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$L_{p;r}$ spaces: Cauchy Singular Integral, Hardy Classes and Riemann-Hilbert Problem in this Framework

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Abstract. In the present work the space $L_{p;r}$ which is continuously embedded into L_p is introduced. The corresponding Hardy spaces of analytic functions are defined as well. Some properties of the functions from these spaces are studied. The analogs of some results in the classical theory of Hardy spaces are proved for the new spaces. It is shown that the Cauchy singular integral operator is bounded in $L_{p;r}$. The problem of basisness of the system $\left\{A(t)\,e^{\mathrm{int}}; B(t)\,e^{-\mathrm{int}}\right\}_{n\in\mathbb{Z}_+}$, is also considered. It is shown that under an additional condition this system forms a basis in $L_{p;r}$ if and only if the Riemann-Hilbert problem has a unique solution in corresponding Hardy class $H^+_{p;r}\times H^+_{p;r}$.

1. Introduction

During the last two decades, non-standard function spaces became an extremely popular subject because of their appearance in modern problems of analysis and qualitative theory of PDEs. Introduction of Lebesgue spaces with variable exponents at the end of last century and variety of extraordinary results obtained therein were the main motivation and the inception of this new tendency in analysis. For original results in the theory of Lebesgue spaces with variable exponent the reader may consult the books [9, 10] and references therein. Another kind of non-standard function spaces-small and grand Lebesgue spaces were defined motivated by C.B. Morrey's seminal work, in which Morrey

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space has been defined and proved to be extremely useful tool in qualitative theory of elliptic equations(for further discussions, see, [1, 14] and references therein). We only mention few recent monographs with a comprehensive bibliography, where in-depth treatment of these issues can be found: [1, 9, 14]. We mention the works [3–7, 12] because of their closeness to the spirit of the present paper.

In [13] Y. Katznelson considered the class of functions, whose Fourier coefficients are p-th power summable and proved a uniqueness theorem for the Fourier series of the functions of this class. In the present work equipping this class of Katznelson by a norm, the $L_{p;r}$ function space which is continuously embedded into L_p is introduced. The corresponding Hardy spaces of analytic functions are defined as well. Some properties of the functions from these spaces are studied. The analogs of some results in the classical theory of Hardy spaces are proved for the new spaces. It is shown that the Cauchy singular integral operator is bounded in $L_{p;r}$.

2. $L_{p;r}$ SPACES

Let $L_p \equiv L_p(-\pi,\pi)$, $1 \le p \le +\infty$ and l_r, $1 \le r \le +\infty$ be the usual spaces of p-th power summable functions and r-th power summable sequences of scalars, respectively; $\hat{f}$ denotes the sequence of Fourier coefficients of the function$f$:

$$\hat{f} \equiv \{f_n\}_{n\in\mathbb{Z}}, \qquad f_n \equiv \frac{1}{\sqrt{2\pi}}\int_{-\pi}^{\pi} f(t)\, e^{-\mathrm{int}}dt, \quad n \in \mathbb{Z}.$$

Denote the set $\left\{f \in L_p : \hat{f} \in l_r\right\}$ by $L_{p;r}$. It is evident that $L_{p;r}$ is a linear space with respect to pointwise operations and $\|f\|_{p;r} = \|f\|_p + \left\|\hat{f}\right\|_{l_r}$ defines a norm in $L_{p;r}$ here and thereafter $\|f\|_p = \|f\|_{L_p}$. We show that the space $L_{p;r}$ is a Banach space. Indeed, if $\left\{\hat{f}_m\right\}_{m\in\mathbb{N}} \subset L_{p;r}$ is any fundamental sequence, then the sequences $\{f_m\}_{m\in\mathbb{N}}$ and $\left\{\hat{f}_m\right\}_{m\in\mathbb{N}}$ are fundamental in the spaces L_p and l_r, respectively. Hence there exist $f \in L_p$ and $\hat{a} \in l_r$ such that $\left\{\hat{f}_m\right\}_{m\in\mathbb{N}} \to \{f_m\}_{m\in\mathbb{N}}$ *and* $\hat{f}_m \to \hat{a}$, as $m \to \infty$. It is easy to observe that $\hat{f} = \hat{a}$.

Take any $g \in L_q$ $\left(\frac{1}{p}+\frac{1}{q}=1\right)$ and consider the linear functional

$$l_g(f) = \frac{1}{\sqrt{2\pi}}\int_{-\pi}^{\pi} f(t)\,\overline{g(t)}dt$$

defined in $L_{p;r}$. As l_g is bounded, $L_q \subset (L_{p;r})^*$. We shall identify the function from L_q with the linear functional generated by itself. In that sense the system $E \equiv \left\{e^{\text{int}}\right\}_{n\in\mathbb{Z}}$ is biorthogonal to the system $\frac{1}{2\pi}E = \left\{\frac{1}{2\pi}e^{\text{int}}\right\}_{n\in\mathbb{Z}} \subset L_{p;r}$.

Consider the problem of basicity of E in $L_{p;r}$. First, let $p \in (1, +\infty)$. It is known that in that case E forms a basis in L_p. Take any $f \in L_{p;r}$. We can write

$$\|f - T_m f\|_{L_{p;r}} = \|f - T_m f\|_p + \left(\sum_{|n|\geq m+1}^{+\infty} |f_n|^r\right)^{\frac{1}{r}},$$

where $T_m f = \sum_{|n|\leq m} f_n e^{int}$. Since E is a basis in L_p, $\|f - T_m f\| \to 0$, as $m \to \infty$. Also, since $\hat{f} \in l_r$, the second term of above sum also has a zero limit. The uniqueness of expansion is obvious.

Now consider the case when $p = 1$ and $r \in [1\,,\,2]$. Take any $f \in L_{1;r}$. Then $\hat{f} \in l_2$, and therefore, it implies that $f \in L_2$. We can write

$$\begin{aligned}\|f - T_m f\|_{L_{p;r}} &= \left\|\hat{f} - \hat{T}_m f\right\|_{l_r} + \|f - T_m f\|_1 \\ &\leq \left(\sum_{|n|\geq m+1}^{+\infty} |f_n|^r\right)^{\frac{1}{r}} + c\cdot\|f - T_m f\|_2 \to 0\end{aligned}$$

as $m \to \infty$.

It proves the following

Proposition 2.1. *The system E forms a basis in $L_{p;r}$ for any $p \in (1, +\infty)$ and $r \in [1, +\infty]$; the same property holds in $L_{1;r}$ for any $r \in [1\,,\,2]$.*

3. Cauchy Singular Integral

Throughout the paper ω will denote the open unit disc $\omega = \{z \in C : |z| < 1\}$ and $\gamma = \partial\omega$ will denote the unit circle $\omega = \{z \in C : |z| = 1\}$. Let $f \in L_1(\gamma)$. Consider the Cauchy-type integral

$$F(z) = \frac{1}{2\pi i}\int_\gamma \frac{f(\tau)}{\tau - z}d\tau, \quad z \notin \gamma,$$

and the singular integral

$$(Sf)(\tau) = \frac{1}{2\pi i}\int_\gamma \frac{f(\xi)\,d\xi}{\xi - \tau}, \quad \tau \in \gamma$$

corresponding to it.

It is well known that Sf exists a.e. on γ (see, e.g. [8, 11]). In the sequel, we will use the following space of analytic functions generalized by $L_{p;\nu}$. Denote

$$H_{p;\nu}^{+}=\left\{f:f\text{ is analytic on }\omega\text{ and }\|f\|_{H_{p;\nu}^{+}}=\sup_{0<r<1}\|f_r(\cdot)\|_{L_{p;\nu}}<+\infty\right\},$$

where $f_r(t)=f\left(re^{it}\right)$.
Let $f\in H_{p;\nu}^{+}$, $1<p<+\infty$, $1\leq\nu\leq+\infty$, and

$$f\left(re^{it}\right)=\sum_{n=0}^{\infty}f_n r^n e^{int}.$$

Then we have

$$\|f\|_{H_{p;\nu}^{+}}=\sup_{0<r<1}\left(\left(\sum_{n=0}^{\infty}|f_n|^{\nu}r^{\nu n}\right)^{\frac{1}{\nu}}+\|f_r(\cdot)\|_{L_p}\right).$$

Denote by $f^{+}(\tau)$ the nontangential boundary values of $f(\tau)$ on γ. By a classical theorem $f^{+}(\tau)$ exists a.e. on γ and $\sup\limits_{0<r<1}\|f_r(\cdot)\|_{L_p}=\|f^{+}(\cdot)\|_{L_p}$(see, e.g. [15]). As each summand on the right-hand side of the above equality is monotonic increasing function of r, we get

$$\|f\|_{H_{p;\nu}^{+}}=\left(\sum_{n=0}^{\infty}|f_n|^{\nu}\right)^{\frac{1}{\nu}}+\left\|f^{+}\right\|_{L_p}=\left\|f^{+}\right\|_{L_{p;\nu}}.$$

Now define the set ${}_mH_{p;\nu}^{-}$ for fixed integer m. Let $f(z)$ be a function, analytic outside ω and

$$f(z)=\sum_{n=-\infty}^{k}f_n z^n \tag{3.1}$$

with some $k\leq m$. Write f as

$$f(z)=P_k(z)+f_1(z),$$

where $P_k(\cdot)$ and $f_1(\cdot)$ are the analytic, if any, and principal parts of the expansion (3.1), respectively. We will say that $f(\cdot)$ belongs to ${}_mH_{p;\nu}^{-}$ if $\overline{f_1\left(\frac{1}{\bar{z}}\right)}\in H_{p;\nu}^{+}$.

Let $f\in H_{p;\nu}^{+}$, $1\leq p<+\infty$, $1\leq\nu\leq+\infty$. It immediately follows that $f\in H_p^{+}$, and therefore, we have the Cauchy formula

$$f(z)=\frac{1}{2\pi i}\int_{\gamma}\frac{f^{+}(\tau)}{\tau-z}dz,\quad\forall z\in\omega. \tag{3.2}$$

As shown above, $f^{+}(\cdot)\in L_{p;\nu}$. It is clear that, $f(\cdot)\in H_1^{+}$.

Conversely, suppose $f(\cdot) \in H_1^+$ and $f^+(\cdot) \in L_{p;\nu}$. From here we have $f^+ \in L_p$, and as a result the representation (3.2) is true. Then, from the equality

$$\|f\|_{H^+_{p;\nu}} = \left\|f^+\right\|_{L_{p;\nu}}, \tag{3.3}$$

it follows that $f \in H^+_{p;\nu}$.

Hence, the following theorem was proved

Theorem 3.1. *The function $f(\cdot)$ belongs to $H^+_{p;\nu}$, $1 \le p < +\infty$, $1 \le \nu \le +\infty$ iff $f^+(\cdot) \in L_{p;\nu}$; in that case the Cauchy formula* (3.2) *is valid.*

By (3.3), we deduce the following theorem.

Theorem 3.2. *$H^+_{p;\nu}$ and ${}_mH^-_{p;\nu}$, $1 \le p < +\infty$, $1 \le \nu \le \infty$ are Banach spaces.*

Now consider the singular integral S in $L_{p;\nu}$, for $1 < p < +\infty$ and $1 \le \nu \le \infty$. Let $F(z)$ be the corresponding Cauchy-type integral with $f(\cdot)$ as its density. Therefore, the following Sokhotski-Plemely formula is true

$$F^{\pm}(\tau) = \pm\frac{1}{2}f(\tau) + (Sf)(\tau), \quad \tau \in \gamma,$$

where $F^+(\cdot)$ $((F^-(\cdot))$ is the interior(exterior) non-tangential boundary values of $F(z)$ along γ. Hence

$$f(\tau) = F^+(\tau) - F^-(\tau), \quad \tau \in \gamma.$$

Let

$$f(\tau) = \sum_{n\in\mathbb{Z}} f_n \tau^n$$

be the Fourier expansion of $f(\cdot) \in L_p(\gamma)$. Then

$$F^+(\tau) = \sum_{n=0}^{\infty} f_n \tau^n, \qquad F^-(\tau) = -\sum_{n=1}^{\infty} f_{-n}\tau^{-n}.$$

We have

$$\begin{aligned}(Sf)(\tau) &= F^+(\tau) - \frac{1}{2}f(\tau) \\ &= \sum_{n\in\mathbb{Z}} g_n \tau^n,\end{aligned}$$

where

$$g_n = \frac{1}{2}\begin{cases} f_n, & n \ge 0, \\ -f_n, & n < 0. \end{cases}$$

Then

$$\|Sf\|_{L_{p;\nu}} = \left(\sum_{n\in\mathbb{Z}} |g_n|^{\nu}\right)^{\frac{1}{\nu}} + \|Sf\|_{L_p}. \tag{3.4}$$

Since, S is bounded in L_p, from the expressions for coefficients $\{g_n\}_{n\in\mathbb{Z}}$ and (3.4) it immediately follows that $\exists A>0$:

$$\|Sf\|_{L_{p;\nu}} \le c\,\|f\|_{L_{p;\nu}}, \quad \forall f\in L_{p;\nu}.$$

Hence, the following theorem was proved

Theorem 3.3. *The singular operator S is bounded in $L_{p;\nu}$, for $1<p<+\infty$,$1\le\nu\le+\infty$.*

The following continuous embedding is true

$$L_{p_2;r_1}\subset L_{p_1;r_2},\quad 1\le p_1<p_2<+\infty,$$
$$1\le r_1<r_2\le+\infty,$$

that follows from the fact that the embedding $L_{p_2}\subset L_{p_1},\ 1\le p_1<p_2<+\infty$, is continuous and the inequality

$$\left(\sum_{1}^{\infty}|a_k|^{r_2}\right)^{\frac{1}{r_2}} \le \left(\sum_{1}^{\infty}|a_k|^{r_1}\right)^{\frac{1}{r_1}}, \tag{3.5}$$

is true for all $1\le r_1<r_2\le+\infty$ (see, e.g. [16, pp. 149]).

The following is an analogue of the classical Smirnov's theorem.

Theorem 3.4. *Let $f\in H^+_{p_1;r_2}$ and $f^+\in L_{p_2;r_1}$, where $1\le p_1<p_2<+\infty$, $1\le r_1<r_2\le+\infty$. Then $f\in H^+_{p_2;r_1}$.*

The proof is a direct consequence of Theorem 3.1.

Thus, each function f from $H^+_{p;\nu}$, $1\le p<+\infty$, $1\le\nu\le+\infty$ is uniquely determined by its boundary values f^+, which belongs to $L_{p;\nu}$. It is obvious that this is in case for the space ${}_mH^-_{p;\nu}$ as well. Denote by $L^+_{p;\nu}$ and ${}_mL^-_{p;\nu}$ the restrictions to the unit circle γ of the functions from $H^+_{p;\nu}$ and ${}_mH^-_{p;\nu}$, respectively. It is easy to see that, $L^+_{p;\nu}$ and ${}_mL^-_{p;\nu}$ are subspaces of $L_{p;\nu}$. Via the restriction isomorphism the spaces $H^+_{p;\nu}$ and $L^+_{p;\nu}$, as well as ${}_mH^-_{p;\nu}$ and ${}_mL^-_{p;\nu}$, can be identified.

Theorem 3.5. *Let $f\in H^+_\delta$, for some $\delta>0$ and $f^+\in L_{p;r}$ for some $1\le p<+\infty$ and $1\le r\le+\infty$. Then $f\in H^+_{p;r}$.*

Indeed, from $f^+\in L_{p;r}$ it follows that $f^+\in L_p(\gamma)$, and then by the Smirnov theorem we get $f\in H^+_p$. Hence, for the function $f(\cdot)$ Cauchy formula (3.2) is valid. Theorem 3.1 completes the rest of proof.

4. Bases of Parts of the System of Exponents in the Subspaces $L^\pm_{p;\nu}$

Let $f\in H^+_{p;r}$, $1<p<+\infty$, $1\le r\le+\infty$. Then $f\in H^+_p$ and $f^+\in L^+_p$. It is clear that the system $\left\{e^{\mathrm{int}}\right\}_{n\in\mathbb{Z}_+}$ is minimal in $L^+_{p;r}$. The system $\left\{e^{\mathrm{int}}\right\}_{n\in\mathbb{Z}}$ forms a basis in $L_{p;r}$ and the system $\left\{\frac{1}{2\pi}e^{int}\right\}_{n\in\mathbb{Z}}$ is its biorthogonal. From $f \in H_p^+$ it follows that, $f^+ \in L_p^+$, and the last implies $\frac{1}{2\pi}\int_{-\pi}^{\pi} f^+\left(e^{it}\right)e^{-\mathrm{int}}dt = 0,\ \forall n \leq -1$. From here we get that $f^+\left(e^{it}\right) = \sum\limits_{n=0}^{\infty} f_n e^{\mathrm{int}}$ (the equality is understood in $L_{p;r}$ norm sense). The uniqueness of above expansion is evident.

The case of the system $\left\{e^{-\mathrm{int}}\right\}_{n=m}^{\infty}$ is considered analogously.

Thus, we proved the following result.

Theorem 4.1. *The system* $\left\{e^{\mathrm{int}}\right\}_{n\in\mathbb{Z}_+}$ $\left(\left\{e^{-\mathrm{int}}\right\}_{n=m}^{\infty}\right)$ *forms a basis in* $L_{p;r}^+$ (${}_mL_{p;r}^-$, *respectively) for* $1 < p < +\infty,\ 1 \leq r \leq +\infty$.

5. The Riemann-Hilbert Problem

Consider the following double exponential system

$$\left\{A(t)\,e^{\mathrm{int}}; B(t)\,e^{-\mathrm{int}}\right\}_{n\in\mathbb{Z}_+}, \tag{5.1}$$

here $A(\cdot); B(\cdot) : [-\pi,\pi] \to C$ are some functions. We will assume that the functions $A(\cdot)$ and $B(\cdot)$ are subjected to the following condition

α) $A^{\pm 1}(\cdot); B^{\pm 1}(\cdot) \in L_\infty(-\pi,\pi)$.

Theorem 5.1. *Under the condition* (α) *the system* (5.1) *forms a basis in* $L_{p;r}$, $1 < p < +\infty,\ 1 \leq r \leq +\infty$, *iff the Riemann-Hilbert problem*

$$a(\tau)\,F^+(\tau) + b(\tau)\,\overline{\Phi^+(\tau)} = g(\tau)\,, \quad \tau \in \gamma, \tag{5.2}$$

has a unique solution in $H_{p;r}^+ \times H_{p;r}^+$. *Here* $a\left(e^{it}\right) = A(t)$, $b\left(e^{it}\right) = B(t)$, $t \in [-\pi,\pi]$.

Proof. Let the system (5.1) forms a basis in $L_{p;r}$. Then for any $f \in L_{p;r}$ there is a unique expansion

$$A(t)\sum_{n=0}^{\infty} f_n^+ e^{\mathrm{int}} + B(t)\sum_{n=0}^{\infty} f_n^- e^{-\mathrm{int}} = f(t)\,.$$

Set

$$f^\pm(\tau) = \sum_{n=0}^{\infty} f_n^\pm \tau^{\pm n}, \quad |\tau| = 1. \tag{5.3}$$

By the condition (α) and the fact that the system (5.1) is a basis, it follows that $f^\pm \in L_1(\partial\omega)$.

We have

$$\int_{\partial\omega} f^+(\tau)\,\tau^n d\tau = \sum_{k=0}^{\infty} f_k^+ i\int_{-\pi}^{\pi} e^{i(k+n+1)t}dt$$

$$= 0, \quad \forall n \geq 0.$$

By Privalov's theorem (see, e.g. [8, 11]), there is $F \in H_1^+$, whose boundary values coincide a.e. on $\partial\omega$ with f^+, i.e. $F^+(\tau) = f^+(\tau)$ a.e..

Similarly, we get

$$\int_{\partial\omega} \overline{f^-(\tau)}\tau^n d\tau = \sum_{k=0}^{\infty} \overline{f_k^-} i \int_{-\pi}^{\pi} e^{i(k+n+1)t} dt$$
$$= 0, \quad \forall n \geq 0.$$

From here by the same reasoning it follows that there is $\Phi \in H_1^+$, such that $\Phi^+(\tau) = \overline{f^-(\tau)}$ a.e. on $\tau \in \partial\omega$. Theorem 3.5 implies that $F, \Phi \in H_{p;r}^+$, and from (5.3) we get that

$$a(\tau) F^+(\tau) + b(\tau) \overline{\Phi^+(\tau)} = g(\tau), \quad \tau \in \gamma,$$

where $g(e^{it}) = f(t)$. Thus, if the system (5.2) forms a basis in $L_{p;r}$, then the problem (5.2) is solvable in $H_{p;r}^+ \times H_{p;r}^+$.

Let's show that the problem (5.3) has a unique solution. It is enough to show that the homogeneous problem

$$a^+(\tau) F^+(\tau) + a^-(\tau) \overline{\Phi^+(\tau)} = 0, \quad \tau \in \gamma, \tag{5.4}$$

has only trivial solution in $H_{p;r}^+ \times H_{p;r}^+$. Assume the contrary. Let there are nonzero functions F and Φ in $H_{p;r}^+ \times H_{p;r}^+$, whose boundary values F^+ and Φ^+ satisfy (5.4). As the system $\left\{e^{\mathrm{int}}\right\}_{n\geq 0}$ forms a basis in $L_{p;r}^+$, by expanding F^+ and Φ^+ upon this system, from (5.4) we get that zero function has a nonzero expansion upon the system (5.2), which contradicts (5.2) to be a basis in $L_{p;r}$. The converse part of the theorem is proved analogously. The proof is over. □

Note that, analogous result in L_p space case has been obtained earlier in [2].

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