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On F-Weak Contraction of Generalized Multivalued Integral Type Mappings with α-admissible

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Abstract. The purpose of this work is to investigate the existence of fixed points of some mappings in fixed point theory by combining some important concepts which are F-weak contractions, multivalued mappings, integral transformations and α-admissible mappings. In fixed point theory, it is important to find fixed points of some classess under F- or F-weak contractions. Also multivalued mappings are the other important classes. Along with that, α-admissible mapping is a different approach in the fixed point theory. According to this method, a single or multivalued mapping does not have a fixed point in general. But, under some restriction on the mapping, a fixed point can be obtained. In this article, we combine four significant notions and also establish fixed point theorem for this mappings in complete metric spaces. Moreover, we give an example to show the interesting of our results according to earlier results in literature.

1. Introduction

Contraction mappings have a very important role in fixed point theory since they are used to solve the problems of existence in many branches of mathematics. Initially, Banach [4] defined the classical contraction principle that guarantees the existence and uniqueness of a fixed point. Due to its usefulness, it has been generalized by using different transformations types and changing the structure of the space. Afterward, Nadler [8] extended the Banach contraction principle for single valued mappings to multivalued mappings by using the Hausdorff metric. This

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idea has led many researchers extensively to investigate the various fixed point theorem results related to multivalued contraction mappings, especially, both the metric space and topological fixed point theory. One can find some related papers in list [6, 10, 12]. In 2012, Wardowski [14] introduced a new type of contraction mapping known as F-contraction and shown that this mapping is a Banach contraction. The concept of multivalued F-contraction mapping was introduced by Altun et al. [3] and they obtained some fixed point theorems for multivalued contraction mappings in a complete metric space. Then, Acar et al. [1] generalized this mapping and achieved some results. Samet et al. [11] introduced α-admissible mappings and a new type of the (α-ϕ) contraction mapping which is a generalization of the Banach principle. Later, Choudhury et al. [5] introduced a multivalued version of the (α-ϕ) contraction and they defined multivalued α-admissible mappings. The applications of contraction which are mentioned above under integral operator have been studied by some authors [2, 7, 13].

The purpose of this paper is to apply F-contractions on generalized multivalued integral type (α-ϕ) contraction mappings which are also α-admissible. We will also establish a fixed point theorem for this mapping in complete metric spaces. Moreover, we will give an example to show the application of our results according to earlier results in literature. Now, give some definitions and theorems that we will use in this article.

Let (X, d) be a metric space. Denote by $P(X)$ the family of all nonempty subsets of X, $CL(X)$ the family of all nonempty and closed subsets of X, $CB(X)$ the family of all nonempty, closed and bounded subsets of X and $K(X)$ the family of all nonempty compact subsets of X.

Definition 1.1. Let (X, d) be a complete metric space. We define the Hausdorff metric on $CB(X)$ by

$$H(A,B) := \max\left\{\sup_{x\in A} D(x,B), \sup_{y\in B} D(y,A)\right\},$$

for all $A, B \in CB(X)$, where $D(x,B) := \inf_{b\in B} d(x,b)$ for all $x \in X$. The mapping H is said to be a Hausdorff metric induced by d.

Definition 1.2 ([8])**.** Let (X, d) be a complete metric space. A map $T : X \to CB(X)$ is said to be a multivalued contraction if there exists $0 \le \lambda < 1$ such that

$$H(Tx, Ty) \le \lambda d(x, y),$$

for all $x, y \in X$.

Definition 1.3 ([8])**.** A point $x_0 \in X$ is said to be a fixed point of a multivalued mapping $T : X \to CB(X)$ if $x_0 \in Tx_0$.

Theorem 1.4 ([9]). *Let (X,d) be a complete metric space. Suppose $T: X \to CB(X)$ is a contraction mapping in the sense that for some $0 \le \lambda < 1$,*

$$\int_0^{H(Tx,Ty)} \varphi(t)dt \le \lambda \int_0^{M(x,y)} \varphi(t)dt,$$

where

$$M(x,y) = \max\left\{d(x,y), D(x,Tx), D(y,Ty), \frac{1}{2}\left[D(x,Ty)+D(y,Tx)\right]\right\},$$

for all $x, y \in X$. Then, there exists a point $x \in X$ such that $x \in Tx$(i.e., x is a fixed point of T).

Wardowski [14] introduced the following definition.

Definition 1.5. Let $F: \mathbb{R}^+ \to \mathbb{R}$ be a mapping satisfying:

(F_1) F is strictly increasing, i.e. for all γ, $\beta \in \mathbb{R}^+$ such that $\gamma < \beta$, $F(\gamma) < F(\beta)$,

(F_2) For each sequence $\{\gamma_n\}_{n\in\mathbb{N}}$ of positive numbers $\lim_{n\to\infty} \gamma_n = 0$ if and only if $\lim_{n\to\infty} F(\gamma_n) = -\infty$,

(F_3) There exists $k \in (0,1)$ such that $\lim_{\alpha\to 0^+} \gamma^k F(\gamma) = 0$.

A mapping $T: X \to X$ is said to be an F-contraction if there exists $\tau > 0$ such that

$$d(Tx,Ty) > 0 \quad \Rightarrow \quad \tau + F(d(Tx,Ty)) \le F(d(x,y)), \tag{1.1}$$

for all $x, y \in X$.

Definition 1.6 ([1]). Let (X,d) be a metric space and $T: X \to CB(X)$ be a mapping. Then T is said to be a generalized multivalued F-contraction if there exists $\tau > 0$ such that

$$H(Tx,Ty) > 0 \quad \Rightarrow \quad \tau + F(H(Tx,Ty)) \le F(M(x,y)),$$

where

$$M(x,y) = \max\left\{d(x,y), D(x,Tx), D(y,Ty), \frac{1}{2}\left[D(x,Ty)+D(y,Tx)\right]\right\},$$

for all $x, y \in X$.

Theorem 1.7 ([1]). *Let (X,d) be a complete metric space and $T: X \to K(X)$ be a generalized multivalued F-contraction. If T or F is continuous, then T has a fixed point in X.*

Definition 1.8 ([13]). Let (X,d) be a complete metric space. The mapping $T: X \to CB(X)$ is a F-contraction of generalized multivalued

integral type mapping if there exists $\tau > 0$ such that, for all $x, y \in X$,
(1.2)

$$H(Tx,Ty) > 0 \quad \Rightarrow \; \tau + F\left(\int_0^{H(Tx,Ty)} \varphi(t)dt\right) \leq F\left(\int_0^{M(x,y)} \varphi(t)dt\right),$$

where

$$M(x,y) = \max\left\{d(x,y), D(x,Tx), D(y,Ty), \frac{1}{2}\left[D(x,Ty) + D(y,Tx)\right]\right\},$$

where $\varphi : [0,+\infty) \to [0,+\infty)$ is a Lebesque-integrable mapping which is summable on each compact subset of $[0,+\infty)$, non-negative and such that for each $\varepsilon > 0$, $\int_0^{\varepsilon} \varphi(t)dt > 0$.

Theorem 1.9 ([13])**.** *Let (X,d) be a complete metric space and let $T : X \to K(X)$ be a generalized multivalued F-contraction mapping of integral type. If T or F is continuous, then T has a fixed point in X.*

Choudhury et al. [5] introduced the concept of multivalued (α-ϕ) contraction and multivalued α-admissible in the following way:

Definition 1.10 ([5])**.** Let (X,d) be a metric space and $\alpha : X \times X \to [0,\infty)$, $\phi : [0,\infty) \to [0,\infty)$ be two mappings such that ϕ is a nondecreasing and continuous function with $\sum \phi^n(t) < \infty$ and $\phi(t) < t$ for each $t > 0$. $T : X \to CL(X)$ be a multivalued mapping. We say that T is a multivalued (α-ϕ) contraction if

$$\alpha(x,y)H(Tx,Ty) \leq \phi(d(x,y)),$$

for all $x, y \in X$.

Definition 1.11 ([5])**.** Let X be any nonempty set and $T : X \to P(X)$ and $\alpha : X \times X \to [0,\infty)$ be two mappings. Then, T is said to be multivalued α-admissible if

$$\alpha(x,y) > 1 \quad \Rightarrow \quad \alpha(a,b) > 1, \text{ for all } a \in Tx \text{ and for all } b \in Ty,$$

for all $x, y \in X$.

Theorem 1.12 ([5])**.** *Let (X,d) be a complete metric space and $T : X \to CL(X)$ be a multivalued (α-ϕ) contraction. Also, suppose T satisfies the following:*

(i) *T is multivalued α-admissible,*
(ii) *For some $x_0 \in X$, $\alpha(x_0, a) > 1$ holds for all $a \in Tx_0$,*
(iii) *If $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) > 1$ for all n, where $x_{n+1} \in Tx_n$ and $x_n \to x$ as $n \to \infty$, then $\alpha(x_n, x) > 1$ for all n.*

Then T has a fixed point.

2. Main Results

Before giving the main results, we define generalized multivalued integral type mapping under F-contraction with α-admissible. Afterwards, we prove that these mappings on complete metric spaces have a fixed point.

Definition 2.1. Let (X, d) be a complete metric space and let $T : X \to CB(X)$ be an F-weak contraction of generalized multivalued integral type mapping. T is a multivalued α-admissible satisfying the conditions of Theorem 1.12 if there exists $\tau > 0$ such that, for all $x, y \in X$

(2.1)
$$H(Tx,Ty) > 0 \Rightarrow \tau + F\left(\int_0^{\alpha(x,y)H(Tx,Ty)} \varphi(t)dt\right) \leq F\left(\phi\left(\int_0^{M(x,y)} \varphi(t)dt\right)\right),$$

where

$$M(x,y) = \max\left\{d(x,y), D(x,Tx), D(y,Ty), \frac{1}{2}\left[D(x,Ty) + D(y,Tx)\right]\right\},$$

where $\varphi : [0,+\infty) \to [0,+\infty)$ is a Lebesque-integrable mapping which is summable on each compact subset of $[0,+\infty)$, non-negative and such that for each $\varepsilon > 0$, $\int_0^{\varepsilon} \varphi(t)dt > 0$.

Example 2.2. Let T be a multivalued α-admissible and it satisfies the conditions of Theorem 1.12. Let $F : \mathbb{R}^+ \to \mathbb{R}$ be a mapping given by $F(t) = \ln t$. It is clear that F satisfies (F_1-F_3). Every F-weak contraction for generalized multivalued integral type mapping satisfies

$$\tau + F\left(\int_0^{\alpha(x,y)H(Tx,Ty)} \varphi(t)dt\right) \leq F\left(\phi\left(\int_0^{M(x,y)} \varphi(t)dt\right)\right).$$

Now, we have

$$\begin{aligned}
\ln e^{\tau} + \ln\left(\int_0^{\alpha(x,y)H(Tx,Ty)} \varphi(t)dt\right) &\leq \ln\left(\phi\left(\int_0^{M(x,y)} \varphi(t)dt\right)\right)\\
\int_0^{\alpha(x,y)H(Tx,Ty)} \varphi(t)dt &\leq e^{-\tau}\phi\left(\int_0^{M(x,y)} \varphi(t)dt\right)\\
&\leq \phi\left(\int_0^{M(x,y)} \varphi(t)dt\right),
\end{aligned}$$

for all $x, y \in X$, $Tx \neq Ty$.

It is clear that for $x, y \in X$ such that $Tx = Ty$ the inequality

$$\int_0^{\alpha(x,y)H(Tx,Ty)} \varphi(t)dt \leq e^{-\tau}\phi\left(\int_0^{M(x,y)} \varphi(t)dt\right),$$

also holds, i.e. T is a contraction.

Theorem 2.3. *Let (X,d) be a complete metric space and let $T : X \to CB(X)$ be an F-weak contraction of generalized multivalued integral type mapping. Also, the α-admissible mapping T satisfies the conditions of Theorem 1.12. Then T has a fixed point.*

Proof. Let $x_0 \in X$ be an arbitrary point. Define a sequence $\{x_n\}$ by $x_{n+1} \in Tx_n$ for $n \in \mathbb{N}$. If $x_1 \in Tx_1$, then this point is a fixed point of T. Suppose that $x_1 \notin Tx_1$. Then by the conditions (ii) in Theorem 1.12, we can choose $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) > 1$. By same idea, let $x_2 \in Tx_1$, then we have

$$\begin{aligned}\int_0^{d(x_1,x_2)} \varphi(t)dt &\le \int_0^{\alpha(x_0,x_1)H(Tx_0,Tx_1)} \varphi(t)dt \\ &\le \phi\left(\int_0^{M(x_0,x_1)} \varphi(t)dt\right).\end{aligned}$$

Since

$$M(x_0,x_1) = \max\left\{d(x_0,x_1), D(x_0,Tx_0), D(x_1,Tx_1), \frac{1}{2}\left[D(x_0,Tx_1) + D(x_1,Tx_0)\right]\right\},$$

we get

$$\begin{aligned}\int_0^{d(x_1,x_2)} \varphi(t)dt &\le \int_0^{\alpha(x_0,x_1)H(Tx_0,Tx_1)} \varphi(t)dt \\ &\le \phi\left(\int_0^{\max\{d(x_0,x_1),D(x_1,Tx_1)\}} \varphi(t)dt\right) \\ &\le \phi\left(\int_0^{d(x_0,x_1)} \varphi(t)dt\right).\end{aligned} \tag{2.2}$$

Moreover, since $x_1 \in Tx_0$ and $x_2 \in Tx_1$, by using Definition 1.11, we have $\alpha(x_1, x_2) > 1$. Now, for $x_2 \in Tx_1$, we can choose $x_3 \in Tx_2$ and then by repeating the process in (2.2), we get

$$\begin{aligned}\int_0^{d(x_2,x_3)} \varphi(t)dt &\le \int_0^{\alpha(x_1,x_2)H(Tx_1,Tx_2)} \varphi(t)dt \\ &\le \phi\left(\int_0^{d(x_1,x_2)} \varphi(t)dt\right).\end{aligned}$$

From Definition 1.11, $\alpha(x_2, x_3) > 1$ and by using (2.2), we have

$$\int_0^{d(x_2,x_3)} \varphi(t)dt \le \int_0^{\alpha(x_1,x_2)H(Tx_1,Tx_2)} \varphi(t)dt$$

$$\leq \phi^2 \left(\int_0^{d(x_0,x_1)} \varphi(t)dt \right).$$

If we continue n times the above mentioned process, we obtain that

$$\int_0^{d(x_n,x_{n+1})} \varphi(t)dt \leq \int_0^{\alpha(x_{n-1},x_n)H(Tx_{n-1},Tx_n)} \varphi(t)dt \tag{2.3}$$
$$\leq \phi^n \left(\int_0^{d(x_0,x_1)} \varphi(t)dt \right).$$

From (ii) in Theorem 1.12, $\alpha(x_{n-1}, x_n) > 1$ for all $n \in N$. Without loosing of generality in (2.3), we can take $\varphi(t) = 1$ for $t > 0$. Then we have

$$d(x_n, x_{n+1}) \leq \alpha(x_{n-1}, x_n)H(Tx_{n-1}, Tx_n) \leq \phi^n \left(d(x_0, x_1)\right).$$

Therefore from Theorem 2.1 in [5], we obtain $\{x_n\}$ is a Cauchy sequence. Also, since X is complete there exists $p \in X$ such that $x_n \to p$ when $n \to \infty$.

Now we show that $p \in Tp$. Let $x_{n+1} \in Tx_n$. Then

$$\int_0^{d(x_{n+1},Tp)} \varphi(t)dt \leq \int_0^{\alpha(x_n,p)H(Tx_n,Tp)} \varphi(t)dt \tag{2.4}$$
$$\leq \phi \left(\int_0^{d(x_n,p)} \varphi(t)dt \right).$$

By taking limit in (2.4), we get

$$\begin{aligned}\int_0^{d(p,Tp)} \varphi(t)dt &\leq \int_0^{\alpha(p,p)H(Tp,Tp)} \varphi(t)dt \\ &\leq \phi \left(\int_0^{d(p,p)} \varphi(t)dt \right) \\ &= \phi\left(0\right).\end{aligned}$$

Also since $\phi(t) < t$ for $t \geq 0$, then $\phi\left(0\right) = 0$. Along with that $\int_0^{\varepsilon} \varphi(t)dt \geq 0$, and $\int_0^{d(p,Tp)} \varphi(t)dt = 0$, that is, $d(p, Tp) = 0$ and we get $p \in Tp$.

After these processes, we prove that all of these processes valid for F-weak contraction.

By considering $\phi(t) < t$ and (F_1), since $x_{n+1} \in Tx_n$ for $n \geq 1$, we have

$$F \left(\int_0^{d(x_1,x_2)} \varphi(t)dt \right) \leq F \left(\int_0^{\alpha(x_0,x_1)H(Tx_0,Tx_1)} \varphi(t)dt \right) \tag{2.5}$$

$$
\begin{aligned}
&\leq F\left(\phi\left(\int_0^{d(x_0,x_1)}\varphi(t)dt\right)\right)-\tau\\
&\leq\ F\left(\int_0^{d(x_0,x_1)}\varphi(t)dt\right)-\tau.
\end{aligned}
$$

Again using (2.5),

$$
\begin{aligned}
F\left(\int_0^{d(x_2,x_3)}\varphi(t)dt\right)&\leq F\left(\phi\left(\int_0^{d(x_1,x_2)}\varphi(t)dt\right)\right)-\tau\\
&\leq F\left(\int_0^{d(x_1,x_2)}\varphi(t)dt\right)-\tau\\
&\leq F\left(\int_0^{d(x_0,x_1)}\varphi(t)dt\right)-2\tau.
\end{aligned}
$$

If repeated this process n times, we get

$$
\begin{aligned}
F\left(\int_0^{d(x_n,x_{n+1})}\varphi(t)dt\right)&\leq F\left(\int_0^{\alpha(x_{n-1},x_n)H(Tx_{n-1},Tx_n)}\varphi(t)dt\right)\\
&\leq F\left(\int_0^{d(x_0,x_1)}\varphi(t)dt\right)-n\tau.
\end{aligned}
\tag{2.6}
$$

Let $\gamma_n=\int_0^{d(x_n,x_{n+1})}\varphi(t)dt$. From (2.6) and by taking limit when $n\to\infty$ we get

$$
\lim_{n\to\infty}F(\gamma_n)=-\infty.
$$

It is easy to see that $\lim\limits_{n\to\infty}\gamma_n=0$ together with (2.4). From (F_3), there exists $k\in(0,1)$ such that

$$
\lim_{n\to\infty}\left(\gamma_n\right)^k F\left(\gamma_n\right)=0.
$$

It is easy to see that all conditions of F-weak contraction hold and therefore mapping (2.1) is F-weak contraction. Hence it has a fixed point. □

Corollary 2.4. *Let (X,d) be an complete metric space and let $T:X\to CB(X)$ be an F-weak contraction of α-admissible multivalued mappings. Also, the α-admissible mapping T satisfies the conditions of Theorem 1.12. Then, T has a fixed point in X.*

Proof. In Definition 2.1, if we take $\varphi(t)=1$ for all $t\in R$, we get the following inequality

$$
H(Tx,Ty)>0\quad\Rightarrow\quad\tau+F\left(\alpha(x,y)H(Tx,Ty)\right)\leq F\left(\phi\left(M(x,y)\right)\right),
$$

where

$$M(x,y)=\max\left\{d(x,y),D(x,Tx),D(y,Ty),\frac{1}{2}\left[D(x,Ty)+D(y,Tx)\right]\right\}.$$

The rest of the proof can be obtained using the same procedures we used in the proof of Theorem 2.3. □

In this step, in order to construct an example for the mapping given in (2.1), we will use the example given in [5].

Example 2.5. Let $X=R$, $d(x,y)=|x-y|$ and $T:R\to CB(R)$ by

$$Tx=\begin{cases}\left\{1,\frac{1}{4x}\right\}, & x>1,\\ \left\{0,\frac{x}{16}\right\}, & 0\le x\le 1,\\ \{2,3\}, & x<0.\end{cases}$$

It is easy to show that $H(T0,Tx)=3>1$ and $M(x,y)=2$. Therefore the contraction mapping given in Theorem 1.4 does not hold due to the condition $k\ge\frac{3}{2}$.

Let $\alpha:R^+\to R^+$ be a map defined by

$$\alpha(x,y)=\begin{cases}2, & x,y\in[0,1],\\ 0, & \text{otherwise}\end{cases}$$

and also $\phi:R^+\to R^+$ defined by $\phi(t)=\frac{1}{2}t$. For $x,y\in[0,1]$, T is a multivalued (α-ϕ) contraction according to integral type mapping. Now we have

$$\begin{aligned}H(Tx,Ty)&=\max\left\{\sup_{x\in Tx}D(x,Ty),\sup_{y\in Ty}D(y,Tx)\right\}\\&=\max\left\{\inf\left\{\frac{|x|}{16},\frac{|x-y|}{16}\right\},\inf\left\{\frac{|y|}{16},\frac{|x-y|}{16}\right\}\right\}\\&=\frac{|x-y|}{16}.\end{aligned}$$

Also we can see that

$$\begin{aligned}M(x,y)&=\max\left\{d(x,y),D(x,Tx),D(y,Ty),\frac{1}{2}\left[D(x,Ty)+D(y,Tx)\right]\right\}\\&=|x-y|.\end{aligned}$$

By using these results, we show that

$$\int_0^{\alpha(x,y)H(Tx,Ty)}\varphi(t)dt\le\phi\left(\int_0^{M(x,y)}\varphi(t)dt\right).$$

Taking $\phi(t)=\frac{1}{2}t$ and $\varphi(t)=1$ for all $t\in R$, we have

$$2\frac{|x-y|}{16}\le\frac{1}{2}|x-y|.$$

Now we show that this mapping is an F-contraction for $\tau \in [0, 1.386]$. Let $F(t) = \ln t$ and $\tau > 0$.

We have the following inequality as in (2.1)

$$H(Tx,Ty) > 0 \quad \Rightarrow \quad \tau + F\left(\int_0^{\alpha(x,y)H(Tx,Ty)} \varphi(t)dt\right) \leq F\left(\phi\left(\int_0^{M(x,y)} \varphi(t)dt\right)\right).$$

By taking $F(t) = \ln t$ in the above inequality, we have

$$\ln\left(e^{\tau}\int_0^{\frac{|x-y|}{8}} dt\right) \leq \ln\left(\frac{1}{2}\int_0^{|x-y|} dt\right)$$

$$\ln\left(e^{\tau}\frac{|x-y|}{8}\right) \leq \ln\left(\frac{|x-y|}{2}\right)$$

$$\frac{|x-y|}{8} \leq e^{-\tau}\frac{|x-y|}{2}.$$

Hence, we get an F-weak contraction of generalized multivalued integral type mapping given in (2.1).

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