(x_k - l, \lambda) \geq \varepsilon \text{ and } \mathcal{W}(x_k - l, \lambda) \geq \varepsilon\}|$$
$$\leq \left(1 + \frac{p(n)}{q(n) - p(n)}\right) \cdot \frac{1}{q(n)} |\{1 \leq k \leq q(n) : \mathcal{R}(x_k - l, \lambda) \leq 1 - \varepsilon, \mathcal{T}(x_k - l, \lambda) \geq \varepsilon \text{ and } \mathcal{W}(x_k - l, \lambda) \geq \varepsilon\}|.$$

Now, since the sequence $\left(\frac{p(n)}{q(n)-p(n)}\right)$ is bounded, letting $n \to \infty$ in both sides of (3.4) we obtain the desired result. □

Theorem 3.9. *Let $p = (p(n))$, $q = (q(n))$, $r = (r(n))$, and $s = (s(n))$ be sequences of non-negative integers satisfying $p(n) \leq r(n) < s(n) \leq q(n)$ for all $n \in \mathbb{N}$. If the sets $\{k \in \mathbb{N} : p(n) < k \leq r(n)\}$ and $\{k \in \mathbb{N} : s(n) < k \leq q(n)\}$ are finite for all $n \in \mathbb{N}$, then for any sequence (x_k) in the NNS V, $x_k \to l\,(\mathcal{N} - DS[r,s])$ implies $x_k \to l\,(\mathcal{N} - DS[p,q])$.*

Proof. Let (x_k) be a sequence in the NNS V such that

$$x_k \to l\,(\mathcal{N} - DS[r,s])\,.$$

Now, for every $0 < \varepsilon < 1$, the following inclusion holds

(3.5)
$$\begin{aligned}
&\{p(n)+1 \leq k \leq q(n) : \mathcal{R}(x_k - l,\lambda) \leq 1-\varepsilon,\ \mathcal{T}(x_k - l,\lambda) \geq \varepsilon \\
&\qquad \text{and } \mathcal{W}(x_k - l,\lambda) \geq \varepsilon\} \\
&\quad = \{p(n)+1 \leq k \leq r(n) : \mathcal{R}(x_k - l,\lambda) \leq 1-\varepsilon,\ \mathcal{T}(x_k - l,\lambda) \geq \varepsilon \\
&\qquad \text{and } \mathcal{W}(x_k - l,\lambda) \geq \varepsilon\} \\
&\quad \cup \{r(n)+1 \leq k \leq s(n) : \mathcal{R}(x_k - l,\lambda) \leq 1-\varepsilon,\ \mathcal{T}(x_k - l,\lambda) \geq \varepsilon \\
&\qquad \text{and } \mathcal{W}(x_k - l,\lambda) \geq \varepsilon\} \\
&\quad \cup \{s(n)+1 \leq k \leq q(n) : \mathcal{R}(x_k - l,\lambda) \leq 1-\varepsilon,\ \mathcal{T}(x_k - l,\lambda) \geq \varepsilon \\
&\qquad \text{and } \mathcal{W}(x_k - l,\lambda) \geq \varepsilon\}.
\end{aligned}$$

Since, the deferred density of a finite set is zero and $x_k \to l\,(\mathcal{N} - DS[r,s])$, so from (3.5), we must have $x_k \to l\,(\mathcal{N} - DS[p,q])$. This completes the proof. □

Theorem 3.10. *Let $p = (p(n))$, $q = (q(n))$, $r = (r(n))$, and $s = (s(n))$ be sequences of non-negative integers satisfying $p(n) \leq r(n) < s(n) \leq q(n)$ for all $n \in \mathbb{N}$. If $\lim\limits_{n\to\infty} \frac{q(n)-p(n)}{s(n)-r(n)} = d > 0$, then for any sequence (x_k) in the NNS V, $x_k \to l\,(\mathcal{N} - DS[p,q])$ implies $x_k \to l\,(\mathcal{N} - DS[r,s])$.*

Proof. Let (x_k) be a sequence in the NNS V such that

$$x_k \to l\,(\mathcal{N} - DS[p,q])\,.$$

Now, for every $0 < \varepsilon < 1$, the following inclusion holds

$$\begin{aligned}
&\{r(n)+1 \leq k \leq s(n) : \mathcal{R}(x_k - l,\lambda) \leq 1-\varepsilon,\ \mathcal{T}(x_k - l,\lambda) \geq \varepsilon \\
&\qquad \text{and } \mathcal{W}(x_k - l,\lambda) \geq \varepsilon\} \\
&\quad \subseteq \{p(n)+1 \leq k \leq q(n) : \mathcal{R}(x_k - l,\lambda) \leq 1-\varepsilon,\ \mathcal{T}(x_k - l,\lambda) \geq \varepsilon \\
&\qquad \text{and } \mathcal{W}(x_k - l,\lambda) \geq \varepsilon\}.
\end{aligned}$$

Consequently we have,

(3.6)

$$\begin{aligned}\frac{1}{s(n)-r(n)}&\left|\{r(n)+1\leq k\leq s(n):\mathcal{R}(x_k-l,\lambda)\leq 1-\varepsilon,\right.\\ &\left.\mathcal{T}(x_k-l,\lambda)\geq\varepsilon \text{ and } \mathcal{W}(x_k-l,\lambda)\geq\varepsilon\}\right|\\ &\leq\frac{q(n)-p(n)}{s(n)-r(n)}\cdot\frac{1}{q(n)-p(n)}\left|\{p(n)+1\leq k\leq q(n):\right.\\ &\left.\mathcal{R}(x_k-l,\lambda)\leq 1-\varepsilon,\ \mathcal{T}(x_k-l,\lambda)\geq\varepsilon \text{ and } \mathcal{W}(x_k-l,\lambda)\geq\varepsilon\}\right|.\end{aligned}$$

Since $x_k \to l\,(\mathcal{N}-DS[p,q])$, so letting $n\to\infty$ on both sides of (3.6), we conclude that $x_k \to l\,(\mathcal{N}-DS[r,s])$. □

Definition 3.11. Let (x_k) be a sequence in a NNS V. Then, (x_k) is a said to be deferred statistical Cauchy if for any $0<\varepsilon<1$, there exists $N=N(\varepsilon)$ such that $\delta_{p,q}(KC_\varepsilon)=0$, where

$$\begin{aligned}KC_\varepsilon=\{p(n)+1\leq k\leq q(n):\mathcal{R}(x_k-x_N,\lambda)\leq 1-\varepsilon,\mathcal{T}(x_k-x_N,\lambda)\geq\varepsilon\\ \text{and } \mathcal{W}(x_k-x_N,\lambda)\geq\varepsilon\}.\end{aligned}$$

Theorem 3.12. *Let (x_k) be a sequence in a NNS V. Then, (x_k) is deferred statistical convergent sequence if and only if it is a deferred statistical Cauchy sequence.*

Proof. Suppose $x_k \to l\,(\mathcal{N}-DS[p,q])$. For a given $0<\varepsilon<1$, we choose $\mu>0$ such that $(1-\varepsilon)\circ(1-\varepsilon)>1-\mu$ and $\varepsilon\bullet\varepsilon<\mu$. Now by definition, for any $0<\varepsilon<1$, $\delta_{p,q}(\mathbb{N}\setminus K_\varepsilon)=1$, where

$$\begin{aligned}K_\varepsilon=\left\{p(n)+1\leq k\leq q(n):\mathcal{R}\left(x_k-l,\frac{\lambda}{2}\right)\leq 1-\varepsilon,\ \mathcal{T}\left(x_k-l,\frac{\lambda}{2}\right)\geq\varepsilon\right.\\ \left.\text{and } \mathcal{W}\left(x_k-l,\frac{\lambda}{2}\right)\geq\varepsilon\right\}.\end{aligned}$$

Thus, the set $\mathbb{N}\setminus K_\varepsilon$ is non-empty. Let $N\in\mathbb{N}\setminus K_\varepsilon$. Then we have,

$$\mathcal{R}\left(x_N-l,\frac{\lambda}{2}\right)>1-\varepsilon,\qquad \mathcal{T}\left(x_N-l,\frac{\lambda}{2}\right)<\varepsilon,\qquad \mathcal{W}\left(x_N-l,\frac{\lambda}{2}\right)<\varepsilon.$$

Now suppose

$$\begin{aligned}KC_\varepsilon=\{p(n)+1\leq k\leq q(n):\mathcal{R}(x_k-x_N,\lambda)\leq 1-\mu,\\ \mathcal{T}(x_k-x_N,\lambda)\geq\mu \text{ and } \mathcal{W}(x_k-x_N,\lambda)\geq\mu\}.\end{aligned}$$

We claim that $KC_\varepsilon\subseteq K_\varepsilon$ because if the inclusion does not hold then we must have some $N_0\in KC_\varepsilon\setminus K_\varepsilon$ which immediately yields

$$\mathcal{R}(x_{N_0}-x_N,\lambda)\leq 1-\mu\ ,\qquad \mathcal{R}\left(x_{N_0}-l,\tfrac{\lambda}{2}\right)>1-\varepsilon.$$

In particular, $\mathcal{R}\left(x_N - l, \frac{\lambda}{2}\right) > 1-\varepsilon$.

But then,

$$\begin{aligned}1-\mu &\geq \mathcal{R}(x_{N_0} - x_N, \lambda)\\ &\geq \mathcal{R}\left(x_{N_0} - l, \frac{\lambda}{2}\right) \circ \mathcal{R}\left(x_N - l, \frac{\lambda}{2}\right)\\ &> (1-\varepsilon)\circ(1-\varepsilon)\\ &> 1-\mu,\end{aligned}$$

which is a contradiction. Further, we have, $\mathcal{T}(x_{N_0} - x_N, \lambda) \geq \mu$ and $\mathcal{T}(x_{N_0} - l, \frac{\lambda}{2}) < \varepsilon$. In particular, $\mathcal{T}(x_N - l, \frac{\lambda}{2}) < \varepsilon$. But then,

$$\begin{aligned}\mu &\leq \mathcal{T}(x_{N_0} - x_N, \lambda)\\ &\leq \mathcal{T}\left(x_{N_0} - l, \frac{\lambda}{2}\right) \bullet \mathcal{T}\left(x_N - l, \frac{\lambda}{2}\right)\\ &< \varepsilon \bullet \varepsilon\\ &< \mu,\end{aligned}$$

which is a contradiction. Finally, we have, $\mathcal{W}(x_{N_0} - x_N, \lambda) \geq \mu$ and $\mathcal{W}\left(x_{N_0} - l, \frac{\lambda}{2}\right) < \varepsilon$. In particular, $\mathcal{W}\left(x_N - l, \frac{\lambda}{2}\right) < \varepsilon$. But then,

$$\begin{aligned}\mu &\leq \mathcal{W}\left(x_{N_0} - x_N, \lambda\right)\\ &\leq \mathcal{W}\left(x_{N_0} - l, \frac{\lambda}{2}\right) \bullet \mathcal{W}\left(x_N - l, \frac{\lambda}{2}\right)\\ &< \varepsilon \bullet \varepsilon\\ &< \mu,\end{aligned}$$

which is a contradiction. Thus all possibilities contradict the existence of an element $N_0 \in KC_\varepsilon \setminus K_\varepsilon$. Therefore, we must have $KC_\varepsilon \subseteq K_\varepsilon$ and as a consequence $\delta_{p,q}(KC_\varepsilon) = 0$. Hence (x_k) is deferred statistical Cauchy.

To prove the converse part, we assume that (x_k) is a deferred statistical Cauchy sequence but not deferred statistical convergent. For a given $0 < \varepsilon < 1$, we choose $\mu > 0$ such that $(1-\varepsilon)\circ(1-\varepsilon) > 1-\mu$ and $\varepsilon \bullet \varepsilon < \mu$. Then, since (x_k) is not deferred statistical convergent, so

$$\begin{aligned}\mathcal{R}(x_k - x_N, \lambda) &\geq \mathcal{R}\left(x_k - l, \frac{\lambda}{2}\right) \circ \mathcal{R}\left(x_N - l, \frac{\lambda}{2}\right)\\ &> (1-\varepsilon)\circ(1-\varepsilon) > 1-\mu,\\ \mathcal{T}\left(x_k - x_N, \lambda\right) &\leq \mathcal{T}\left(x_k - l, \frac{\lambda}{2}\right) \bullet \mathcal{T}\left(x_N - l, \frac{\lambda}{2}\right) < \varepsilon \bullet \varepsilon < \mu,\\ \mathcal{W}(x_k - x_N, \lambda) &\leq \mathcal{W}\left(x_k - l, \frac{\lambda}{2}\right) \bullet \mathcal{W}\left(x_N - l, \frac{\lambda}{2}\right) < \varepsilon \bullet \varepsilon < \mu,\end{aligned}$$

holds for $P(\varepsilon, \mu) = \{k \leq N : \mathcal{T}(x_k - x_N, \lambda) \leq 1 - \mu\}$. Therefore, $\delta_{p,q}(P(\varepsilon, \mu)) = 1$, which is a contradiction to the fact that (x_k) is deferred statistical Cauchy. Hence, (x_k) must be a deferred statistical convergent sequence. This completes the proof. □

4. Conculusion

In this paper, we mainly investigated various fundamental properties of deferred statistical convergence in neutrosophic normed spaces. Theorem 3.8 revealed the connection of this convergence method with the recently introduced statistical convergence in neutrosophic normed spaces by Kirisci and Simsek. Theorem 3.12 proved the equivalency of deferred statistical convergent sequences and deferred statistical Cauchy sequences in a neutrosophic normed space. In future, one may study the deferred statistical boundedness of a sequence and investigate how they are related to deferred statistical convergent sequences in neutrosophic normed spaces. Moreover, this work can be extended over the multisequences to study the structure of the sequence space so formed.

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