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On Periodic Shadowing Property

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Abstract. In this paper, some properties of the periodic shadowing are presented. It is shown that a homeomorphism has the periodic shadowing property if and only if so does every lift of it to the universal covering space. Also, it is proved that continuous mappings on a compact metric space with the periodic shadowing and the average shadowing property also have the shadowing property and then are chaotic in the sense of Li-Yorke. Moreover, any distal homeomorphisms on a compact metric space with the periodic shadowing property do not have the asymptotic average shadowing property.

1. Introduction

A pair (X, f), where X is a metric space and $f : X \to X$ is a continuous map, is called a dynamical system. The theory of shadowing was developed in recent years and became a significant part of the qualitative theory of dynamical systems [13, 14]. This property studies the closeness of approximate and exact orbits of dynamical systems. In other words, the numerical computations reflect the real dynamical behavior of f.

For $\delta \geq 0$, a sequence $\xi = \{x_i : i \in \mathbb{Z}\} \subset X$ with the property

$$dist(f(x_i), x_{i+1}) \leq \delta, \quad \forall i \in \mathbb{Z}, \tag{1.1}$$

is called a δ-pseudo-orbit. Often, pseudo-orbits are obtained as results of numerical studies of dynamical systems. A sequence $\xi = \{x_i : i \in \mathbb{Z}\}$ is said to be ϵ-shadowed by a point $p \in X$ if $dist(f^i(p), x_i) < \epsilon, \forall i \in \mathbb{Z}$. The dynamical system f has the shadowing property or pseudo-orbit tracing property (POTP, for short), if for each $\epsilon > 0$, there exists $\delta > 0$ such that every δ-pseudo-orbit $\xi = \{x_i : i \in \mathbb{Z}\}$ can be ϵ-shadowed by

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some point in X. In the literature, usually the term "shadowing" is used instead of "POTP". This property is fully studied in [13, 14].

Another kind of shadowing property is periodic shadowing property that was introduced by Koscielniak in [10]. A dynamical system f has the periodic shadowing property (PerSh, for short) if for each $\epsilon > 0$ there is a $\delta > 0$ such that if $\xi = \{x_i : i \in \mathbb{Z}\}$ is a periodic δ-pseudo-orbit then there is a periodic point p such that

$$dist(f^i(p), x_i) < \epsilon, \quad \forall i \in \mathbb{Z}. \tag{1.2}$$

This property is well studied in [10] and [12].

Let $x, y \in X$ and $\epsilon > 0$ be given, we define an ϵ-chain from x to y by a finite ϵ-pseudo-orbit $\{x_i\}_{i=0}^n$ such that $x_0 = x$, $x_n = y$. Related to this definition, n is called the length of the chain. A map f is called chain transitive if there is a chain between every two points of X, i.e., for every points $x, y \in X$, there exists a chain from x to y. We say that f is chain mixing if for every two points $x, y \in X$ and $\epsilon > 0$, there is an $N > 0$ such that for every $n \geq N$ there exists a chain from x to y of length n. We say that f is transitive if for each pair of nonempty open sets U, V of X there is $n \in \mathbb{N}$ such that $f^{-n}(U) \cap V \neq \emptyset$. We say that f is weakly mixing if $f \times f : X \times X \to X \times X$ is transitive.

A map f is sensitive dependence on initial conditions if there is $\epsilon > 0$ such that for any $x \in X$ and every $\delta > 0$ there are $y \in X$ with $dist(x, y) < \delta$ and $n \in \mathbb{N}$ such that $dist(f^n(x), f^n(y)) > \epsilon$. A map f is chaotic in the sense of Devaney if (i) f is transitive, (ii) the set of periodic points of f is dense in X, and (iii) f is sensitive dependence on initial conditions. By [1], (i) and (ii) imply (iii):

A subset $A \subset X$ is a scrambled set if for any two points $x, y \in A$ we have

$$\liminf_{n\to\infty} dist(f^n(x), f^n(y)) = 0, \qquad \limsup_{n\to\infty} dist(f^n(x), f^n(y)) > 0.$$

A map f is chaotic in the sense of Li-York if there exists an uncountable scrambled set. Note that in [7] it has proved that the chaos in the sense of Devaney is stronger than that of Li-York.

The orbit of $x \in X$ under f is the set $O_f(x) = \{f^n(x) : n \in \mathbb{Z}\}$. A pair $(x, y) \in X \times X$ is proximal if

$$\liminf_{n\to\infty} dist(f^n(x), f^n(y)) = 0,$$

and distal if it is not proximal, that is

$$\liminf_{n\to\infty} dist(f^n(x), f^n(y)) > 0.$$

A point x is distal if (x, y) is distal for every $y \in \overline{O_f(x)}$ with $y \neq x$. If every point in X is distal, then we say that (X, f) is distal.

Blank [2] introduced the notion of the average-shadowing property in studying chaotic dynamical systems. For $\delta > 0$, a sequence $\{x_i\}_{i=0}^{\infty}$ of points in X is called a δ-average-pseudo-orbit of f if there is a positive integer $N = N(\delta) > 0$ such that for every $n \geq N$ and every nonnegative integer k,

$$\limsup_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} d(f(x_{i+k}), x_{i+k+1}) < \delta.$$

A map f is said to have the average-shadowing property if for any $\epsilon > 0$ there is a $\delta > 0$ such that every δ-average-pseudo-orbit $\{x_i\}$ is ϵ-shadowed on average by some point $z \in X$, that is,

$$\limsup_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} d(f^i(z), x_i) < \epsilon.$$

The asymptotic average shadowing property was introduced by Gu [5]. A sequence $\{x_i\}_{i=0}^{\infty}$ of points in X with

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} d(f(x_i), x_{i+1}) = 0,$$

is called asymptotic-average pseudo-orbit. A sequence $\{x_i\}_{i=0}^{\infty}$ is said to be asymptotically shadowed in average by a point $z \in X$ if

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} d(f^i(z), x_i) = 0.$$

A dynamical system (X, f) is said to have the asymptotic average shadowing property, if every asymptotic-average pseudo-orbit of f can be asymptotically shadowed in average by some point in X.

If $\mathcal{H}(M)$ is the space of homeomorphisms defined on a manifold M then the following metric induces the C^0-topology. It should be noted that this metric is complete

$$d(f,g) = \max\left\{\max_{x\in M} dist\left(f(x), g(x)\right), \max_{x\in M} dist\left(f^{-1}(x), g^{-1}(x)\right)\right\}.$$

Koscielniak [10] has proved that the periodic shadowing property is generic in $\mathcal{H}(M)$ whenever the manifold M is smooth compact. That is, there is a residual subset $\mathcal{R}$ in $\mathcal{H}(M)$ where every element of it has the periodic shadowing property. One of the approaches in the study of shadowing's problems is the passage to C^1-interior in C^1-topology. Osipov et. al, in [12, Theorem 1] used this approach and proved $Int^1(\text{PerSh}) = \Omega S$, where ΩS denotes the Ω-stable diffeomorphisms.

There are systems without the periodic shadowing property (see [4]). The authors in [4] have investigated the periodic shadowing property and its relation to the standard shadowing property. They proved that chain transitivity and topological transitivity are equivalent in the presence of the periodic shadowing property. Also, in chain transitive systems the periodic shadowing implies the shadowing property. In this study, some other results of the periodic shadowing property are obtained. More precisely, in the next section, we prove that the product of two systems also has the periodic shadowing property. The converse case holds for expansive systems. In Sec. 3, we prove that a homeomorphism has the periodic shadowing property if and only if every lift of it to the covering space is so. Also, we extend a result in [4, Proposition 2.1], and obtain two results about distal homeomorphism with the periodic shadowing property.

2. Periodic Shadowing

The periodic shadowing property means that near a periodic pseudo-orbit (numerically computed periodic orbit) there exists an exact periodic orbit. That is, the numerically found periodic orbits approximate the real periodic orbits of f in the past and future very well.

Note that it can be shown that if f has the periodic shadowing property on $\mathbb{Z}^+$, then it also has the periodic shadowing property on $\mathbb{Z}$. Additionally, it is easy to see that this property is independent of choosing a compatible metric for X. In fact, let d_1 and d_2 induce the same topology on X and f has the periodic shadowing property with d_1. Suppose $\epsilon > 0$ is given, there exists $\epsilon_1 > 0$ so that $B_{d_1}(x, \epsilon_1) \subset B_{d_2}(x, \epsilon)$. Take δ_1 by the periodic shadowing property. Choose δ_2 such that $B_{d_2}(x, \delta_2) \subset B_{d_1}(x, \delta_1)$. Now if $\{x_i : i \in \mathbb{Z}\}$ is a periodic δ_2-pseudo-orbit with metric d_2 then it is also δ_1-pseudo-orbit with metric d_1. So, there is a periodic point $p \in X$ with relation $d_1(f^i(p), x_i) < \epsilon_1, \forall i \in \mathbb{Z}$. Therefore, $d_2(f^i(p), x_i) < \epsilon, \forall i \in \mathbb{Z}$.

Let X and Y be metric spaces and $X \times Y$ be the product topological space with the metric

$$d\left((x_1, y_1), (x_2, y_2)\right) = \max\left\{d_X(x_1, x_2), d_Y(y_1, y_2)\right\},$$

where d_X and d_Y are the metrics for X and Y, respectively. Let $f : X \to X$ and $g : Y \to Y$ be homeomorphisms and let $f \times g$ be the map defined on $X \times Y$ by

$$(f \times g)(x, y) = (f(x), g(y)).$$

In the following, we prove that $f \times g$ has the periodic shadowing property whenever f and g have it. Note that the converse implication holds in case of expansivity of both f and g.

Proposition 2.1. *Let f and g be homeomorphisms on metric spaces X and Y. If f and g have the periodic shadowing property, then $f \times g$ also has the periodic shadowing property.*

Proof. Suppose that $\epsilon > 0$ is given. Then there exists $\delta > 0$ by the periodic shadowing property such that (1.2) holds for both f and g. Let the sequence $\{(x_i, y_i) : i \in \mathbb{Z}\}$ be a periodic δ-pseudo-orbit for $f \times g$, then

$$\begin{aligned} d\left((f \times g)(x_i, y_i), (x_{i+1}, y_{i+1})\right) &= \max\left\{d_X\left(f(x_i), x_{i+1}\right), d_Y\left(g(y_i), y_{i+1}\right)\right\} \\ &< \delta. \end{aligned}$$

So, the sequences $\{x_i : i \in \mathbb{Z}\}$ and $\{y_i : i \in \mathbb{Z}\}$ are periodic δ-pseudo-orbit for f and g, respectively. Therefore, there exists periodic points p and q so that

$$d_X(f^i(p), x_i) < \epsilon, \qquad d_Y(g^i(q), y_i) < \epsilon, \quad \forall i \in \mathbb{Z}.$$

Hence,

$$d\left((f \times g)^i(p, q), (x_i, y_i)\right) < \epsilon, \forall i \in \mathbb{Z}.$$

Thus $f \times g$ has the periodic shadowing property. □

Proposition 2.2. *Let f and g be expansive homeomorphisms on compact metric spaces X and Y, respectively. If $f \times g$ has the periodic shadowing property on $X \times Y$, then f and g have the periodic shadowing property on X and Y, respectively.*

Proof. Assume that $\delta_1 > 0$ and $\delta_2 > 0$ are expansive constants for f and g, respectively. Put $\delta = \min\{\delta_1, \delta_2\}$. Without loss of generality, it is enough to take $0 < \epsilon \leq \frac{\delta}{2}$. Suppose that $0 < \epsilon \leq \frac{\delta}{2}$ is given, there is $\eta > 0$ (if necessary, choose $\eta < \delta$) by the periodic shadowing property for the mapping $f \times g$. Now if $\{x_i\}_{i \in \mathbb{Z}}$ and $\{y_i\}_{i \in \mathbb{Z}}$ are η-pseudo-orbits, μ and ν periodic respectively for f and g, then $\{(x_i, y_i)\}_{i \in \mathbb{Z}}$ is also η-pseudo-orbit with respect to the metric for product space. Note that the sequence $\{(x_i, y_i)\}_{i \in \mathbb{Z}}$ is periodic with the period equal to the least common multiple of μ and ν. So, we can find a point (p, q) such that

$$d\left((f \times g)^i(p, q), (x_i, y_i)\right) < \epsilon, \quad \forall i \in \mathbb{Z}.$$

Therefore,

$$d_X\left(f^i(p), x_i\right) < \epsilon, \qquad d_Y\left(g^i(q), y_i\right) < \epsilon, \quad \forall i \in \mathbb{Z}. \tag{2.1}$$

To complete the proof, it is enough to show that the points p and q are periodic. We show the first inequality of (2.1), the other one is similar. Since $\{x_i\}$ is periodic there exists μ so that $x_{i+\mu} = x_i, i \in \mathbb{Z}$, and inequality (2.1) implies

$$d_X\left(f^{i+\mu}(p), x_i\right) < \epsilon, \quad i \in \mathbb{Z}.$$

Putting $f^{\mu}(p) = z$, it follows

$$d_X\left(f^i(z), x_i\right) < \epsilon, \quad i \in \mathbb{Z}. \tag{2.2}$$

Inequalities (2.1) and (2.2) imply

$$d_X\left(f^i(p), f^i(z)\right) < 2\epsilon \leq \delta, \quad i \in \mathbb{Z}.$$

Now, the expansivity of f yields $p = z = f^{\mu}(p)$. □

Note that in [10] it is stated that the periodic shadowing property is stronger than the shadowing property. However, this might not be correct. Because there is an Ω-stable diffeomorphism on the torus without the shadowing property [15], but with the periodic shadowing by the above result. Furthermore, the authors in [4] proved that in a chain transitive dynamical system the periodic shadowing property implies the shadowing.

3. Consequences of the Periodic Shadowing

In this section, we prove a list of results about the periodic shadowing property, which are not necessarily connected to each other. First, we prove that if a homeomorphism on a compact metric space (X, d) has the periodic shadowing property then so does f^n for $n \in \mathbb{Z} \setminus \{0\}$. The authors in [4] proved the following result.

Proposition 3.1 ([4], Proposition 2.1)**.** *If f has the periodic shadowing property, then so does f^n for all $n > 0$.*

Lemma 3.2. *Let $f : X \to X$ be a homeomorphism, then f has the periodic shadowing property if and only if f^{-1} has so.*

Proof. It is enough to prove the necessary condition. Suppose that f has the periodic shadowing property. Assume that $\epsilon > 0$ is given and let $\delta > 0$ be the number of the periodic shadowing property of f. There exists $\delta_1 > 0$ by the uniform continuity of f such that

$$d(x, y) < \delta_1 \quad \Rightarrow \quad d(f(x), f(y)) < \delta.$$

Let $\{y_i : i \in \mathbb{Z}\}$ be a periodic δ_1-pseudo-orbit for f^{-1}. So, we have

$$d(y_i, f(y_{i+1})) < \delta.$$

For each $i \in \mathbb{Z}$ put $x_i = y_{-i}$. Obviously, $\{x_i\}$ is periodic and

$$d(f(x_i), x_{i+1}) < \delta.$$

So, there exists a point y by periodic shadowing property for f such that

$$d\left(f^i(y), x_i\right) < \epsilon, \quad i \in \mathbb{Z}.$$

Hence,

$$d\left(f^{-i}(y), x_{-i}\right) < \epsilon,$$

and this means that

$$d\left((f^{-1})^i(y), y_i\right) < \epsilon, \quad i \in \mathbb{Z}.$$

This completes the proof. □

Here, using the above lemma we extend Proposition 3.1 for $n \in \mathbb{Z}\backslash\{0\}$.

Proposition 3.3. *If f is a homeomorphism on a compact metric space which has the periodic shadowing property, then so does f^n for $n \in \mathbb{Z} \setminus \{0\}$.*

In the following, we show that the periodic shadowing property like other type of shadowings is invariant of topological conjugacy.

Proposition 3.4. *The periodic shadowing is invariant of topological conjugacy provided that the conjugacy and its inverse are Lipschitz.*

Proof. Suppose the bijective map h is a conjugacy between f and g, i.e. $h \circ f = g \circ h$, and let h and h^{-1} be Lipschitz. If f has the periodic shadowing property we show that so does g. Suppose $\epsilon > 0$ is given and let $\delta > 0$ be given such that every δ-pseudo-orbit can be ϵ-shadowed by a periodic point. Assume that K is a Lipschitz constant for h^{-1} and $\{x_i\}_{i\in\mathbb{Z}}$ be a $\frac{\delta}{K}$-pseudo-orbit for g. Then $\{h^{-1}(x_i)\}_{i\in\mathbb{Z}}$ is a δ-pseudo-orbit for f. Since

$$\begin{aligned} d\left(f(h^{-1}(x_i)), h^{-1}(x_{i+1})\right) &= d\left(h^{-1}(g(x_i)), h^{-1}(x_{i+1})\right) \\ &< Kd(g(x_i), x_{i+1}) < \delta, \end{aligned}$$

therefore, there exists a periodic point z so that

$$d(f^i(z), h^{-1}(x_i)) < \epsilon.$$

Hence,

$$\begin{aligned} d\left(h(f^i(z)), h(h^{-1}(x_i))\right) &< K'd(f^i(z), h^{-1}(x_i)) \\ &< K'\epsilon, \end{aligned}$$

where K' is the Lipschitz constant of h. So, we have

$$d\left(g^i(h(z)), x_i\right) < K'\epsilon.$$

Clearly, $h(z)$ is a periodic point for g. This completes the proof. □

In [6] and [9] it is proved that a homeomorphism has the shadowing property if and only if so does every lift of it. Here, we prove the same statement for the periodic shadowing property. To do this, we state some notions. If $\tilde{M}$ is the universal covering space of the manifold M, the map $\tilde{f}: \tilde{M} \to \tilde{M}$ is called a lift for f if the following diagram is commutative, where π is the covering map.

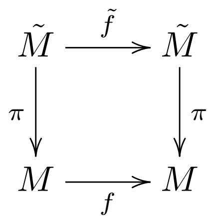

FIGURE 1. Lift's diagram

If $\langle ., . \rangle$ is a Riemannian metric in M which induces the distance d, then it can be lifted to the universal covering space, and again we denote it by $\langle ., . \rangle$. This can be done by setting for $x \in \pi^{-1}(x_0)$ and for $v, w \in T_x\tilde{M}$

$$\langle v, w \rangle_x = \langle D\pi(x)v, D\pi(x)w \rangle_{x_0},$$

where $\langle ., . \rangle_{x_0}$ is the inner product in $T_{x_0}M$. Also, we denote by d the induced distance by the lifted metric. By definition, π is locally isometric. So, there is $\epsilon_0 > 0$ such that for every $x \in \tilde{M}$ the map π projects isometrically ϵ_0-neighborhood of x to ϵ_0-neighborhood of $\pi(x)$. Now, we are going to prove the following proposition.

Proposition 3.5. *A homeomorphism has the periodic shadowing property if and only if any lift to the universal covering space has the periodic shadowing property.*

Proof. Suppose $\tilde{f}$ has the periodic shadowing property. To show that f has also the periodic shadowing property, without loss of generality, it is enough to prove PerSh for $\epsilon \leq \epsilon_0$. Suppose that $\epsilon \leq \epsilon_0$ is given and let $\delta > 0$ be given by the periodic shadowing property for $\tilde{f}$. By decreasing δ, if necessary, we may assume that $\delta \leq \epsilon_0$. Assume that $\{x_k\}_{k\in\mathbb{Z}} \subset M$ be a periodic δ-pseudo-orbit for f. So, there exists a unique sequence $\{y_k\}_{k\in\mathbb{Z}} \subset \tilde{M}$ such that $d(\tilde{f}(y_k), y_{k+1}) < \delta$ and $y_k \in \pi^{-1}(x_k), k \in \mathbb{Z}$ (note the property of ϵ_0). Therefore, we can find a periodic point $z \in \tilde{M}$ for $\tilde{f}$ such that $d(\tilde{f}^k(z), y_k) < \epsilon$. It follows that $d(\pi\tilde{f}^k(z), \pi(y_k)) < \epsilon$ and so $d(f^k(\pi(z)), x_k) < \epsilon$, note that $\pi(z)$ is also a periodic point.

Conversely, suppose that f has the periodic shadowing property and $\epsilon \leq \epsilon_0$ is given. Then there exists $\delta > 0$ by the periodic shadowing property for f. Let $\{x_k\}_{k\in\mathbb{Z}} \subset \tilde{M}$ be a periodic δ-pseudo-orbit for $\tilde{f}$. Then the sequence $\{\pi(x_k)\} \subset M$ is a periodic sequence. In addition, since

$$d(f(\pi(x_k)), \pi(x_{k+1})) = d(\tilde{f}(x_k), x_{k+1}) < \delta,$$

it is a δ-pseudo-orbit for f. We can take $\delta \leq \epsilon_0$, then there exists $z \in M$ periodic for f such that

$$d(f^k(z), \pi(x_k)) < \epsilon,$$

so

$$d(\pi^{-1} f^k(z), \pi^{-1}\pi(x_k)) = d(\tilde{f}^k(\pi^{-1}(z)), x_k) < \epsilon, k \in \mathbb{Z}.$$

Note that $\pi^{-1}(z)$ is also periodic for $\tilde{f}$. This completes the proof. □

In the following, we are going to deduce that in surjective systems on a compact space with the periodic shadowing property, the average shadowing implies chaos in the sense of Li-York. Note that in [11] it is proved that a surjective map on a compact metric space with the average shadowing property is chain mixing. Hence, by [4, Theorem 2.5] readily we get the next result.

Corollary 3.6. *Let $f : X \to X$ be a surjective map which has the periodic shadowing property. If it has the average shadowing property, then it is chain mixing and chaotic in the sense of Li-Yorke.*

Proof. By [4, Theorem 2.5] we get the shadowing property and by [11, Theorem 3.8] weakly mixing is deduced. By [8], if f is weakly mixing then it is chaotic in the sense of Li-Yorke. □

The next two results are dealt with distal homeomorphisms which have the periodic shadowing property. In this regard, we use the following lemma in [5, Lemma 5.2].

Lemma 3.7. *Suppose X is a compact metric space with at least two points and $f : X \to X$ is a distal homeomorphism. If for every $n > 0$, f^n is chain transitive, then f does not have the shadowing property.*

Corollary 3.8. *Suppose X and f are as in the above lemma. If f is chain mixing, then it does not have the periodic shadowing property.*

Proof. By contrary suppose that f has the periodic shadowing property. Then since f is chain mixing, [4, Proposition 3.5] implies that f^n is chain transitive for each $n > 0$, and by [4, Theorem 2.5] the shadowing property will be result. However, this contradicts Lemma 3.7. □

Corollary 3.9. *Suppose X and f are as in the above lemma. Then if f has the periodic shadowing property then it does not have the asymptotic average shadowing property.*

Proof. By contrary suppose that f has the asymptotic average shadowing property. Then by [5, Proposition 2.2] f^n, for every $n > 0$, has the asymptotic average shadowing property. So, by [5, Theorem 3.1], f is chain mixing. However, Corollary 3.8 implies that f does not have the periodic shadowing property. This is a contradiction. □

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