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On Fuzzy e-open Sets, Fuzzy e-continuity and Fuzzy e-compactness in Intuitionistic Fuzzy Topological Spaces

Veerappan Chandrasekar[1], Durairaj Sobana[2], and Appachi Vadivel[3*]

Abstract. The purpose of this paper is to introduce and study the concepts of fuzzy e-open set, fuzzy e-continuity and fuzzy e-compactness in intuitionistic fuzzy topological spaces. After giving the fundamental concepts of intuitionistic fuzzy sets and intuitionistic fuzzy topological spaces, we present intuitionistic fuzzy e-open sets and intuitionistic fuzzy e-continuity and other results related topological concepts. Several preservation properties and some characterizations concerning intuitionistic fuzzy e-compactness have been obtained.

1. Introduction

The fuzzy concept has invaded almost all branches of Mathematics since the introduction of the concept of fuzzy set by Zadeh [16]. Fuzzy sets have applications in many fields such as information [13] and control [14]. The theory of fuzzy topological spaces was introduced and developed by Chang [3] and since then various notions in classical topology have been extended to fuzzy topological spaces. The initiations of e-open sets, e^*-open sets, a-open sets, e-continuity and e-compactness in topological spaces are due to Ekici [6–10]. In fuzzy topology, e-open sets were introduced by Seenivasan in 2014 [12]. In this paper we generalize this notion to intuitionistic fuzzy spaces and also the concepts of intuitionistic fuzzy e-open sets, intuitionistic fuzzy e-continuity and intuitionistic fuzzy e-compactness and study their properties in detail.

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* Corresponding author.

Intuitionistic fuzzy e-open set are weaker than intuitionistic fuzzy δ-pre open set, intuitionistic fuzzy δ-semi open set and stronger then intuitionistic fuzzy β-open sets. It may be possible to obtain stronger forms of the existing results in ordinary topological spaces as well as in fuzzy topological spaces. Our motivation in this paper, using intuitionistic fuzzy sets, we put this concepts in the intuitionistic fuzzy setting, then defining intuitionistic fuzzy e-open set, intuitionistic fuzzy e-continuity and intuitionistic fuzzy e-compactness are studied. Several preservation properties and some characterizations concerning intuitionistic fuzzy e-compactness have been obtained.

2. Preliminaries

First we shall present the fundamental definitions obtained by K. Atanassov and D. Coker.

Definition 2.1 ([2])**.** Let X be a nonempty fixed set. An intuitionistic fuzzy set (IFS, for short) A is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where the functions $\mu_A : X \to I$ and $\nu_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\nu_A(x)$) of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Obviously, every fuzzy set A on a nonempty set X is an IFS having the form

$$A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}.$$

Definition 2.2 ([2])**.** Let X be a nonempty set and the IFS's A and B be in the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\},$$
$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\},$$

and let $A = \{A_j : j \in J\}$ be an arbitrary family of IFS's in X, then

(i) $A \leq B$ iff $\forall x \in X\, [\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)]$;
(ii) $\overline{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$;
(iii) $\wedge A_j = \left\{\left\langle x, \wedge \mu_{A_j}(x), \vee \nu_{A_j}(x)\right\rangle : x \in X\right\}$;
(iv) $\vee A_j = \left\{\left\langle x, \vee \mu_{A_j}(x), \wedge \nu_{A_j}(x)\right\rangle : x \in X\right\}$;
(v) $\underset{\sim}{1} = \{\langle x, 1, 0\rangle : x \in X\}$ and $\underset{\sim}{0} = \{\langle x, 0, 1\rangle : x \in X\}$;
(vi) $\overline{\overline{A}} = A$, $\overline{\underset{\sim}{0}} = \underset{\sim}{1}$ and $\overline{\underset{\sim}{1}} = \underset{\sim}{0}$.

Definition 2.3 ([4])**.** Let X and Y be two nonempty sets and $f : X \to Y$ be a function.

(i) If $B = \{\langle y, \mu_B(y), \nu_B(y)\rangle : y \in X\}$ is an IFS in Y, then the preimage of B under f denoted and defined by

$$f^{-1}(B) = \left\{\left\langle f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x) : x \in X\right\rangle\right\};$$

(ii) If $A = \{\langle x, \lambda_A(x), \nu_A(x)\rangle : x \in X\}$ is an IFS in X, then the image of A under f denoted and defined by
$$f(A) = \{\langle y, f(\lambda_A)(y), f_{\_}(\nu_A)(y) : y \in Y\rangle\},$$
where
$$f(\lambda_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \lambda_A(x), & f^{-1}(y) \neq 0, \\ 0, & \text{otherwise,} \end{cases}$$
and
$$\begin{aligned} f_{\_}(\nu_A)(y) &= 1 - f(1 - \nu_A)(y) \\ &= \begin{cases} \inf_{x \in f^{-1}(y)} \nu_A(x), & f^{-1}(y) \neq 0, \\ 1, & \text{otherwise.} \end{cases} \end{aligned}$$

Corollary 2.4 ([4])**.** *Let $A, A_j (j \in J)$ be IFS's in X, $B, B_j (j \in J)$ be IFS's in Y and $f : X \to Y$ be a function. Then*

(i) $A_1 \leq A_2 \quad \Rightarrow \quad f(A_1) \leq f(A_2)$;
(ii) $B_1 \leq B_2 \quad \Rightarrow \quad f^{-1}(B_1) \leq f^{-1}(B_2)$;
(iii) $A \leq f^{-1}(f(A))$ *(If f is one-to-one, then $A = f^{-1}(f(A))$);*
(iv) $f(f^{-1}(B)) \leq B$ *(If f is onto, then $f(f^{-1}(B)) = B$);*
(v) $f^{-1}(1_{\sim}) = 1_{\sim}$ *and* $f^{-1}(0_{\sim}) = 0_{\sim}$;
(vi) $f(1_{\sim}) = 1_{\sim}$ *if f is onto and* $f(0_{\sim}) = 0_{\sim}$;
(vii) $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$.

Now, we mention the definition of intuitionistic fuzzy points and also some basic results related to it.

Definition 2.5 ([5])**.** Let X be a nonempty set and $c \in X$ a fixed element in X. If $a \in (0, 1]$ and $b \in [0, 1)$ are two fixed real numbers such that $a + b \leq 1$, then the IFS $c(a, b) = \langle x, c_a, 1 - c_{1-b}\rangle$ is called an intuitionistic fuzzy point (IFP, for short) in X, where α denotes the degree of membership of $c(a, b)$, and $c \in X$ the support of $c(a, b)$.

Definition 2.6 ([5])**.** Let $c(a, b)$ be an IFP in X and $A = \langle x, \mu_A, \nu_A\rangle$ an IFS in X. Suppose further that $a, b \in (0, 1)$, $c(a, b)$ is said to be properly contained in A ($c(a, b) \in A$ for short) iff $a < \mu_A(c)$ and $b > \nu_A(c)$.

Definition 2.7 ([5])**.** (i) An IFP $c(a, b)$ in X is said to be quasi-coincident with the IFS $A = \langle x, \mu_A, \nu_A\rangle$, denoted by $c(a, b)qA$, iff $a > \nu_A(c)$ or $b < \mu_A(c)$.

(ii) Let $A = \langle x, \mu_A, \nu_A\rangle$ and $B = \langle x, \mu_B, \nu_B\rangle$ be two IFS's in X. Then, A and B are said to be quasi-coincident, denoted by AqB, iff there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Proposition 2.8. *Let $f : X \to Y$ be a function and $c(a, b)$ be an IFP in X.*

(i) *If for IFS B in Y we have $f(c(a,b))qB$, then $c(a,b)qf^{-1}(B)$.*
(ii) *If for IFS A in X we have $c(a,b)qA$, then $f(c(a,b))qf(A)$.*

Proof. (i) Let $f(c(a,b))qB$ for IFS B in Y. Then $a > \nu_B(f(c))$ or $b < \mu_B(f(c))$. (equivalently, $(f(c)_a > \nu_B$ or $1 - f(c)_{1-b} < \mu_B)$. This gives that $a > f^{-1}(\nu_B)(c)$ or $b < f^{-1}(\mu_B)(c)$ (equivalently, $c_a > f^{-1}(\nu_B)$ or $1 - c_{1-b} < f^{-1}(\mu_B))$ which implies $c(a,b)qf^{-1}(B)$.

(ii) Let $c(a,b)qA$, for IFS A in X. Then $a > \nu_A(c)$ or $b < \mu_A(c)$. This implies $a > \inf_{x\in f^{-1}(f(c))} \nu_A(x)$ or $b < \sup_{x\in f^{-1}(f(c))} \mu_A(x)$ which gives $a > f_{-}(\nu_A)(f(c))$ or $b < f(\mu_A)(f(c))$. Thus we have $f(c(a,b))qf(A)$.

□

Proposition 2.9. *Let A be an IFS in IFTS in X and $c(a,b)$ be an IFP in X. If $c(a,b) \in A$, then $c(a,b)qA$.*

Proof. Let $c(a,b) \in A$, then $a < \mu_A(c)$ and $b > \nu_A(c)$ which implies $c(b,a)qA$. □

Here we give the definitions of intuitionistic fuzzy topological space and some types of intuitionistic fuzzy continuity introduced by Coker [4]. Also, some of results is of interest.

Definition 2.10 ([4])**.** An intuitionistic fuzzy topology (IFT, for short) on a nonempty set X is a family Ψ of IFS's in X satisfying the following axioms:

(i) $\underset{\sim}{0}, \underset{\sim}{1} \in \Psi$;
(ii) $A_1 \wedge A_2 \in \Psi$ for any $A_1, A_2 \in \Psi$;
(iii) $\vee A_j \in \Psi$ for any $\{A_j : j \in J\} \subseteq \Psi$.

In this case the pair (X, Ψ) is called an intuitionistic fuzzy topological space (IFTS, for short) and each IFS in Ψ is known as an intuitionistic fuzzy open set (IFOS, for short) in X.

Definition 2.11 ([4])**.** The complement $\overline{A}$ of IFOS A in IFTS(X, Ψ) is called an intuitionstic fuzzy closed set (IFCS, for short).

Definition 2.12 ([4])**.** Let (X, Ψ) be an IFTS and $A = \langle x, \mu_A(x), \nu_A(x)\rangle$ be an IFS in X. Then the fuzzy closure and fuzzy interior of A are denoted and defined by: $cl(A) = \wedge\{K : K$ is an IFCS in X and $A \leq K\}$ and $int(A) = \vee\{G : G$ is an IFOS in X and $G \leq A\}$.

Definition 2.13 ([15])**.** Let A be IFS in an IFTS (X, Ψ). A is called an

(i) intuitionistic fuzzy regular open set (briefly $IFROS$) if $A = intcl(A)$.

(ii) intuitionistic fuzzy regular closed set (briefly $IFRCS$) if $A = clint(A)$.

Definition 2.14. Let (X, Ψ) be an IFTS and $A = \langle x, \mu_A(x), \nu_A(x)\rangle$ be a IFS in X. Then the fuzzy δ closure of A are denoted and defined by $cl_\delta(A) = \wedge\{K : K$ is an IFRCS in X and $A \leq K\}$ and $int_\delta(A) = \vee\{G : G$ is an IFROS in X and $G \leq A\}$.

Definition 2.15. Let (X, Ψ) and (Y, ϕ) be IFTS's. A function $f : (X, \Psi) \to (Y, \Phi)$ is called intuitionistic fuzzy continuous [4] if $f^{-1}(B)$ is an IFOS for every $B \in \Phi$.

Definition 2.16 ([4])**.** Let X be an IFTS. A family $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle\, |i \in I\}$ of IFOS's in X satisfying the condition $\bigcup\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle\, |i \in I\} = 1\!\!1$ is called an intuitionistic fuzzy open cover of X.

A finite subfamily of an intuitionistic fuzzy open cover $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle\, |i \in I\}$ which is also an intuitionistic fuzzy open cover of X is called a finite subcover of $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle\, |i \in I\}$.

An IFTS X is called intuitionistic fuzzy compact if and only if every intuitionistic fuzzy open cover has a finite subcover.

Definition 2.17 ([4])**.** Let A be an IFS in an IFTS X. A family $\{\langle x, \mu_{G_i},\ \nu_{G_i}\rangle |i \in I\}$ of IFOS's in X satisfying the condition $A \subseteq \bigcup\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle\, |i \in I\}$ is called an intuitionistic fuzzy open cover of A.

A finite subfamily of an intuitionistic fuzzy open cover $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle\, |i \in I\}$ of A which is also an intuitionistic fuzzy open cover of A is called a finite subcover of $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle\, |i \in I\}$.

An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS X is called intuitionistic fuzzy compact if and only if every intuitionistic fuzzy open cover of A have a finite subcover.

3. Intuitionistic Fuzzy e-open Sets

Now we introduce the following definition.

Definition 3.1. Let A be an IFS in an IFTS (X, Ψ). A is called an intuitionistic fuzzy δ-semiopen (resp. δ-preopen, β-open) set (IFδSO (resp. IFδPO, IFβO), for short), if $A \leq cl(\mathrm{int}_\delta(A))$ (resp. $A \leq \mathrm{int}(cl_\delta(A))$, $A \leq cl(\mathrm{int}(cl(A))))$. A is called an intuitionistic fuzzy δ-semiclosed (resp. δ-preclosed, β-closed) set (IFδSC (resp. IFδPC, IFβC) (for short)) if $A \geq \mathrm{int}(cl_\delta(A))$ (resp. $A \geq cl(\mathrm{int}_\delta(A))$, $A \geq \mathrm{int}(cl(int(A))))$.

Definition 3.2. Let A be an IFS in an IFTS(X, Ψ). The intuitionistic fuzzy δ-semi-closure (δ-semi-interior) (resp. δ-pre-closure (δ-pre-interior)) of A is denoted by $scl_\delta(A)$ ($sint_\delta(A)$) (resp. $pcl_\delta(A)(pint_\delta(A)))$ and defined as follows:

(i) $scl_\delta(A)(pcl_\delta(A)) = \wedge\{K : K$ is an IFδSCS (resp. IFδPCS) in X and $A \leq K\}$,
(ii) $sint_\delta(A)(pint_\delta(A)) = \vee\{G : G$ is an IFδSOS (resp. IFδPOS) in X and $G \leq A\}$.

It is clear that A is an IFδSCS (resp. IFδPCS, IFδSOS, IFδPOS) iff $A = scl_\delta(A)$ (resp. $A = pcl_\delta(A)$, $A = sint_\delta(A)$, $A = pint_\delta(A)$).

Theorem 3.3. *Let A be an IFS in an IFTS(X, Ψ), then*
(i) $pcl_\delta(A) \geq A \vee cl\,(int_\delta(A))$ *and* $pint_\delta(A) \leq A \wedge int\,(cl_\delta(A))$*;*
(ii) $scl_\delta(A) \geq A \vee int\,(cl_\delta(A))$ *and* $sint_\delta(A) \leq A \wedge cl\,(int_\delta(A))$*.*

Proof. We will prove only the first statement of (i) and the second is similar. Since $pcl_\delta(A)$ is IFδPCS, we have

$$cl(int_\delta(A)) \leq clint_\delta(pcl_\delta(A)) \leq pcl_\delta(A).$$

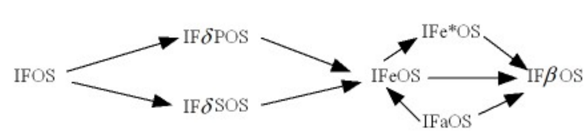
Thus $A \vee cl(int_\delta(A)) \leq pcl_\delta(A)$. □

Definition 3.4. Let (X, Ψ) and (Y, ϕ) be IFTS's. A function $f : (X, \Psi) \to (Y, \Phi)$ is called intuitionistic fuzzy δ-semicontinuous (resp. δ-precontinuous, β-continuous) (IFδsemi-cont. (resp. IFδpre-cont., IFβ-cont. for short)) if $f^{-1}(B)$ is an IFδSO (resp. IFδPO, IFβOS) for every $B \in \Phi$.

In the sequel, we introduce and study in IFTS's the concept of fuzzy e-open (closed) sets which generalized the concepts of IFOS's (IFCS's).

Definition 3.5. Let A be an IFS in an IFTS(X, Ψ). A is called
(i) an intuitionistic fuzzy e-open set (IFeOS, for short) in X if $A \leq clint_\delta(A) \vee intcl_\delta(A)$,
(ii) an intuitionistic fuzzy e-closed set (IFeCS, for short) in X if $A \geq clint_\delta(A) \wedge intcl_\delta(A)$.
(iii) an intuitionistic fuzzy e^*-open set (IFe^*OS, for short) in X if $A \leq clintcl_\delta(A)$.
(iv) an intuitionistic fuzzy a-open set (IFaOS, for short) in X if $A \leq intclint_\delta(A)$.

Remark 3.6. From the above definition and some types of IFOS's, we have the following diagram:



The converse of the above implications need not be true in general as shown by the following examples:

Example 3.7. Let $X = \{a, b\}$ and

$$A = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.2}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}\right) \right\rangle,$$
$$B = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.5}\right), \left(\frac{a}{0.7}, \frac{b}{0.2}\right) \right\rangle.$$

Then the family $\Psi = \{0_\sim, 1_\sim, A\}$ is an IFT on X. Since $B \leq clintcl(B) = \overline{A}$ and $B \leq clintcl_\delta(B) = \overline{A}$ then B is an IFβOS and IFe^*OS in X, but not IFeOS since $B \not\leq clint_\delta(B) \vee intcl_\delta(B) = 0_\sim \vee A = A$.

Example 3.8. Let $X = \{a, b\}$ and

$$A = \left\langle x, \left(\frac{a}{0.2}, \frac{b}{0.1}\right), \left(\frac{a}{0.7}, \frac{b}{0.5}\right) \right\rangle,$$
$$B = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.5}\right), \left(\frac{a}{0.7}, \frac{b}{0.2}\right) \right\rangle.$$

Then the family $\Psi = \{0_\sim, 1_\sim, A\}$ is an IFT on X. Since $B \leq clint_\delta(B) \vee intcl_\delta(B) = \overline{A} \vee A = \overline{A}$, B is an IFeOS,and IFβOS but not IFδPOS, and IFaOS hence $B \not\leq intcl_\delta(B) = A$ and $B \not\leq intclint_\delta(B) = A$

Example 3.9. Let $X = \{a, b, c, d\}$ and

$$A = \left\langle x, \left(\frac{a}{1}, \frac{b}{0}, \frac{c}{0.2}, \frac{d}{0}\right), \left(\frac{a}{0}, \frac{b}{1}, \frac{c}{0.7}, \frac{d}{1}\right) \right\rangle,$$
$$B = \left\langle x, \left(\frac{a}{0}, \frac{b}{1}, \frac{c}{0}, \frac{d}{0}\right), \left(\frac{a}{1}, \frac{b}{0}, \frac{c}{1}, \frac{d}{0.1}\right) \right\rangle,$$
$$C = \left\langle x, \left(\frac{a}{1}, \frac{b}{0}, \frac{c}{0}, \frac{d}{1}\right), \left(\frac{a}{0}, \frac{b}{0.2}, \frac{c}{0}, \frac{d}{0}\right) \right\rangle.$$

Then the family $\Psi = \{0_\sim, 1_\sim, A, B, A \vee B\}$ is an IFT on X. Since $C \leq clint_\delta(C) \vee intcl_\delta(C) = 0_\sim \vee 1_\sim = 1_\sim$, C is an IFeOS, but not IFδSOS, hence $C \not\leq clint_\delta(C) = 0_\sim$.

Example 3.10. Let $X = \{a, b\}$ and

$$A = \left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.5}\right), \left(\frac{a}{0.3}, \frac{b}{0.5}\right) \right\rangle,$$
$$B = \left\langle x, \left(\frac{a}{0.7}, \frac{b}{0.2}\right), \left(\frac{a}{0.3}, \frac{b}{0.2}\right) \right\rangle.$$

Then the family $\Psi = \{0_\sim, 1_\sim, A\}$ is an IFT on X. Since $B \leq clintcl(B) = 1_\sim$ then B is an IFβOS in X but not IFe^*OS. Since $B \not\leq clintcl_\delta(B) = 0_\sim$

Example 3.11. Refer to Example 3.9, $C \leq intcl_\delta(C) = 1_\sim$, C is an IFδPOS, but not IFOS.

Example 3.12. Refer to Example 3.8, $B \leq clint_\delta(B) = \overline{A}$, B is an IFδSOS, but not IFOS.

Remark 3.13. (i) It is clear that the union of any family of IFeOS's is IFeOS.

(ii) The intersection of two IFeOS's need not be IFeOS as illustrated by the following example.

Example 3.14. Refer to Example 3.8, B is an $IFeOS$ and also

$$C = \left\langle x, \left(\frac{a}{0.1}, \frac{b}{0.2}\right), \left(\frac{a}{0.1}, \frac{b}{0.1}\right)\right\rangle,$$

is an IFeOS, since $C \leq clint_\delta(C) \vee intcl_\delta(C) = \overline{A} \vee A = \overline{A}$. But

$$B \wedge C = \left\langle x, \left(\frac{a}{0.1}, \frac{b}{0.2}\right), \left(\frac{a}{0.7}, \frac{b}{0.2}\right)\right\rangle,$$

is not IFeOS, since $B \wedge C \not\leq clint_\delta(B \wedge C) \vee intcl_\delta(B \wedge C) = 0\!\!\!\sim \vee A = A$.

Proposition 3.15. *Let A be an IFS in an IFTS(X, Ψ).*

(i) *If A is an IFeOS and $int_\delta(A) = 0\!\!\!\sim$, then A is an IFδPOS.*

(ii) *If A is an IFeOS and $cl_\delta(A) = 0\!\!\!\sim$, then A is an IFδSOS.*

(iii) *If A is an IFeOS and IFδCS, then A is an IFδSOS.*

(iv) *If A is an IFδSOS and IFδCS, then A is an IFeOS.*

Proof. (i) Let A be an IFeOS, that is

$$A \leq clint_\delta(A) \vee intcl_\delta(A) = 0\!\!\!\sim \vee intcl_\delta(A) = intcl_\delta(A).$$

Hence A is an IFδPOS.

(ii) Follows from (i).

(iii) Let A be an IFeOS and IFδCS, that is

$$A \leq clint_\delta(A) \vee intcl_\delta(A) = clint_\delta(A) \vee int(A) = clint_\delta(A).$$

Hence A is an IFδSOS.

(iv) Let A be an IFδSOS and IFδCS, that is

$$A \leq clint_\delta(A) \leq clint_\delta(A) \vee intcl_\delta(A).$$

Hence A is an IFeOS.

□

Theorem 3.16. *Let A be an IFS in an IFTS(X, Ψ), A is an IFeOS if and only if $A = pint_\delta(A) \vee sint_\delta(A)$.*

Proof. Let A be an IFeOS. Then $A \leq cl(int_\delta(A)) \vee int(cl_\delta(A))$. By Theorem 3.3, we have

$$\begin{aligned} pint_\delta(A) \vee sint_\delta(A) &= (A \wedge int(cl_\delta(A))) \vee (A \wedge cl\,(int_\delta(A))) \\ &= A \wedge (int(cl_\delta(A)) \vee cl\,(int_\delta(A))) \\ &= A. \end{aligned}$$

Conversely, if $A = pint_\delta(A) \vee sint_\delta(A)$ then, by Theorem 3.3

$$\begin{aligned} A &= pint_\delta(A) \vee sint_\delta(A) \\ &= (A \wedge int(cl_\delta(A))) \vee (A \wedge cl(int_\delta(A))) \\ &= A \wedge (int(cl_\delta(A)) \vee cl(int_\delta(A))) \\ &\leq int\,(cl_\delta(A)) \vee cl\,(int_\delta(A))\,, \end{aligned}$$

and hence A is an IFeOS. □

Definition 3.17. Let (X, Ψ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the intuitionistic fuzzy e-interior and intuitionistic fuzzy e-closure are defined and denoted by:

$$cl_e(A) = \wedge\{K : K \text{ is an IFeCS in } X \text{ and } A \leq K\},$$

and

$$int_e(A) = \vee\{G : G \text{ is an IFeOS in } X \text{ and } G \leq A\}.$$

It is clear that A is an IFeCS (IFeOS) in X iff $A = cl_e(A)(A = int_e(A))$.

Proposition 3.18. *For any IFS A in an IFTS(X, Ψ) we have:*

(i) $cl_e(\overline{A}) = \overline{int_e(A)}$, $int_e(\overline{A}) = \overline{cl_e(A)}$.
(ii) $cl_e(A \vee B) \geq cl_e(A) \vee cl_e(B)$, $int_e(A \vee B) \geq int_e(A) \vee int_e(B)$.
(iii) $cl_e(A \wedge B) \leq cl_e(A) \wedge cl_e(B)$, $int_e(A \wedge B) \leq int_e(A) \wedge int_e(B)$.

Remark 3.19. The inclusion of the results (ii) and in the above Proposition can not be replaced by equality. In the following example we shall shown one of them.

Example 3.20. Let $X = \{a, b, c, d\}$ and

$$\begin{aligned} A &= \left\langle x, \left(\frac{a}{1}, \frac{b}{0}, \frac{c}{0.2}, \frac{d}{0}\right), \left(\frac{a}{0}, \frac{b}{1}, \frac{c}{0.7}, \frac{d}{1}\right)\right\rangle, \\ B &= \left\langle x, \left(\frac{a}{0}, \frac{b}{1}, \frac{c}{0}, \frac{d}{0}\right), \left(\frac{a}{1}, \frac{b}{0}, \frac{c}{1}, \frac{d}{0.1}\right)\right\rangle, \\ C &= \left\langle x, \left(\frac{a}{1}, \frac{b}{0}, \frac{c}{0}, \frac{d}{1}\right), \left(\frac{a}{0}, \frac{b}{0.2}, \frac{c}{0}, \frac{d}{0}\right)\right\rangle, \\ D &= \left\langle x, \left(\frac{a}{0}, \frac{b}{0.9}, \frac{c}{0.3}, \frac{d}{1}\right), \left(\frac{a}{1}, \frac{b}{0}, \frac{c}{0.2}, \frac{d}{0}\right)\right\rangle. \end{aligned}$$

Then the family $\Psi = \{0_\sim, 1_\sim, A, B,\ A \vee B\}$ is an IFT on X. Notice that C and D are IFeCS's in X, then $cl_e(C) = C$ and $cl_e(D) = D$. But $cl_e(C \vee D) = 1_\sim$ (obviously, $C \vee D$ is not IFeCS). Then

$$1_\sim = cl_e(C \vee D) \not\leq cl_e(C) \vee cl_e(D) = C \vee D.$$

Proposition 3.21. *For any IFS A in an $IFTS(X, \Psi)$, we have:*

(i) $cl_e(A) \geq clint_\delta(A) \wedge intcl_\delta(A)$.

(ii) $int_e(A) \leq clint_\delta(A) \vee intcl_\delta(A)$.

Proof. (i) $cl_e(A)$ is an IFeCS and $A \leq cl_e(A)$, then

$$cl_e(A) \geq clint_\delta cl_e(A) \wedge intcl_\delta cl_e(A) \geq clint_\delta(A) \wedge intcl_\delta(A).$$

(ii) Follows from (i) by taking the complementation.

□

Theorem 3.22. *Let A be an IFS in an IFTS(X, Ψ), then*

$$cl_e(A) = pcl_\delta(A) \wedge scl_\delta(A).$$

Proof. It is obvious that, $cl_e(A) \leq pcl_\delta(A) \wedge scl_\delta(A)$. Conversely, from Definition 3.17 we have

$$cl_e(A) \geq cl(int_\delta(cl_e(A))) \wedge int(cl_\delta(cl_e(A))) \geq cl(int_\delta(A)) \wedge int(cl_\delta(A)).$$

Since $cl_e(A)$ is IFeOS, by Theorem 3.3, we have

$$\begin{aligned} pcl_\delta(A) \wedge scl_\delta(A) &= (A \vee cl(int_\delta(A))) \wedge (A \vee int(cl_\delta(A))) \\ &= A \vee (cl(int_\delta(A)) \wedge int(cl_\delta(A))) \\ &= A \leq cl_e(A). \end{aligned}$$

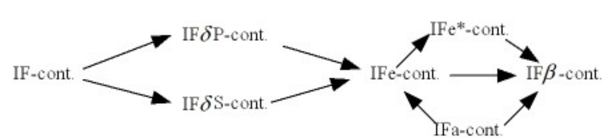
□

4. Intuitionistic Fuzzy e-continuity

Definition 4.1. A function $f : (X, \Psi) \to (Y, \Phi)$ is called an

(i) intuitionistic fuzzy e-continuous (IFe-cont., for short) if $f^{-1}(B)$ is an IFeOS in X, for every $B \in \Phi$.
(ii) intuitionistic fuzzy e^*-continuous (IFe^*-cont., for short) if $f^{-1}(B)$ is an IFe^*OS in X, for every $B \in \Phi$.
(iii) intuitionistic fuzzy a-continuous (IFa-cont., for short) if $f^{-1}(B)$ is an IFaOS in X, for every $B \in \Phi$.

From the above definition and some known types of intuitionistic fuzzy continuity, one can show the following diagram:



Now, the following examples shows that the converses of these implications are not true in general.

Example 4.2. Let $X = \{a, b, c\}$, $Y = \{1, 2, 3\}$ and

$$A = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.1}, \frac{c}{0.4}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right)\right\rangle,$$
$$B = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.5}\right), \left(\frac{a}{0.2}, \frac{b}{0.2}, \frac{c}{0.4}\right)\right\rangle,$$
$$C = \left\langle y, \left(\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.3}\right), \left(\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.4}\right)\right\rangle.$$

Now, the family $\Psi = \{0_\sim, 1_\sim, A, B\}$ of IFS's in X is an IFT on X and the family $\Phi = \{0_\sim, 1_\sim, C\}$ of IFS's in Y is an IFT on Y. If we define the function $f : X \to Y$ by $f(a) = 3$, $f(b) = 1$, $f(c) = 2$ then

$$f^{-1}(C) = \left\langle y, \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.5}\right)\right\rangle,$$
$$f^{-1}(C) \leq clintcl\left(f^{-1}(C)\right) = \overline{A},$$
$$f^{-1}(C) \leq clintcl_\delta\left(f^{-1}(C)\right) = \overline{A}.$$

But

$$f^{-1}(C) \not\leq clint_\delta\left(f^{-1}(C)\right) \vee intcl_\delta\left(f^{-1}(C)\right) = 0_\sim \vee A = A,$$

and

$$f^{-1}(C) \not\leq intclint_\delta\left(f^{-1}(C)\right) = 0_\sim.$$

Thus f is IFβ-cont. and IFe^*-continuous but not IFe-cont. and IFa-continous.

Example 4.3. Let $X = Y = \{a, b\}$ and

$$A = \left\langle x, \left(\frac{a}{0.2}, \frac{b}{0.1}\right), \left(\frac{a}{0.7}, \frac{b}{0.5}\right)\right\rangle,$$
$$B = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.5}\right), \left(\frac{a}{0.7}, \frac{b}{0.2}\right)\right\rangle.$$

Consider the IFT's $\Psi = \{0_\sim, 1_\sim, A\}$ and $\Phi = \{0_\sim, 1_\sim, B\}$ on X. Then the identity function $f : (X, \Psi) \to (Y, \Phi)$ is IFe-continuous, but not IFδp cont. and IFa-continous (Indeed, $B \leq intcl_\delta(B) \vee clint_\delta(B) = A \vee \overline{A} = \overline{A}$, but $B \not\leq intcl_\delta(B) = A$ and $B \not\leq intclint_\delta(B) = A$).

Example 4.4. Let $X = Y = \{a, b\}$ and

$$A = \left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.5}\right), \left(\frac{a}{0.3}, \frac{b}{0.5}\right)\right\rangle,$$
$$B = \left\langle x, \left(\frac{a}{0.7}, \frac{b}{0.2}\right), \left(\frac{a}{0.3}, \frac{b}{0.2}\right)\right\rangle.$$

Consider the IFT's $\Psi = \{0_\sim, 1_\sim, A\}$ and $\Phi = \{0_\sim, 1_\sim, B\}$ on X. Then the identity function $f : (X, \Psi) \to (Y, \Phi)$ is $IF\beta$-continuous, but not IFe^*-cont. (Indeed, $B \leq clintcl(B) = 1_\sim$, but $B \nleq clintcl_\delta(B) = 0_\sim$).

Example 4.5. Let $X = Y = \{a, b, c, d\}$ and

$$A = \left\langle x, \left(\frac{a}{0.1}, \frac{b}{0}, \frac{c}{0.2}, \frac{d}{0}\right), \left(\frac{a}{0}, \frac{b}{1}, \frac{c}{0.7}, \frac{d}{1}\right)\right\rangle,$$
$$B = \left\langle x, \left(\frac{a}{0}, \frac{b}{1}, \frac{c}{0}, \frac{d}{0}\right), \left(\frac{a}{1}, \frac{b}{0}, \frac{c}{1}, \frac{d}{0.1}\right)\right\rangle,$$
$$C = \left\langle x, \left(\frac{a}{1}, \frac{b}{1}, \frac{c}{0.2}, \frac{d}{0}\right), \left(\frac{a}{0}, \frac{b}{0}, \frac{c}{0.7}, \frac{d}{0.1}\right)\right\rangle.$$

Now, the family $\Psi = \{0_\sim, 1_\sim, A, B, A \vee B\}$ of IFS's in X is an IFT on X and the family $\Phi = \{0_\sim, 1_\sim, C\}$ of IFS's in Y is an IFT on Y. Then the identity function $f : (X, \Psi) \to (Y, \Phi)$ is IFe-continuous, but not IFδS cont. (Indeed, $C \leq intcl_\delta(C) \vee clint_\delta(C) = 1_\sim \vee 0_\sim = 1_\sim$, but $C \nleq clint_\delta(C) = 0_\sim$).

Example 4.6. Refer to Example 4.5, the identity function $f : (X, \Psi) \to (Y, \Phi)$ is IFδP-continuous, but not IF cont. (Indeed, $C \leq intcl_\delta(C) = 1_\sim$, but C is not open in (X, Ψ)).

Example 4.7. Refer to Example 4.3, the identity function $f : (X, \Psi) \to (Y, \Phi)$ is IFδS-continuous, but not IF cont. (Indeed, $B \leq clint_\delta(B) = \overline{A}$, but B is not open in (X, Ψ)).

Definition 4.8. Let (X, Ψ) be an IFTS on X and $c(a, b)$ an IFP in X. An IFS N is called ϵe-nbd (ϵeq-nbd) of $c(a, b)$ if there exists an IFeOS G in X such that $c(a, b) \in G \leq N$ ($c(a, b)qG \leq A$).

The family of all ϵe-nbd (ϵeq-nbd) of $c(a, b)$ will be denoted by N_ϵ^e $(N_\epsilon^{eq})(c(a, b))$.

Theorem 4.9. *An IFS A of an IFTS(X, Ψ) is an IFeOS iff for every IFP$c(a, b)qA$, $A \in N_\epsilon^{eq}(c(a, b))$.*

Proof. $A = \langle x, \mu_A, \nu_A\rangle$ be an IFeOS of X and $c(a, b)qA$. Then $c(a, b)qA \leq A$. Hence $A \in N_\epsilon^{eq}(c(a, b))$.

Conversely, let $c(a, b) \in A$, this implies $a < \mu_A(c)$ and $b > \nu_A(c)$. Since $a, b \in (0, 1)$ and $a + b \leq 1$, we have $c(a, b)qA$ and by hypothesis $A \in N_\epsilon^{eq}(c(b, a))$, then there exists an IFeOS G such that $c(a, b)qG \leq A$ which implies $c(a, b) \in G \leq A$. Hence by Remark 3.13 (i), we have that A is an IFeOS. □

Theorem 4.10. *Let $f : (X, \Psi) \to (Y, \Phi)$ be a function. Then the following are equivalent:*

(i) *f is IFe-continuous.*
(ii) *for every $B \in N_{\epsilon}^{eq}(c(a,b))$, there exists $A \in N_{\epsilon}^{eq}(c(a,b))$ such that $f(A) \leq B$.*

Proof.(i) $\Rightarrow$(ii) Let $c(a,b)$ be any IFP in X and $B \in N_{\epsilon}^{eq}(f(c(a,b)))$. Then there exists an IFOS G of Y such that $f(c(a,b))qG \leq B$. Since f is IFe-continuous, $f^{-1}(G)$ is an IFeOS of X with $c(a,b)qf^{-1}(G)$ (by Proposition 2.8). Let $A = f^{-1}(G)$ then $A \in N_{\epsilon}^{eq}(c(a,b))$ such that $f(A) = ff^{-1}(G) \leq G \leq B$.

(ii) $\Rightarrow$ (i) Let B be an IFOS in Y and $c(a,b) \in f^{-1}(B)$. This implies that $f(c(a,b)) \in B$. Thus by Proposition 2.9 $f(c(a,b))qB$, i.e., $B \in N_{\epsilon}^{q}(f(c(a,b)))$. So there exists $A \in N_{\epsilon}^{eq}(c(b,a))$ such that $f(A) \leq B$. Then there exists an IFeOS H of X such that $c(a,b)qH \leq A \leq f^{-1}(B)$. This implies that $c(a,b) \in H \leq f^{-1}(B)$. Hence by Remark 3.13 (i), $f^{-1}(B)$ is an IFeOS.

□

Theorem 4.11. *Let $f : (X,\Psi) \to (Y,\Phi)$ be a function. Then the following are equivalent:*

(i) *f is an IFe-continuous.*
(ii) *$f^{-1}(B)$ is an IFeCS in X, for every $B \in \Phi$.*
(iii) *$f(cl_e(A)) \leq cl(f(A))$ for every IFS A in X.*
(iv) *$cl_e(f^{-1}(B)) \leq f^{-1}(cl(B))$, for every IFS B in Y.*

Proof.(i) $\Rightarrow$ (ii) Obvious.

(ii) $\Rightarrow$ (iii) Let A be an IFS in X. Then $cl(f(A))$ is an IFCS in Y. By (ii), $f^{-1}(cl(f(A)))$ is an IFeCS in X, and so

$$f^{-1}(cl(f(A))) = cl_e\left(f^{-1}\left(cl\left(f(A)\right)\right)\right).$$

Since $A \leq f^{-1}f(A)$, we have

$$\begin{aligned} cl_e(A) &\leq cl_e(f^{-1}f(A)) \\ &\leq cl_e(f^{-1}(clf(A))) \\ &= f^{-1}(clf(A)). \end{aligned}$$

Hence $f(cl_e(A)) \leq cl(f(A))$.

(iii) $\Rightarrow$ (iv) Let B be an IFOS in Y. By (iii), we have

$$f(cl_e(f^{-1}(B))) \leq cl(ff^{-1}(B)).$$

Hence

$$cl_e\left(f^{-1}(B)\right) \leq f^{-1}\left(cl\left(ff^{-1}(B)\right)\right) \leq f^{-1}\left(cl(B)\right).$$

(iv) $\Rightarrow$ (i) Let B be an IFOS in Y. Then B is an IFCS. By (iv), we have

$$cl_e\left(f^{-1}(B)\right) \leq f^{-1}\left(cl(\overline{B})\right) = f^{-1}\left(\overline{B}\right)$$

which implies

$$f^{-1}(B) \geq cl_e\left(f^{-1}(\overline{B})\right) = \overline{int_e\left(f^{-1}(B)\right)}.$$

Hence $f^{-1}(B)$ is an IFeOS in X.

□

Theorem 4.12. *Let $f : (X, \Psi) \to (Y, \Phi)$ be a function. Then the following are equivalent:*

(i) *f is an IFe-continuous.*
(ii) *$clint_\delta(f^{-1}(B)) \wedge intcl_\delta(f^{-1}(B)) \leq f^{-1}(cl(B))$, for every IFS B in Y.*

Proof. (i) $\Rightarrow$ (ii) Let B be an IFS in Y. Then $cl(B)$ is an IFCS. By (i) and using Theorem 4.11, we have $f^{-1}(cl(B))$ is an IFeCS in X. Hence

$$\begin{aligned} f^{-1}(cl(B)) &\geq clint_\delta\left(f^{-1}(cl(B))\right) \wedge intcl_\delta\left(f^{-1}(cl(B))\right) \\ &\geq clint_\delta\left(f^{-1}(B)\right) \wedge intcl_\delta\left(f^{-1}(B)\right). \end{aligned}$$

(ii) $\Rightarrow$ (i) Let B be an IFCS in Y. Then by (ii)

$$clint_\delta\left(f^{-1}(B)\right) \wedge intcl_\delta\left(f^{-1}(B)\right) \leq f^{-1}(cl(B)) = f^{-1}(B),$$

which implies $f^{-1}(B)$ is an IFeCS in X.

□

Theorem 4.13. *Let (X, Ψ), (Y, Φ) and (Z, Ω) be IFTS's. If $f : X \to Y$ is IFe-continuous and $g : Y \to Z$ is IF-continuous, then $g \circ f$ is IFe-continuous.*

Proof. Obvious. □

Remark 4.14. The composition of two IFe-continuous functions need not be IFe-continuous as shown by the following example.

Example 4.15. Let $X = Y = Z = \{a, b, c\}$ and

$$A = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.1}, \frac{c}{0.4}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right)\right\rangle,$$
$$B = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.5}\right), \left(\frac{a}{0.2}, \frac{b}{0.2}, \frac{c}{0.4}\right)\right\rangle,$$
$$C = \langle x, \left(\frac{a}{0.4}, \frac{b}{0.3}, \frac{c}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.5}\right)\rangle,$$
$$D = \left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}\right)\right\rangle,$$
$$E = \left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}\right), \left(\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.5}\right)\right\rangle,$$
$$F = \left\langle x, \left(\frac{a}{0.4}, \frac{b}{0.3}, \frac{c}{0.4}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}\right)\right\rangle,$$
$$G = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.5}\right)\right\rangle.$$

Then the family $\Psi = \{0_\sim, 1_\sim, A, B\}$, $\Phi = \{0_\sim, 1_\sim, C, D, E, F\}$ and $\Omega = \{0_\sim, 1_\sim, G\}$ are IFS's in X respectively. If we define the identity functions $f : X \to Y$ and $g : Y \to Z$, it is clear that f and g is IFe-cont., but $g \circ f$ is not IFe-cont..

5. Intuitionistic Fuzzy e-compact Spaces

Definition 5.1. Let X be an IFTS. A family $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \, | i \in I\}$ of IFeOS's in X satisfying the condition

$$\bigcup \{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \, | i \in I\} = 1_\sim,$$

is called an intuitionistic fuzzy e-open cover of X.

A finite subfamily of an intuitionistic fuzzy e-open cover

$$\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \, | i \in I\},$$

which is also an intuitionistic fuzzy e-open cover of X is called a finite subcover of $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \, | i \in I\}$.

Definition 5.2. Let X be an IFTS. A family $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \, | i \in I\}$ of IFeCS's in X has the finite intersection property if every finite sub-family $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \, | i = 1, 2, \ldots, n\}$ satisfies the condition

$$\bigcap_{k=1}^{n} \{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \, | i \in I\} \neq 0_\sim.$$

Definition 5.3. An IFTS X is called intuitionistic fuzzy e-compact if and only if every intuitionistic fuzzy e-open cover has a finite subcover.

Example 5.4. Consider the IFTS(X,τ), where $X=\{a,b\}$,

$$A_n=\left\langle x,\left(\frac{a}{\frac{n}{n+1}},\frac{b}{\frac{n+1}{n+2}}\right),\left(\frac{a}{\frac{1}{n+2}},\frac{b}{\frac{1}{n+3}}\right)\right\rangle$$

and $\tau=\{0_\sim,1_\sim\}\cup\{A_n:n\in N\}$. Note that $\bigcup_{n\in N}A_n$ is an open cover for X, but this cover has no finite subcover. Consider

$$A_1=\left\langle x,\left(\frac{a}{0.5},\frac{b}{0.6}\right),\left(\frac{a}{0.3},\frac{b}{0.25}\right)\right\rangle,$$
$$A_2=\left\langle x,\left(\frac{a}{0.6},\frac{b}{0.75}\right),\left(\frac{a}{0.25},\frac{b}{0.2}\right)\right\rangle,$$
$$A_3=\left\langle x,\left(\frac{a}{0.75},\frac{b}{0.8}\right),\left(\frac{a}{0.2},\frac{b}{0.16}\right)\right\rangle,$$

and observe that $A_1\cup A_2\cup A_3=A_3$. So, for any finite subcollection $\{A_{n_i}:i\in I$, where I is a finite subset of $N\}$, $\bigcup_{n_i\in I}A_{n_i}=A_m\neq 1_\sim$, where $m=max\{n_i:n_i\in I\}$. Therefore IFTS(X,τ) is not compact.

Theorem 5.5. *An IFTS X is intuitionistic fuzzy e-compact if and only if every family $\{\langle x,\mu_{G_i},\nu_{G_i}\rangle\,|i\in I\}$ of IFeCS's with the finite intersection property has a nonempty intersection.*

Proof. Suppose X is intuitionistic fuzzy e-compact and $\{\langle x,\mu_{G_i},\nu_{G_i}\rangle\,|i\in I\}$ is any family of IFeCS's in X such that

$$\bigcap\{\langle x,\mu_{G_i},\nu_{G_i}\rangle\,|i\in I\}=0_\sim.$$

Therefore $\bigwedge\{\langle\mu_{G_i}\rangle\,|i\in I\}=0_\sim$ and $\bigcup\{\langle\nu_{G_i}\rangle\,|i\in I\}=1_\sim$. Then

$$\bigvee\{\langle x,\mu_{G_i},\nu_{G_i}\rangle\,|i\in I\}=1_\sim,$$

so $\{\langle x,\mu_{G_i},\nu_{G_i}\rangle\,|i\in I\}$ is a intuitionistic fuzzy e-open cover of X. Since X is intuitionistic fuzzy e-compact there is a finite subcover

$$\{\langle x,\mu_{G_i},\nu_{G_i}\rangle\,|i=1,2,\ldots,n\}.$$

Then

$$\bigcup_{k=1}^{n}\{\langle x,\mu_{G_i},\nu_{G_i}\rangle\,|i\in I\}=1_\sim.$$

Hence

$$\bigvee\left\{\nu_{G_{i(x)}}|i=1,2,\ldots,n\right\}=1_\sim,$$

and

$$\bigwedge\left\{\mu_{G_i}(x)|i=1,2,\ldots,n\right\}=0_\sim.$$

Finally

$$\bigcap_{k=1}^{n}\{\langle x,\mu_{G_i},\nu_{G_i}\rangle\,|i\in I\}=0_\sim.$$

We have proved that if X is intuitionistic fuzzy e-compact space, then given any family $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,|i \in I\}$ of IFeCS's whose intersection is empty, the intersection of some finite subfamily is empty. Conversely, let X has the finite intersection property. It means that if the intersection of any family of IFeCS's is empty, the intersection of each finite subfamily is empty. Suppose $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,|i \in I\}$ is any intuitionistic fuzzy e-open cover of X. Then

$$\bigcup \{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,|i \in I\} = 1\!\!\!\sim.$$

Therefore,

$$\bigvee \{\mu_{G_i}(x)|i \in I\} = 1\!\!\!\sim, \qquad \bigwedge \{\nu_{G_i}(x)|i \in I\} = 0\!\!\!\sim.$$

Hence

$$\bigcap\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,|i \in I\} = 0\!\!\!\sim,$$

so $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,|i \in I\}$ is a family of IFeCS's whose intersection is empty. According to the assumption, we can find finite subfamily

$$\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,|i = 1, 2, \ldots, n\},$$

such that

$$\bigcap_{k=1}^{n} \{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,|i \in I\} = 0\!\!\!\sim.$$

Then

$$\bigcup_{k=1}^{n} \{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,|i \in I\} = 1\!\!\!\sim,$$

so $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,|i = 1, 2, \ldots, n\}$ is a finite subcover of

$$\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,|i \in I\}.$$

Therefore, X is intuitionistic fuzzy e-compact. $\square$

Remark 5.6. Since every IFOS is an IFeOS, from the definition above we may conclude that every intuitionistic fuzzy e-compact IFTS is intuitionistic fuzzy compact.

Theorem 5.7. *Let $f : X \to Y$ be an intuitionistic fuzzy e-irresolute mapping from an IFTS X onto IFTS Y. If X is intuitionistic fuzzy e-compact, then Y is intuitionistic fuzzy e-compact, as well.*

Proof. Let $\{\langle y, \mu_{G_i}, \nu_{G_i}\rangle \,|i \in I\}$ be any intuitionistic fuzzy e-open cover of Y. Then

$$\bigcup\{\langle y, \mu_{G_i}, \nu_{G_i}\rangle|i \in I\} = 1\!\!\!\sim.$$

From the relation

$$f^{-1}\left(\bigcup\{\langle y, \mu_{G_i}, \nu_{G_i}\rangle|i \in I\}\right) = 1\!\!\!\sim,$$

follows that

$$\bigcup\{\langle y, \mu_{G_i}, \nu_{G_i}\rangle \,|i \in I\} = 1,$$

so $\{f^{-1}(\{\langle y, \mu_{G_i},\, \nu_{G_i}\rangle | i \in I\}$ is a intuitionistic fuzzy e-open cover of X. Since X is intuitionistic fuzzy e-compact, there exists a finite subcover

$$\{f^{-1}(\langle x, \mu_{G_i}, \nu_{G_i}\rangle)|i = 1, 2, \ldots, n\}.$$

Therefore

$$\bigcup\{f^{-1}(\langle y, \mu_{G_i}, \nu_{G_i}\rangle)|i = 1, 2, \ldots, n\} = 1.$$

Hence

$$f\left(\bigcup\{f^{-1}(\langle y, \mu_{G_i}, \nu_{G_i}\rangle)|i = 1, 2, \ldots, n\}\right) = 1,$$

so

$$\bigcup\{f(f^{-1}(\langle y, \mu_{G_i}, \nu_{G_i}\rangle))|i = 1, 2, \ldots, n\} = 1.$$

From

$$\bigcup\{(\langle y, \mu_{G_i}, \nu_{G_i}\rangle)|i = 1, 2, \ldots, n\} = 1,$$

follows that Y is intuitionistic fuzzy e-compact. □

Theorem 5.8. *Let $f : X \to Y$ be an intuitionistic fuzzy e-continuous mapping from an IFTS X onto IFTS Y. If X is intuitionistic fuzzy e-compact, then Y is fuzzy compact.*

Proof. It is similar to the proof of the Theorem 5.7. □

Definition 5.9. Let A be an IFS in an IFTS X. A family

$$\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,|i \in I\},$$

of IFeOS's in X satisfying the condition $A \subseteq \bigcup\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,|i \in I\}$ is called intuitionistic fuzzy e-open cover of A.

A finite subfamily of a intuitionistic fuzzy e-open cover

$$\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,|i \in I\}$$

of A which is also a intuitionistic fuzzy e-open cover of A is called a finite subcover of $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,|i \in I\}$.

Definition 5.10. An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS X is called intuitionistic fuzzy e-compact if and only if every intuitionistic fuzzy e-open cover of A has a finite subcover.

Theorem 5.11. *An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS X is intuitionistic fuzzy e-compact if and only if for each family $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,|i \in I\}$ of IFeOS's with properties*

$$\mu_A \leq \vee\{\mu_{G_i}|i \in I\}, \qquad 1 - \nu_A \leq \vee\{1 - \nu_{G_i}|i \in I\},$$

there exists a finite subfamily $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,|i = 1, 2, \ldots, n\}$ such that

$$\mu_A = \vee\{\mu_{G_i}|i = 1, 2, \ldots, n\}, \qquad 1 - \nu_A = \vee\{1 - \nu_{G_i}|i = 1, 2, \ldots, n\}.$$

Proof. Suppose $A = \langle x, \mu_A, \nu_A \rangle$ is a intuitionistic fuzzy e-compact set in IFTS X and $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle \, | i \in I\}$ be any family of IFeCS's in X satisfies the condition

$$\mu_A \leq \vee\{\mu_{G_i} | i \in I\}, \qquad 1 - \nu_A \leq \vee\{1 - \nu_{G_i} | i \in I\}.$$

Then $1 - \nu_A \leq 1 - \wedge\{\nu_{G_i} | i \in I\}$, so $\nu_A \geq \wedge\{\nu_{G_i} | i \in I\}$. Hence

$$A \subseteq \bigcup\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle \, | i \in I\}.$$

According to the assumption there exists finite subfamily

$$\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle \, | i = 1, 2, \ldots, n\},$$

such that

$$A \subseteq \bigcup\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle \, | i = 1, 2, \ldots, n\}.$$

It follows that

$$\mu_A = \vee\{\mu_{G_i} | i = 1, 2, \ldots, n\}, \qquad 1 - \nu_A = \vee\{1 - \nu_{G_i} | i = 1, 2, \ldots, n\}.$$

Conversely, let $A = \langle x, \mu_{G_i}, \nu_{G_i} \rangle$ be any IFS in IFTS X and let $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle \, | i \in I\}$ be any family of IFeCS's in X satisfies the condition

$$\mu_A \leq \vee\{\mu_{G_i} | i \in I\}, \qquad 1 - \nu_A \leq \vee\{1 - \nu_{G_i} | i \in I\}.$$

From $1 - \nu_A \leq 1 - \wedge\{\nu_{G_i} | i \in I\}$ follows that $\mu_A \geq \wedge\{\nu_{G_i} | i \in I\}$, so

$$A \subseteq \bigcup\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle \, | i \in I\}.$$

Hence $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle \, | i \in I\}$ is a intuitionistic fuzzy e-open cover of IFS A. According to the assumption there exists finite subfamily

$$\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle \, | i = 1, 2, \ldots, n\},$$

such that

$$\mu_A = \vee\{\mu_{G_i} | i = 1, 2, \ldots, n\}, \qquad 1 - \nu_A \leq \vee\{1 - \nu_{G_i} | i = 1, 2, \ldots, n\}.$$

From

$$\mu_A \leq \vee\{\mu_{G_i} | i = 1, 2, \ldots, n\}, \qquad \nu_A \geq \wedge\{\mu_{G_i} | i = 1, 2, \ldots, n\},$$

we obtain that

$$A \subseteq \bigcup\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle \, | i = 1, 2, \ldots, n\}.$$

Therefore, A is intuitionistic fuzzy e-compact. □

Remark 5.12. From the definition above it is not difficult to conclude that every intuitionistic fuzzy e-compact IFS in an IFTS is fuzzy compact.

Theorem 5.13. *Let $f : X \to Y$ be an intuitionistic fuzzy e-irresolute mapping from an IFTS X onto IFTS Y. If A is intuitionistic fuzzy e-compact, then $f(A)$ is intuitionistic fuzzy e-compact.*

Proof. Let $\{\langle y, \mu_{G_i}, \nu_{G_i}\rangle \,|i \in I\}$ be any intuitionistic fuzzy e-open cover of $f(A)$. Then

$$f(A) \subseteq \bigcup\{\langle y, \mu_{G_i}, \nu_{G_i}\rangle \,|i \in I\}.$$

From the relation

$$A \subseteq f^{-1}\left(\bigcup\{\langle y, \mu_{G_i}, \nu_{G_i}\rangle \,|i \in I\}\right),$$

follows that

$$A \subseteq \bigcup\{f^{-1}(\langle y, \mu_{G_i}, \nu_{G_i}\rangle)|i \in I\},$$

so $\{f^{-1}(\langle y, \mu_{G_i}, \nu_{G_i}\rangle)|i \in I\}$ is an intuitionistic fuzzy e-open cover of A. Since A is intuitionistic fuzzy e-compact, there exists a finite subcover $\left\{f^{-1}(\langle y, \mu_{G_i}, \nu_{G_i}\rangle)|i = 1, 2, \ldots, n\right\}$. Therefore

$$A \subseteq \bigcup\{f^{-1}(\langle y, \mu_{G_i}, \nu_{G_i}\rangle)|i = 1, 2, \ldots, n\}.$$

Hence

$$\begin{aligned} f(A) &\subseteq f\left(\bigcup\{f^{-1}(\langle y, \mu_{G_i}, \nu_{G_i}\rangle)|i = 1, 2, \ldots, n\}\right) \\ &= \bigcup\left\{f(f^{-1}\left(\langle y, \mu_{G_i}, \nu_{G_i}\rangle\right))\,|i = 1, 2, \ldots, n\right\} \\ &= \bigcup\{(\langle y, \mu_{G_i}, \nu_{G_i}\rangle)|i = 1, 2, \ldots, n\} \end{aligned}$$

so $f(A)$ is intuitionistic fuzzy e-compact. □

Theorem 5.14. *Let $f : X \to Y$ be an intuitionistic fuzzy e-continuous mapping from an IFTS X onto IFTS Y. If A is intuitionistic fuzzy e-compact, then $f(A)$ is fuzzy compact.*

Definition 5.15. An IFTS X is called intuitionistic fuzzy e-Lindelöf (fuzzy Lindelöf) if and only if every intuitionistic fuzzy e-open (fuzzy open) cover of X has a countable subcover.

Definition 5.16. An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS X is called intuitionistic fuzzy e-Lindelöf (fuzzy Lindelöf) if and only if every intuitionistic fuzzy e-open (fuzzy open) cover of X has a countable subcover.

Definition 5.17. An IFTS X is called countable intuitionistic fuzzy e-compact (countably fuzzy compact) if and only if every countable intuitionistic fuzzy e-open (fuzzy open) cover of X has a finite subcover.

Definition 5.18. An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS X is called countable intuitionistic fuzzy e-compact (countably fuzzy compact) if and only if every countable intuitionistic fuzzy e-open (fuzzy open) cover of A has a finite subcover.

Remark 5.19. From the definitions above we may conclude that

(i) Every intuitionistic fuzzy e-Lindelöf of IFTS (IFS in IFTS) is fuzzy Lindelöf;
(ii) Every countably intuitionistic fuzzy e-compact of IFTS (IFS in IFTS) is countably fuzzy compact;
(iii) Every countably intuitionistic fuzzy e-compact of IFTS (IFS in IFTS) is intuitionistic fuzzy e-compact.

Theorem 5.20. *If an IFTS X is both intuitionistic fuzzy e-Lindelöf and countably intuitionistic fuzzy e-compact, then it is intuitionistic fuzzy e-compact.*

Theorem 5.21. *If an IFS A in an IFTS X is both intuitionistic fuzzy e-Lindelöf and fuzzy countably intuitionistic fuzzy e-compact, then A is intuitionistic fuzzy e-compact.*

Theorem 5.22. *Let X be an intuitionistic fuzzy e-Lindelöf IFTS. Then X is countably intuitionistic fuzzy e-compact if and only if X is intuitionistic fuzzy e-compact.*

Proof. In the Remark 5.19 it is mentioned that if X is intuitionistic fuzzy e-compact, then it is countably intuitionistic fuzzy e-compact. Conversely, let $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,| i \in I\}$ be any intuitionistic fuzzy e-open cover of X. Since X is intuitionistic fuzzy e-Lindelöf, there exists a countable subcover $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,| i = 1, 2, \ldots\}$ of $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,| i \in I\}$. Therefore $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,| i = 1, 2, \ldots\}$ is countably intuitionistic fuzzy e-open cover of X, so there exists subcover $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,| i = 1, 2, \ldots, n\}$ of $\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle \,| i = 1, 2, \ldots\}$. Hence X is intuitionistic fuzzy e-compact. □

Theorem 5.23. *Let an IFeOS A be intuitionistic fuzzy e-Lindelöf in an IFTS. Then A is countably intuitionistic fuzzy e-compact if and only if A is intuitionistic fuzzy e-compact.*

Proof. The proof is similar to the proof of the previous theorem. □

Theorem 5.24. *Let $f : X \to Y$ be an intuitionistic fuzzy e-irresolute mapping from an IFTS X onto IFTS Y. If X is intuitionistic fuzzy e-Lindelöf (countably intuitionistic fuzzy e-compact), then Y is intuitionistic fuzzy e-Lindelöf (countably intuitionistic fuzzy e-compact), as well.*

Proof. It is similar to the proof of the Theorem 5.7. □

Theorem 5.25. *Let $f : X \to Y$ be an intuitionistic fuzzy e-continuous mapping from an IFTS X onto IFTS Y. If X is intuitionistic fuzzy e-Lindelöf (countably intuitionistic fuzzy e-compact), then Y is fuzzy Lindelöf (countably fuzzy compact).*

Proof. It is similar to the proof of the Theorem 5.8. □

Theorem 5.26. *Let $f : X \to Y$ be an intuitionistic fuzzy e-irresolute mapping from an IFTS X onto IFTS Y. If A is intuitionistic fuzzy e-Lindelöf (countably intuitionistic fuzzy e-Lindelöf (countably intuitionistic fuzzy e-compact)), then $f(A)$ is intuitionistic fuzzy e-Lindelöf (countably intuitionistic fuzzy e-compact), as well.*

Proof. It is similar to the proof of the Theorem 5.13. □

Theorem 5.27. *Let $f : X \to Y$ be an intuitionistic fuzzy e-continuous mapping from an IFTS X onto IFTS Y. If A is intuitionistic fuzzy e-Lindelöf (countably intuitionistic fuzzy e-compact), then $f(A)$ is fuzzy Lindelöf (countably fuzzy compact).*

Proof. It is similar to the proof of the Theorem 5.14. □

6. Conclusion

The initiations of e - open sets, e - continuity, e - compactness and related studies in topological spaces are due to [6–10]. This present paper contains the next steps of fuzzy e-open sets, fuzzy e^*-open sets, fuzzy a-open sets, fuzzy e-continuity and fuzzy e-compactness in intuitionistic fuzzy topological spaces are studied. After giving the fundamental concepts of intuitionistic fuzzy sets and intuitionistic fuzzy topological spaces, we present intuitionistic fuzzy e-open sets and intuitionistic fuzzy e-continuity and other results related topological concepts. Several preservation properties and some characterizations concerning intuitionistic fuzzy e-compactness have been obtained.

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[1] Department of Mathematics, Kandaswami Kandar's College, P-velur-638 182, Tamil Nadu, India.

E-mail address: vckkc3895@gmail.com

[2] Department of Mathematics, Kandaswami Kandar's College, P-velur-638 182, Tamil Nadu, India.

E-mail address: slmsobana@gmail.com

[3] Department of Mathematics, Annamalai University, Annamalainagar, Tamil Nadu-608 002.

E-mail address: avmaths@gmail.com