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Somewhat pairwise fuzzy α-irresolute continuous mappings

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Abstract. The concept of somewhat pairwise fuzzy α-irresolute continuous mappings and somewhat pairwise fuzzy irresolute α-open mappings have been introduced and studied. Besides, some interesting properties of those mappings are given.

1. Introduction

The fundamental concept of fuzzy sets was introduced by L.A. Zadeh [9] provided a natural foundation for building new branches. In 1968 C.L. Chang [2] introduced the concept of fuzzy topological spaces as a generalization of topological spaces. The class of somewhat continuous mappings was first introduced by K.R. Gentry and others in [3]. Later, the concept of somewhat in classical topology has been extented to fuzzy topological spaces. In 1989, A. Kandil [5] introduced the concept of fuzzy bitopological spaces. The product related spaces and the graph of a function were found in Azad [1].

The concept of fuzzy α-continuous mappings on a fuzzy topological space was introduced and studied by M.K. Singal and N. Rajvanshi in [7]. Also, the concept of fuzzy α-irresolute continuous mappings and fuzzy irresolute α-open mappings on a fuzzy topological space was introduced and studied by R. Prasad et. al. in [6].

The concept of somewhat fuzzy α-continuous mappings on a fuzzy topological space were introduced and studied by G. Thangaraj and G. Balasubramanian in [8].

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Recently, the concept of a somewhat fuzzy α-irresolute continuous mapping and a somewhat fuzzy irresolute α-open mappings were introduced in [4].

In this paper, the concepts of somewhat pairwise fuzzy α-irresolute continuous mappings and somewhat pairwise fuzzy irresolute α-open mappings on a fuzzy bitopological space are introduced and studied their properties.

2. Somewhat pairwise fuzzy α-irresolute continuous mappings

In this section, I introduce a somewhat pairwise fuzzy α-irresolute continuous mappings which are stronger than a somewhat pairwise fuzzy α-continuous mappings. Also, we characterize a somewhat pairwise fuzzy α-irresolute continuous mapping.

Definition 2.1. A mapping $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ is called pairwise fuzzy α-continuous if $f^{-1}(\nu)$ is a τ_1-fuzzy α-open or τ_2-fuzzy α-open set on (X, τ_1, τ_2) for any η_1-fuzzy open or η_2-fuzzy open set ν on (Y, η_1, η_2).

Definition 2.2. A mapping $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ is called pairwise fuzzy α-irresolute continuous if $f^{-1}(\nu)$ is a τ_1-fuzzy α-open or τ_2-fuzzy α-open set on (X, τ_1, τ_2) for any η_1-fuzzy α-open or η_2-fuzzy α-open set ν on (Y, η_1, η_2).

Definition 2.3. A mapping $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ is called somewhat pairwise fuzzy α-continuous if there exists a τ_1-fuzzy α-open or τ_2-fuzzy α-open set $\mu \neq 0_X$ on (X, τ_1, τ_2) such that $\mu \leq f^{-1}(\nu) \neq 0_X$ for any η_1-fuzzy open or η_2-fuzzy open set $\nu \neq 0_Y$ on (Y, η_1, η_2).

Definition 2.4. A mapping $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ is called somewhat pairwise fuzzy α-irresolute continuous if there exists a τ_1-fuzzy α-open or τ_2-fuzzy α-open set $\mu \neq 0_X$ on (X, τ_1, τ_2) such that $\mu \leq f^{-1}(\nu) \neq 0_X$ for any η_1-fuzzy α-open or η_2-fuzzy α-open set $\nu \neq 0_Y$ on (Y, η_1, η_2).

From the definitions, it is clear that every pairwise fuzzy α-irresolute continuous mapping is a somewhat pairwise fuzzy α-irresolute continuous mapping, and every somewhat pairwise fuzzy α-irresolute continuous mapping is a pairwise fuzzy α-continuous mapping. Also, every pairwise fuzzy α-continuous mapping is a somewhat pairwise fuzzy α-continuous mapping from the above definition. But the converses are not true in general as the following examples show.

Example 2.5. Let λ_1,λ_2 and λ_3 be fuzzy sets on $X = \{a, b, c\}$ and let σ_1,σ_2 and σ_3 be fuzzy sets on $Y = \{x, y, z\}$. Then

$$\lambda_1 = \tfrac{0.1}{a} + \tfrac{0.1}{b} + \tfrac{0.1}{c}, \quad \sigma_1 = \tfrac{0.3}{x} + \tfrac{0.1}{y} + \tfrac{0.3}{z},$$

$$\lambda_2 = \tfrac{0.4}{a} + \tfrac{0.4}{b} + \tfrac{0.4}{c}, \quad \sigma_2 = \tfrac{0.5}{x} + \tfrac{0.4}{y} + \tfrac{0.5}{z},$$

$$\lambda_3 = \tfrac{0.5}{a} + \tfrac{0.5}{b} + \tfrac{0.5}{c}, \quad \sigma_3 = \tfrac{0.5}{x} + \tfrac{0.5}{y} + \tfrac{0.5}{z},$$

are defined as follows: Consider

$$\tau_1 = \{0_X, \lambda_1, 1_X\}, \quad \eta_1 = \{0_Y, \sigma_1, \sigma_2, 1_Y\},$$

$$\tau_2 = \{0_X, \lambda_1^c, 1_X\}, \quad \eta_2 = \{0_Y, \sigma_3, 1_Y\}.$$

Then, (X, τ_1, τ_2) and (Y, η_1, η_2) are fuzzy bitopologies and $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ defined by $f(a) = y$, $f(b) = y$, $f(c) = y$. Then we have $f^{-1}(\sigma_1) = \lambda_1$, $\lambda_1 \leq f^{-1}(\sigma_2) = \lambda_2$ and $\lambda_1 \leq f^{-1}(\sigma_3) = \lambda_3$. Since λ_1 is a τ_1-fuzzy α-open set on (X, τ_1, τ_2), f is somewhat pairwise fuzzy α-irresolute continuous. But $f^{-1}(\sigma_2) = \lambda_2$ and $f^{-1}(\sigma_3) = \lambda_3$ are not τ_1-fuzzy α-open or τ_2-fuzzy α-open sets on (X, τ_1, τ_2). Hence f is not a pairwise fuzzy α-irresolute continuous mapping.

Example 2.6. Let λ_1 and λ_2 be fuzzy sets on $X = \{a, b, c\}$ and let σ_1,σ_2 and σ_3 be fuzzy sets on $Y = \{x, y, z\}$. Then

$$\lambda_1 = \tfrac{0.4}{a} + \tfrac{0.4}{b} + \tfrac{0.4}{c}, \quad \sigma_1 = \tfrac{0.4}{x} + \tfrac{0.0}{y} + \tfrac{0.4}{z},$$

$$\lambda_2 = \tfrac{0.5}{a} + \tfrac{0.5}{b} + \tfrac{0.5}{c}, \quad \sigma_2 = \tfrac{0.5}{x} + \tfrac{0.0}{y} + \tfrac{0.5}{z},$$

$$\sigma_3 = \tfrac{0.5}{x} + \tfrac{0.5}{y} + \tfrac{0.5}{z},$$

are defined as follows: Consider

$$\tau_1 = \{0_X, \lambda_1, 1_X\}, \quad \eta_1 = \{0_Y, \sigma_1, \sigma_3, 1_Y\},$$

$$\tau_2 = \{0_X, \lambda_2, 1_X\}, \quad \eta_2 = \{0_Y, \sigma_2, 1_Y\}.$$

Then, (X, τ_1, τ_2) and (Y, η_1, η_2) are fuzzy bitopologies and $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ defined by $f(a) = y$, $f(b) = y$, $f(c) = y$. We have $f^{-1}(\sigma_1) = 0_X$, $f^{-1}(\sigma_2) = 0_X$ and $f^{-1}(\sigma_3) = \lambda_2$ are τ_2-fuzzy α-open sets on (X, τ_1, τ_2), the f is pairwise fuzzy α-continuous. But for an η_1-fuzzy α-open set $\sigma_2 \neq 0_Y$ on (Y, η_1, η_2), $f^{-1}(\sigma_2) = 0_X$. Hence f is not a somewhat pairwise fuzzy α-irresolute continuous mapping.

Example 2.7. Let λ_1 and λ_2 be fuzzy sets on $X = \{a, b, c\}$ and let σ_1,σ_2 and σ_3 be fuzzy sets on $Y = \{x, y, z\}$. Then

$$\lambda_1 = \tfrac{0.4}{a} + \tfrac{0.4}{b} + \tfrac{0.4}{c}, \quad \sigma_1 = \tfrac{0.7}{x} + \tfrac{0.7}{y} + \tfrac{0.7}{z},$$

$$\lambda_2 = \tfrac{0.5}{a} + \tfrac{0.5}{b} + \tfrac{0.5}{c}, \quad \sigma_2 = \tfrac{0.8}{x} + \tfrac{0.8}{y} + \tfrac{0.8}{z},$$

are defined as follows: Consider

$$\tau_1 = \{0_X, \lambda_1, 1_X\}, \quad \eta_1 = \{0_Y, \sigma_1, 1_Y\},$$

$$\tau_2 = \{0_X, \lambda_2, 1_X\}, \quad \eta_2 = \{0_Y, \sigma_2, 1_Y\}.$$

Then, (X, τ_1, τ_2) and (Y, η_1, η_2) are fuzzy bitopologies and $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ is the identity map. We have $\lambda_1 \leq f^{-1}(\sigma_1) = \sigma_1$ and $\lambda_1 \leq f^{-1}(\sigma_2) = \sigma_2$. Since λ_1 is a τ_1-fuzzy α-open set on (X, τ_1, τ_2), f is somewhat pairwise fuzzy α-continuous. But, $f^{-1}(\sigma_1) = \sigma_1$ and $f^{-1}(\sigma_2) = \sigma_2$ are not τ_1-fuzzy α-open or τ_2-fuzzy α-open on (X, τ_1, τ_2). Hence f is not a pairwise fuzzy α-continuous mapping.

Definition 2.8. A fuzzy set μ on a fuzzy bitopological space (X, τ_1, τ_2) is called pairwise α-dense fuzzy set if there exists no τ_1-fuzzy α-closed or τ_2-fuzzy α-closed set ν in (X, τ_1, τ_2) such that $\mu < \nu < 1$.

Theorem 2.9. *Let $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ be a mapping. Then the following are equivalent:*

(1) *f is somewhat pairwise fuzzy α-irresolute continuous.*
(2) *If ν is an η_1-fuzzy α-closed or η_2-fuzzy α-closed set of (Y, η_1, η_2) such that $f^{-1}(\nu) \neq 1_X$, then there exists a τ_1-fuzzy α-closed or τ_2-fuzzy α-closed set $\mu \neq 1_X$ of (X, τ_1, τ_2) such that $f^{-1}(\nu) \leq \mu$.*
(3) *If μ is a pairwise α-dense fuzzy set on (X, τ_1, τ_2), then $f(\mu)$ is a pairwise α-dense fuzzy set on (Y, η_1, η_2).*

Proof. (1) $\Rightarrow$ (2) Let ν be an η_1-fuzzy α-closed or η_2-fuzzy α-closed set on (Y, η_1, η_2) such that $f^{-1}(\nu) \neq 1_X$. Then ν^c is an η_1-fuzzy α-open or η_2-fuzzy α-open set on (Y, η_1, η_2) and $f^{-1}(\nu^c) = (f^{-1}(\nu))^c \neq 0_X$. Since f is somewhat pairwise fuzzy α-irresolute continuous, there exists a τ_1-fuzzy α-open or τ_2-fuzzy α-open set $\lambda \neq 0_X$ on (X, τ_1, τ_2) such that $\lambda \leq f^{-1}(\nu^c)$. Let $\mu = \lambda^c$. Then $\mu \neq 1_X$ is a τ_1-fuzzy α-closed or τ_2-fuzzy α-closed set such that

$$f^{-1}(\nu) = 1 - f^{-1}(\nu^c) \leq 1 - \lambda = \lambda^c = \mu.$$

(2) $\Rightarrow$ (3) Let μ be a pairwise α-dense fuzzy set on (X, τ_1, τ_2) and suppose $f(\mu)$ is not a pairwise α-dense fuzzy set on (Y, η_1, η_2). Then there exists an η_1-fuzzy α-closed or η_2-fuzzy α-closed set ν, (Y, η_1, η_2) such that $f(\mu) < \nu < 1$. Since $\nu < 1$ and

$f^{-1}(\nu) \neq 1_X$, there exists a τ_1-fuzzy α-closed or τ_2-fuzzy α-closed set $\delta \neq 1_X$ such that $\mu \leq f^{-1}(f(\mu)) < f^{-1}(\nu) \leq \delta$. This contradicts to the assumption that μ is a pairwise α-dense fuzzy set on (X, τ_1, τ_2). Hence $f(\mu)$ is a pairwise α-dense fuzzy set on (Y, η_1, η_2).

(3) $\Rightarrow$ (1) Let $\nu \neq 0_Y$ be an η_1-fuzzy α-open or η_2-fuzzy α-open set on (Y, η_1, η_2) and let $f^{-1}(\nu) \neq 0_X$. Suppose that there exists no τ_1-fuzzy α-open or τ_2-fuzzy α-open set $\mu \neq 0_X$ on (X, τ_1, τ_2) such that $\mu \leq f^{-1}(\nu)$. Then $(f^{-1}(\nu))^c$ is a τ_1-fuzzy set or τ_2-fuzzy set on (X, τ_1, τ_2) such that there is no τ_1-fuzzy α-closed or τ_2-fuzzy α-closed set δ on (X, τ_1, τ_2) with $(f^{-1}(\nu))^c < \delta < 1$. In fact, if there exists a τ_1-fuzzy α-open or τ_2-fuzzy α-open set δ^c such that $\delta^c \leq f^{-1}(\nu)$, then it is a contradiction. So $(f^{-1}(\nu))^c$ is a pairwise α-dense fuzzy set on (X, τ_1, τ_2). Then $f\left((f^{-1}(\nu))^c\right)$ is a pairwise α-dense fuzzy set on (Y, η_1, η_2). But $f\left((f^{-1}(\nu))^c\right) = f\left(f^{-1}(\nu^c)\right) \neq \nu^c < 1$. This is a contradiction to the fact that $f\left((f^{-1}(\nu))^c\right)$ is a pairwise α-dense fuzzy set on (Y, η_1, η_2). Hence there exists a τ_1-fuzzy α-open or τ_2-fuzzy α-open set $\mu \neq 0_X$ on (X, τ_1, τ_2) such that $\mu \leq f^{-1}(\nu)$. Consequently, f is somewhat pairwise fuzzy α-irresolute continuous.

□

Theorem 2.10. *Let $(X_1, \tau_1, \tau_2), (X_2, \omega_1, \omega_2), (Y_1, \eta_1, \eta_2), (Y_2, \sigma_1, \sigma_2)$ be fuzzy bitopological spaces. Let (X_1, τ_1, τ_2) be the product related to $(X_2, \omega_1, \omega_2)$ and let (Y_1, η_1, η_2) be the product related to $(Y_2, \sigma_1, \sigma_2)$. If $f_1 : (X_1, \tau_1, \tau_2) \to (Y_1, \eta_1, \eta_2)$ and $f_2 : (X_2, \omega_1, \omega_2) \to (Y_2, \sigma_1, \sigma_2)$ are somewhat pairwise fuzzy α-irresolute continuous mappings, then the product*

$$f_1 \times f_2 : (X_1, \tau_1, \tau_2) \times (X_2, \omega_1, \omega_2) \to (Y_1, \eta_1, \eta_2) \times (Y_2, \sigma_1, \sigma_2),$$

is also somewhat pairwise fuzzy α-irresolute continuous.

Proof. Let $\lambda = \bigvee_{i,j}(\mu_i \times \nu_j)$ be η_i-fuzzy α-open or σ_j-fuzzy α-open set on $(Y_1, \eta_1, \eta_2) \times (Y_2, \sigma_1, \sigma_2)$ where $\mu_i \neq 0_{Y_1}$ is η_i-fuzzy α-open set and $\nu_j \neq 0_{Y_2}$ is σ_j-fuzzy α-open set on (Y_1, η_1, η_2) and $(Y_2, \sigma_1, \sigma_2)$, respectively. Then

$$(f_1 \times f_2)^{-1}(\lambda) = \bigvee_{i,j} \left(f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j)\right).$$

Since f_1 is somewhat pairwise fuzzy α-irresolute continuous, there exists a τ_1-fuzzy α-open or τ_2-fuzzy α-open set $\delta_i \neq 0_{X_1}$ such that $\delta_i \leq f_1^{-1}(\mu_i) \neq 0_{X_1}$. Since f_2 is somewhat pairwise fuzzy α-irresolute continuous, there exists a ω_1-fuzzy α-open or ω_2-fuzzy α-open set $\alpha_j \neq 0_{X_2}$

such that $\alpha_j \leq f_2^{-1}(\nu_j) \neq 0_{X_2}$. Now

$$\delta_i \times \alpha_j \leq f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j) = (f_1 \times f_2)^{-1}(\mu_i \times \nu_j),$$

and $\delta_i \times \alpha_j \neq 0_{X_1 \times X_2}$ is a δ_i-fuzzy α-open or ν_j-fuzzy α-open set on $(X_1, \tau_1, \tau_2) \times (X_2, \omega_1, \omega_2)$. Hence $\bigvee_{i,j}(\delta_i \times \alpha_j) \neq 0_{X_1 \times X_2}$ is a τ_i-fuzzy α-open or ω_j-fuzzy α-open set on $(X_1, \tau_1, \tau_2) \times (X_2, \omega_1, \omega_2)$ such that

$$\begin{aligned}\bigvee_{i,j}(\delta_i \times \alpha_j) &\leq \bigvee_{i,j} \left(f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j)\right) \\ &= (f_1 \times f_2)^{-1}\left(\bigvee_{i,j}(\mu_i \times \nu_j)\right) \\ &= (f_1 \times f_2)^{-1}(\lambda) \\ &\neq 0_{X_1 \times X_2}.\end{aligned}$$

Therefore, $f_1 \times f_2$ is somewhat pairwise fuzzy α-irresolute continuous. □

Theorem 2.11. *Let $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ be a mapping. If the graph $g : (X, \tau_1, \tau_2) \to (X, \tau_1, \tau_2) \times (Y, \eta_1, \eta_2)$ of f is a somewhat pairwise fuzzy α-irresolute continuous mapping, then f is also somewhat pairwise fuzzy α-irresolute continuous.*

Proof. Let ν be an η_1-fuzzy α-open or η_2-fuzzy α-open set on (Y, η_1, η_2). Then $f^{-1}(\nu) = 1 \wedge f^{-1}(\nu) = g^{-1}(1 \times \nu)$. Since g is somewhat pairwise fuzzy α-irresolute continuous and $1 \times \nu$ is a τ_i-fuzzy α-open or η_j-fuzzy α-open set on $(X, \tau_1, \tau_2) \times (Y, \eta_1, \eta_2)$, there exists a τ_1-fuzzy α-open or τ_2-fuzzy α-open set $\mu \neq 0_X$ on (X, τ_1, τ_2) such that $\mu \leq g^{-1}(1 \times \nu) = f^{-1}(\nu) \neq 0_X$. Therefore, f is somewhat pairwise fuzzy α-irresolute continuous. □

3. Somewhat pairwise fuzzy irresolute α-open mappings

In this section, We introduce a somewhat pairwise fuzzy irresolute α-open mappings which are stronger than a somewhat pairwise fuzzy α-open mapping. Also, we characterize a somewhat pairwise fuzzy irreolute α-open mapping.

Definition 3.1. A mapping $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ is called pairwise fuzzy α-open if $f(\mu)$ is an η_1-fuzzy α-open or η_2-fuzzy α-open set on (Y, η_1, η_2) for any τ_1-fuzzy open or τ_2-fuzzy open set μ on (X, τ_1, τ_2).

Definition 3.2. A mapping $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ is called pairwise fuzzy irresolute α-open if $f(\mu)$ is an η_1-fuzzy α-open or η_2-fuzzy α-open set on (Y, η_1, η_2) for any τ_1-fuzzy α-open or τ_2-fuzzy α-open set μ on (X, τ_1, τ_2).

Definition 3.3. A mapping $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ is called somewhat pairwise fuzzy α-open if there exists an η_1-fuzzy α-open or η_2-fuzzy α-open set $\nu \neq 0_Y$ on (Y, η_1, η_2) such that $\nu \leq f(\mu) \neq 0_Y$ for any τ_1-fuzzy open or τ_2-fuzzy open set μ on (X, τ_1, τ_2).

Definition 3.4. A mapping $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ is called somewhat pairwise fuzzy irresolute α-open if there exists an η_1-fuzzy α-open or η_2-fuzzy α-open set $\nu \neq 0_Y$ on (Y, η_1, η_2) such that $\nu \leq f(\mu) \neq 0_Y$ for any τ_1-fuzzy α-open or τ_2-fuzzy α-open set $\mu \neq 0_X$ on (X, τ_1, τ_2).

From the definitions, it is clear that every pairwise fuzzy irresolute α-open mapping is a somewhat pairwise fuzzy irresolute α-open mapping. Moreover, every somewhat pairwise fuzzy irresolute α-open mapping is a pairwise fuzzy α-open mapping. Also, every pairwise fuzzy α-open mapping is a somewhat pairwise fuzzy α-open mapping. But, the converses are not true in general as the following examples show.

Example 3.5. Let λ_1,λ_2 and λ_3 be fuzzy sets on $X = \{a, b, c\}$ and let σ_1,σ_2 and σ_3 be fuzzy sets on $Y = \{x, y, z\}$. Then

$$\lambda_1 = \tfrac{0.3}{a} + \tfrac{0.1}{b} + \tfrac{0.3}{c}, \quad \sigma_1 = \tfrac{0.0}{x} + \tfrac{0.1}{y} + \tfrac{0.0}{z},$$

$$\lambda_2 = \tfrac{0.5}{a} + \tfrac{0.4}{b} + \tfrac{0.5}{c}, \quad \sigma_2 = \tfrac{0.0}{x} + \tfrac{0.4}{y} + \tfrac{0.0}{z},$$

$$\lambda_3 = \tfrac{0.5}{a} + \tfrac{0.5}{b} + \tfrac{0.5}{c}, \quad \sigma_3 = \tfrac{0.0}{x} + \tfrac{0.5}{y} + \tfrac{0.0}{z},$$

are defined as follows: Consider

$$\tau_1 = \{0_X, \lambda_1, \lambda_2, 1_X\}, \quad \eta_1 = \{0_Y, \sigma_1, 1_Y\},$$

$$\tau_2 = \{0_X, \lambda_2, 1_X\}, \qquad \eta_2 = \{0_Y, \sigma_2, 1_Y\}.$$

Then, (X, τ_1, τ_2) and (Y, η_1, η_2) are fuzzy bitopologies and $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ defined by $f(a) = y$, $f(b) = y$, $f(c) = y$. We have $\sigma_1 \leq f(\lambda_1) = \sigma_1$, $\sigma_1 \leq f(\lambda_2) = \sigma_2$ and $\sigma_1 \leq f(\lambda_3) = \sigma_3$. Since σ_1 is an η_1-fuzzy α-open set on (Y, η_1, η_2), f is somewhat pairwise fuzzy irresolute α-open mapping. But, $f(\lambda_3) = \sigma_3$ is not an η_1-fuzzy α-open or η_2-fuzzy α-open set on (Y, η_1, η_2). Hence, f is not a pairwise fuzzy irresolute α-open mapping.

Example 3.6. Let λ_1,λ_2 and λ_3 be fuzzy sets on $X = \{a, b, c\}$ and let σ_1 and σ_2 be fuzzy sets on $Y = \{x, y, z\}$. Then

$$\lambda_1 = \tfrac{0.4}{a} + \tfrac{0.0}{b} + \tfrac{0.4}{c}, \quad \sigma_1 = \tfrac{0.0}{x} + \tfrac{0.5}{y} + \tfrac{0.0}{z},$$

$$\lambda_2 = \tfrac{0.5}{a} + \tfrac{0.0}{b} + \tfrac{0.5}{c}, \quad \sigma_2 = \tfrac{1.0}{x} + \tfrac{0.5}{y} + \tfrac{1.0}{z},$$

$$\lambda_3 = \tfrac{0.5}{a} + \tfrac{0.5}{b} + \tfrac{0.5}{c},$$

are defined as follows: Consider

$$\tau_1 = \{0_X, \lambda_1, \lambda_3, 1_X\}, \quad \eta_1 = \{0_Y, \sigma_1, 1_Y\},$$

$$\tau_2 = \{0_X, \lambda_2, 1_X\}, \qquad \eta_2 = \{0_Y, \sigma_2, 1_Y\}.$$

Then, (X, τ_1, τ_2) and (Y, η_1, η_2) are fuzzy bitopologies and $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ defined by $f(a) = y$, $f(b) = y$, $f(c) = y$. We have $f(\lambda_1) = 0_Y$, $f(\lambda_2) = 0_Y$ and $f(\lambda_3) = \sigma_1$ are η_1-fuzzy α-open sets on (Y, η_1, η_2), since f is a pairwise fuzzy α-open mapping. But, λ_1 is a τ_1-fuzzy α-open set on (X, τ_1, τ_2) and $f(\lambda_1) = 0_Y$. Hence, f is not a somewhat pairwise fuzzy irresolute α-open mapping.

Example 3.7. Let λ_1,λ_2 and λ_3 be fuzzy sets on $I = [0, 1]$ with

$$\begin{aligned} \lambda_1 &= 0.1, \quad 0 \le x \le 1, \\ \lambda_2 &= 0.3, \quad 0 \le x \le 1, \\ \lambda_3 &= 0.7, \quad 0 \le x \le 1, \\ \lambda_4 &= 0.8, \quad 0 \le x \le 1. \end{aligned}$$

Let

$$\tau_1 = \{0_I, \lambda_3, 1_I\}, \quad \eta_1 = \{0_I, \lambda_1, \lambda_2, 1_I\},$$

$$\tau_2 = \{0_I, \lambda_4, 1_I\}, \quad \eta_2 = \{0_I, \lambda_1^c, 1_I\}.$$

Then, (I, τ_1, τ_2) and (I, η_1, η_2) are fuzzy bitopologies on I. Consider an identity mapping $f : (I, \tau_1, \tau_2) \to (I, \eta_1, \eta_2)$ defined by $f(x) = x$, $0 \le x \le 1$. We have $\lambda_1 \le f(\lambda_3) = \lambda_3$, $\lambda_1 \le f(\lambda_4) = \lambda_4$. Since λ_1 is a τ_1-fuzzy α-open set on (I, τ_1, τ_2), f is somewhat pairwise fuzzy α-open. But, $f(\lambda_3) = \lambda_3$ and $f(\lambda_4) = \lambda_4$ are not η_1-fuzzy α-open or η_2-fuzzy α-open sets on (I, η_1, η_2). Hence f is not a pairwise fuzzy α-open mapping.

Theorem 3.8. *Let $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ be a bijection. Then the following are equivalent:*

(1) *f is somewhat pairwise fuzzy irresolute α-open.*

(2) *If μ is a τ_1-fuzzy α-closed or τ_2-fuzzy α-closed set on (X, τ_1, τ_2) such that $f(\mu) \ne 1_Y$, then there exists an η_1-fuzzy α-closed or η_2-fuzzy α-closed set $\nu \ne 1_Y$ on (Y, η_1, η_2) such that $f(\mu) < \nu$.*

Proof. (1) $\Rightarrow$ (2) Let μ be a τ_1-fuzzy α-closed or τ_2-fuzzy α-closed set on (X, τ_1, τ_2) such that $f(\mu) \ne 1_Y$. Since f is bijective and μ^c is a τ_1-fuzzy α-open or τ_2-fuzzy α-open set on (X, τ_1, τ_2), $f(\mu^c) = (f(\mu))^c \ne 0_Y$. Moreover, since f is a somewhat pairwise fuzzy irresolute α-open mapping, there exists an η_1-fuzzy α-open or η_2-fuzzy α-open set $\delta \ne 0_Y$ on (Y, η_1, η_2) such that $\delta < f(\mu^c) = (f(\mu))^c$. Consequently, $f(\mu) < \delta^c = \nu \ne 1_Y$ and ν is an η_1-fuzzy α-closed or η_2-fuzzy α-closed set on (Y, η_1, η_2).

(2) $\Rightarrow$ (1) Let μ be a τ_1-fuzzy α-open or τ_2-fuzzy α-open set on (X,τ_1,τ_2) such that $f(\mu)\neq 0_Y$. Then μ^c is a τ_1-fuzzy α-closed or τ_2-fuzzy α-closed set on (X,τ_1,τ_2) and $f(\mu^c)\neq 1_Y$. Hence, there exists an η_1-fuzzy α-closed or η_2-fuzzy α-closed set $\nu\neq 1_Y$ on (Y,η_1,η_2) such that $f(\mu^c)<\nu$. Since f is bijective, $f(\mu^c)=(f(\mu))^c<\nu$. Hence $\nu^c<f(\mu)$ and $\nu^c\neq 0_X$ is an η_1-fuzzy α-open or η_2-fuzzy α-open set on (Y,η_1,η_2). Therefore, f is somewhat pairwise fuzzy irresolute α-open.

□

Theorem 3.9. *Let $f:(X,\tau_1,\tau_2)\to(Y,\eta_1,\eta_2)$ be a surjection. Then the following are equivalent:*

(1) *f is somewhat pairwise fuzzy irresolute α-open.*
(2) *If ν is a pairwise α-dense fuzzy set on (Y,η_1,η_2), then $f^{-1}(\nu)$ is a pairwise α-dense fuzzy set on (X,τ_1,τ_2).*

Proof. (1) $\Rightarrow$ (2) Let ν be a pairwise α-dense fuzzy set on (Y,η_1,η_2). Suppose $f^{-1}(\nu)$ is not a pairwise α-dense fuzzy set on (X,τ_1,τ_2). Then, there exists a τ_1-fuzzy α-closed or τ_2-fuzzy α-closed set μ on (X,τ_1,τ_2) such that $f^{-1}(\nu)<\mu<1$. Since f is somewhat pairwise fuzzy irreolute α-open and μ^c is a τ_1-fuzzy α-open or τ_2-fuzzy α-open set on (X,τ_1,τ_2), there exists an η_1-fuzzy α-open or η_2-fuzzy α-open set $\delta\neq 0_Y$ on (Y,η_1,η_2) such that $\delta\leq f(Int\mu^c)\leq f(\mu^c)$. Since f is surjective, $\delta\leq f(\mu^c)<f(f^{-1}(\nu^c))=\nu^c$. Thus, there exists an η_1-fuzzy α-closed or η_2-fuzzy α-closed set δ^c on (Y,η_1,η_2) such that $\nu<\delta^c<1$. This is a contradiction. Hence, $f^{-1}(\nu)$ is a pairwise α-dense fuzzy set on (X,τ_1,τ_2).

(2) $\Rightarrow$ (1) Let μ be a τ_1-fuzzy open or τ_2-fuzzy open set on (X,τ_1,τ_2) and $f(\mu)\neq 0_Y$. Suppose there exists no η_1-fuzzy α-open or η_2-fuzzy α-open set $\nu\neq 0_Y$ on (Y,η_1,η_2) such that $\nu\leq f(\mu)$. Then, $(f(\mu))^c$ is an η_1-fuzzy set or η_2-fuzzy set δ on (Y,η_1,η_2) such that there exists no η_1-fuzzy α-closed or η_2-fuzzy α-closed set δ on (Y,η_1,η_2) with $(f(\mu))^c<\delta<1$. This means that $(f(\mu))^c$ is a pairwise α-dense fuzzy set on (Y,η_1,η_2). Thus $f^{-1}((f(\mu))^c)$ is a pairwise α-dense fuzzy set on (X,τ_1,τ_2). But, $f^{-1}\left((f(\mu))^c\right)=\left(f^{-1}(f(\mu))\right)^c\leq\mu^c<1$. This is a contradiction to the fact that $f^{-1}(f(\nu))^c$ is pairwise α-dense fuzzy set on (X,τ_1,τ_2). Hence, there exists an η_1-fuzzy α-open or η_1-fuzzy α-open set $\nu\neq 0_Y$ on (Y,η_1,η_2) such that $\nu\leq f(\mu)$. Therefore, f is somewhat pairwise fuzzy irresolute α-open.

□

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