Sahand Communications in Mathematical Analysis (SCMA) Vol. 17 No. 4 (2020), 49-59
http://scma.maragheh.ac.ir
DOI: 10.22130/scma.2020.117584.713

Using Copulas to Model Dependence Between Crude Oil Prices of West Texas Intermediate and Brent-Europe

Vadoud Najjari

Abstract. In this study the main endeavor is to model dependence structure between crude oil prices of West Texas Intermediate (WTI) and Brent - Europe. The main activity is on concentrating copula technique which is powerful technique in modeling dependence structures. Beside several well known Archimedean copulas, three new Archimedean families are used which have recently presented to the literature. Moreover, convex combination of these copulas also are investigated on modeling of the mentioned dependence structure. Modeling process is relied on 318 data which are average of the monthly prices from Jun-1992 to Oct-2018.

1. Introduction

Providing a model for dependence structure between random variables plays an important role in any field of science. Of course correlation is a scalar measure of dependence and it is not able to explain much more about the dependence structure. Moreover, correlation is related with linear dependency and cannot capture non-linear dependence relationships. Also we cannot rely on the correlation results if the related random variables are not normal distributed.

Copulas allow us to combine univariate distributions to obtain a joint distribution with a particular dependence structure. In this way, the scaling and the shape are entirely determined by the marginals, while the dependence relationship is entirely determined by the copula. To see more related with correlation and dependence, readers are referred to see Bertail et al. [4], Mari and Kotz [19], Song [26].

2020 *Mathematics Subject Classification.* 62H05, 62H12, 62H20.
Key words and phrases. Akaike information criterion (AIC), Copulas, Goodness of fit test (GOF), Linear convex combination, Parameter estimation.
Received: 26 November 2019, Accepted: 13 June 2020.

During the recent decades, the use of copulas has been a powerful technique in modeling dependence structure in many fields of science. For example, McNeil et al. [8], Clemen and Reilly [5] and a lot of other people used copulas in finance, Najjari et al. [21] applied copulas in meteorological data, Çelebioğlu [6], provided copulas in modeling of students grades. Bacigál and Komorníková [2] proposed copulas in modeling dependence structure of geodetic data. Najjari et al. [20] used copulas in modeling dependence between flood peak and volume. Many other authors have been worked on copulas like, Genest and MacKay [9, 10], Hua and Joe [14], among them.

Hui-Ming et al. [15], try on modelling dynamic dependence between crude oil prices and Asia-Pacific stock market returns by using unconditional and conditional copula models. They show that time-varying copulas best capture the tail dependence and that taking the tail correlation into account leads to improved accuracy of VaR estimates. Al-Harthy et al. [1], prepared copulas to model dependence in petroleum decision making. They, after providing several preliminaries, show that copula technique has an important role in model dependency of the petroleum industry as well as Envelope method and also Iman-Conover method.

In this study we use copulas to model the dependency between crude oil prices of West Texas Intermediate (WTI) and Brent-Europe. We use the mean of monthly prices from Jun-1992 to Oct-2018 which are 318 data. During process beside several well known Archimedean copulas, we have also used three new Archimedean families which have recently presented to the literature. These new families have trigonometric and hyperbolic generators.

To estimate copulas parameters, we rely on nonparametric estimation (Genest et al. [12]) and also maximum likelihood estimation. And to select the right copula we use nonparametric and semi-parametric procedure (Genest et al. [11, 12]), GOF test (Genest et al. [13]), also Akaike information criterion (AIC) of maximum likelihood estimate goodness.

The rest of the paper is organized as follows. Section 2 is preliminaries. Fitting copulas to data is discussed in Section 3. Section 4 describes modeling of the mentioned crude oil prices by copulas and Section **??** summarizes the conclusion of our study.

2. Preliminaries

A bivariate copula (or 2-copula) is a function $C : [0,1]^2 \to [0,1]$ which satisfies:

(a) for every u, v in $[0,1]$, $C(u,0) = 0 = C(0,v)$ and $C(u,1) = u$ and $C(1,v) = v$;

(b) for every u_1, u_2, v_1, v_2 in [0, 1] such that $u_1 \leq u_2$ and $v_1 \leq v_2$, $V_C(R) = C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0$ (in other word, for all rectangles $R = [u_1, u_2] \times [v_1, v_2]$ whose vertices lie in $[0, 1]^2$, $V_C(R) \geq 0$).

Copulas are multivariate distributions (restricted to the unit hypercube) in modeling the dependence structure between variables, irrespective of their marginal distributions, readers are referred to see Schweizer [25]. One of the most important classes of copulas is known as Archimedean copulas (AC) which are also used in this study to model the dependency of the mentioned crude oil prices . These copulas are very easy to construct, many parametric families belong to this class and have a great variety of different dependence structures. In addition, the Archimedean representation allows us to reduce the study of a multivariate copula to a single univariate function. AC originally appeared not in statistics, but rather in the study of probabilistic metric spaces, where they were studied as part of the development of a probabilistic version of the triangle inequality. For details see Schweizer [25] and Nelsen [23].

Basic properties of AC are presented below and more information could be found in Nelsen [23]. Let φ be a continuous, strictly decreasing function from I to $[0, \infty]$ such that $\varphi(1) = 0$. The pseudo-inverse of φ is the function $\varphi^{[-1]}$ given by

$$\varphi^{[-1]}(t) = \begin{cases} \varphi^{(-1)}(t), & 0 \leq t \leq \varphi(0), \\ 0, & \varphi(0) \leq t \leq \infty. \end{cases} \tag{2.1}$$

Copulas of the form $C(u, v) = \varphi^{[-1]}(\varphi(u) + \varphi(v))$, for every u, v in $[0, 1]$ are called AC and the function φ is called a generator of the copula. If $\varphi(0) = \infty$ we say that φ is a strict generator. In this case, $\varphi^{[-1]} = \varphi^{(-1)}$ and $C(u, v) = \varphi^{(-1)}(\varphi(u) + \varphi(v))$ is said to be a strict Archimedean copula.

3. Fitting Copulas to Data

In this section we review several methods for identifying the right copula that fits to data. Several methods of copulas parameters estimation and identifying the right copula have been developed, including the methods of concordance Genest et al.[11, 12], fully maximum likelihood (ML), pseudo maximum likelihood (PML), Genest et al. [11], inference function of margins (IFM), Joe [16, 17] and minimum distance (MD) Tsukahara [27], etc. Nonetheless in this paper we rely on two methods in estimating copulas parameters and also three criterion in identifying the right copula which are reviewed in below subsections.

3.1. **Estimation of Copulas Parameters.** In this study we review two methods in estimating copulas parameters. First one is nonparametric estimation which proposed by Genest et al. [12] and it is usually estimated by dependence measures such as Kendall's tau, Spearman's rho and Gini's gamma. The second one is the estimating copula parameters by maximizing a pseudo log-likelihood function. Assume that we have a random sample of bivariate observations (X_i, Y_i) for $i = 1, ..., n$ available and joint distribution function H has associated copula C, we look for $\hat{\theta}$ which maximizes pseudo log-likelihood function

$$L(\theta) = \sum_{i=1}^{n} \log(c_\theta(F_n(x), G_n(y))) \tag{3.1}$$

where F_n and G_n are defined similarly as follows

$$F_n(x) = \frac{1}{n+1} \sum_{i=1}^{n} \text{if}[X_i \leq x, 1, 0] \tag{3.2}$$

and

$$c_\theta(u, v) = \frac{\partial}{\partial u}\frac{\partial}{\partial v} C_\theta(u, v). \tag{3.3}$$

3.2. **Identifying the Right Copula.** In selecting the right copula we use three methods which are reviewed in the following. Firstly, we review nonparametric and semi-parametric procedures which are proposed by Genest et al. [11, 12]. Assume that we have a random sample of bivariate observations (X_i, Y_i) for $i = 1, \ldots, n$ available and joint distribution function H has associated Archimedean copula C_φ, we wish to identify the form of φ. First to begin with, define an intermediate (unobserved) random variable $Z_i = H(X_i, Y_i)$ that has distribution function $K(z) = Prob[Z_i \leq z]$. This distribution function is related to the generator of an Archimedean copula through the expression,

$$K(z) = K_\varphi(z) = z - \frac{\varphi(z)}{\varphi'(z)}. \tag{3.4}$$

In order to identify φ,

(i) find Kendall's tau using the usual (nonparametric or distribution-free) estimate

$$\tau_n = \binom{n}{2}^{-1} \sum_{i=2}^{n} \sum_{j=1}^{i-1} \text{sign}[(X_i - X_j)(Y_i - Y_j)]. \tag{3.5}$$

(ii) Construct a nonparametric estimation of K, as follows:

- First, define the pseudo-observations,

$$Z_i = (n-1)^{-1} \sum_{j=1}^{n} \text{if}[X_j < X_i \text{ and } Y_j < Y_i, 1, 0],\ i = 1, 2, \ldots, n. \tag{3.6}$$

- Second, construct the estimate of K,

$$K_n(z) = n^{-1} \sum_{i=1}^{n} \text{if}[Z_i \leq z, 1, 0]. \tag{3.7}$$

(iii) Now construct a parametric estimate K_φ by using (3.4). Illustratively, $\tau_n \to \theta_n \to \varphi_n(t) \to K_{\varphi_n}(z)$, where subscript n denotes estimation.

The step (3) is to be repeated for every copula family that we wish to compare. The best choice of generator then corresponds to the parametric estimate $K_{\varphi_n}(z)$, that most closely resembles the nonparametric estimate $K_n(z)$. Measuring closeness can be done either by a (L_2-norm) distance such as

$$\int_0^1 [K_{\varphi_n}(z) - K_n(z)]^2 dz, \tag{3.8}$$

or graphically by (i) plotting of $z - K(z)$ versus z or (ii) corresponding quantile-quantile (Q-Q) plots. We recall that Q-Q plots are used to determine whether two data sets come from populations with a common distribution. If the points of the plot, which are formed from the quantiles of the data, are roughly on a line with a slope of 1, then the distributions are the same (see also [7]).

Akaike information criterion (AIC) is the second criterion which we use it. We recall, it is defined by

$$AIC = -2(\text{log\_ liklihood}) + 2k, \tag{3.9}$$

where k is the number of parameters in the model. The lowest AIC value means that the copula has the best fitness.

The third criterion which it is used in this study, is standard GOF test. Readers are referred to see Najjari et al. [20, 24]. In using this method the range of two variables transformed into four uniform intervals each, therefore $df = 9$ and $\chi^2_{0.05,df} = 16.9190$ is the critical point. Matlab software has been used in calculations.

4. Modeling Dependency of the Crude Oil Prices

The main aim of this section is to model the dependency between crude oil prices of West Texas Intermediate (WTI) and Brent-Europe which are available online at Federal Reserve Bank of St. Louis[1]. We

[1] https://research.stlouisfed.org/

concentrate on the mean of monthly prices from Jun-1992 to Oct-2018 which are 318 data. Data set was tested for serial (temporal) independence (not simply by checking significance of autocorrelations but) primarily by a test based on empirical copulas which is described in Kojadinovic and Yan [18]. To avoid decision about univariate distributions, the observations were transformed to unit interval by their corresponding empirical distribution functions, see Figure 1 for the resulting pairs. Also Table 2 shows the estimated measures from data.

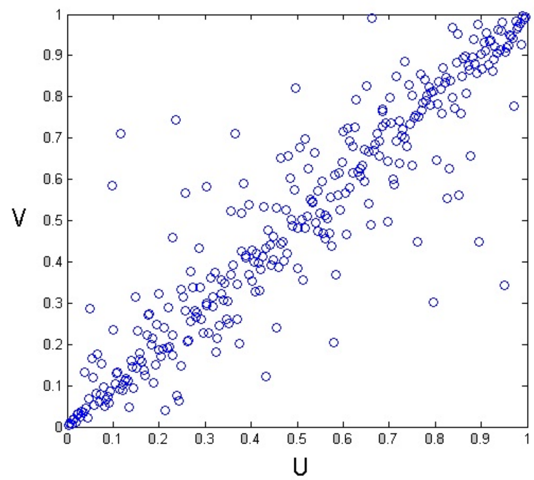

FIGURE 1. Scatterplots of the crude oil prices of U:WTI and V:Brent-Europe.

TABLE 1. Details of the selected copula families in this study

Family	Generator	Kendall's tau	λ_L	λ_U	θ interval
Clayton	$\frac{1}{\theta}(\frac{1}{t^\theta}-1)$	$\frac{\theta}{\theta+2}$	$2^{\frac{-1}{\theta}}$	0	$(0,\infty)$
Gumbel	$(-lnt)^\theta$	$\frac{\theta-1}{\theta}$	0	$2-2^{\frac{1}{\theta}}$	$[1,\infty)$
A12	$(\frac{1}{t}-1)^\theta$	$1-\frac{2}{3\theta}$	$2^{\frac{-1}{\theta}}$	$2-2^{\frac{1}{\theta}}$	$[1,\infty)$
A14	$(t^{-1/\theta}-1)^\theta$	$1-\frac{2}{1+2\theta}$	$\frac{1}{2}$	$2-2^{\frac{1}{\theta}}$	$[1,\infty)$
A15	$(1-t^{\frac{1}{\theta}})^\theta$	$1+\frac{2}{1+2\theta}$	0	$2-2^{\frac{1}{\theta}}$	$[1,\infty)$
cot-copula	$\cot^\theta(\frac{\pi t}{2})$	$1-\frac{8}{\pi^2\theta}$	$2^{\frac{-1}{\theta}}$	$2-2^{\frac{1}{\theta}}$	$[1,\infty)$
coth-copula	$\coth(\theta t)-\coth(\theta)$	$1+\frac{2}{\theta^2}-\frac{2}{\theta}\coth(\theta)$	$\frac{1}{2}$	0	$[1,\infty)$
csch-copula	$\text{csch}(t^\theta)-\text{csch}(1)$	$\frac{\theta}{\theta+2}$	$2^{\frac{-1}{\theta}}$	0	$(0,\infty)$

Note: A12, A14 and A15 families are numbered as 4.2.12, 4.2.14 and 4.2.15 in Table 4.1 Nelsen's book [23]

Copula families which are used in modeling dependency of the mentioned crude oil prices, are summarized in Table 1. As it is seen, there

Table 2. Estimated measures from data

Data	Kendall's τ	Spearman's ρ	Gini's γ
Crude Oil Prices	0.7802	0.916	0.7886

are three new Archimedean families which are recently presented to the literature. cot-copula family has trigonometric generator and proposed by Pirmoradian and Hamzah [24]. Also coth-copula and csch-copula families have hyperbolic generators and have proposed by Najjari et al. [20], Bal and Najjari [3] respectively.

As it was discussed in the Section 3, to estimate copulas parameters, we use nonparametric estimation (Genest et al. [12]) and maximum likelihood estimation. Note that all copulas families in Table 1 have only one parameter, so in the proposed nonparametric method by Genest et al. [12] we just use Kendall's τ which its value calculated as 0.7802.

Table 3 consists of closeness measure and GOF test statistic values by nonparametric estimation of copulas parameters. It is seen that A12 and cot-copula families are in the minimum closeness to the mentioned crude oil prices data by 0.0671 and 0.0867 values respectively. A12 and cot-copula parameters nonparametric estimation values are 3.03 and 3.69 respectively. By the GOF test, since the critical point is $\chi^2_{0.05,df} = 16.9190$, none of the mentioned copulas in Table 1 are suitable to model the dependency of crude oil prices data. It is notable that A12 and cot-copula families have the minimum GOF test statistic vales which are 21.7133 and 19.1106 respectively.

Table 3. Results of the nonparametric estimation and the GOF test

Families	Parameter	$d(K_\varphi, K_n)$	GOF
Gumbel	4.55	0.1466	37.6212
Clayton	7.10	0.6562	126.7121
A12	3.03	0.0671	21.7133
A14	4.05	14.2561	31.7156
A15	5.05	0.2217	53.9645
cot-copula	3.69	0.0867	19.1106
coth-copula	7.96	0.1786	61.5989
csch-copula	7.10	14.3396	126.4086

As convex combination of copulas are also copula, so we decided to check it for several copula families in Table 1 which seems they improve fitness. Note that value of weight a (in the convex combination) is arranged such that the GOF test statistic to get its minimum value. Results of calculations are summarized in Table 4. It is seen that none of the mentioned convex combination of copulas are suitable to model the dependency of crude oil prices data, but convex combinations have improved GOF test statistic values. As an example, in 0.91 A12+0.09 A14 copula, GOF test value is 18.0486, while GOF test values for A12, A14 (in Table 3) are 21.7133 and 31.7156 respectively. Although the mentioned convex combination is not a right copula to model the dependency of crude oil prices data by GOF test, but its GOF test statistic value is less than the GOF test statistic values of A12 and A14 families.

TABLE 4. Results of copulas convex combination by the nonparametric estimation and the GOF test

$aC_{\theta_1} + (1-a)C_{\theta_2}$	θ_1	θ_2	a	GOF
Clayton, Gumbel	7.10	4.55	0.46	27.1601
Clayton, A12	7.10	3.03	0.01	21.6374
Clayton, A14	7.10	4.05	0.92	28.7528
Clayton, csch-copula	7.10	7.10	0.01	126.5237
Clayton, coth-copula	7.10	7.96	0.31	51.8123
Clayton, cot-copula	7.10	3.69	0.01	19.1840
A12, A14	3.03	4.05	0.91	18.0486
A12, Gumbel	3.03	4.55	0.99	21.6785
A12, cot-copula	3.03	3.69	0.01	19.1789

Table 5 consists of Akaike information criterion and GOF test statistic values by MLE of copulas parameters. MLE parameter of A15 family are not available since it has not closed form in the second derivative. csch-copula family with parameter $\theta = 4.01$ is selected as a right family to the mentioned crude oil prices since it has the minimum AIC value equal to -823.58. By the GOF test only Clayton family with parameter $\theta = 4.96$ is selected as a reight copula to model the dependency of crude oil prices data.

TABLE 5. Results of the MLE estimation and GOF test

Families	Parameter	AIC	GOF
Gumbel	4.16	-610.55	29.0269
Clayton	4.96	-569.07	16.0366
A12	2.99	-662.7	20.2576
A14	3.73	-132.75	23.425
A15	-	-	-
cot-copula	6.82	-376.12	98.5829
coth-copula	8.12	-633.2	68.9936
csch-copula	4.01	-823.58	29.6509

By considering convex combination of copulas by MLE of copulas parameters, Table 6 shows there are several families which fit to model the dependency of crude oil prices data. Convex combination of Clayton

TABLE 6. Results of copulas convex combination by the MLE estimation and the GOF test

$aC_{\theta_1} + (1-a)C_{\theta_2}$	θ_1	θ_2	a	GOF
Clayton, Gumbel	4.96	4.16	0.64	10.9009
Clayton, A12	4.96	2.99	0.69	9.2518
Clayton, A14	4.96	3.73	0.87	11.3295
Clayton, csch-copula	4.96	6.82	0.90	16.1664
Clayton, coth-copula	4.96	8.11	0.90	16.0820
Clayton, cot-copula	4.96	4.01	0.96	16.0289
A12, A14	2.99	3.73	0.88	17.5999
A12, Gumbel	2.99	4.16	0.93	20.1785
A12, cot-copula	2.99	4.01	0.99	20.3172

copula by Gumbel, A12, A14, csch-copula, coth-copula and cot-copula families are again selected as right copulas to fit these data. It is seen that 0.69Clayton+0.31A12 copula has the minimum GOF test statistic value (=9.2518), so it is the best one to model the dependency of crude oil prices data. Clearly, in the convex combination of copulas, quota of the combined copulas are affected by the GOF test statistic values in Table 5. As an example, in Table 5, GOF test statistic values for Clayton and A12 families are 16.0366 and 20.2576 respectively, hence in the convex combination of them quota of the Clayton family is more than A12 family. This fact in the convex combination of A12 and cot-copula families is evident. By results of Table 3, Table 4, Table 5 and Table 6 we summarize that convex combination of copulas improves fitness of them to model dependence of crude oil prices data.

5. Conclusion

By using copulas we try on modeling the dependency between crude oil prices of West Texas Intermediate (WTI) and Brent-Europe. The modeling is considered by several Archimedean copulas and also their convex combinations. To estimate copulas parameters, we rely on non-parametric estimation (Genest et al. [12]) and also maximum likelihood estimation. And to select the right copula we use nonparametric and semi-parametric procedure (Genest et al. [11, 12]), GOF test (Genest et al. [13]) and also Akaike information criterion (AIC). GOF test results show that by the nonparametric estimation, all of the selected AC in Table 3 and also their convex combinations in Table 4 are not able to model the dependency of crude oil prices. While by the maximum likelihood estimation, Clayton family in Table 5 and also convex combinations of Clayton and several other families in Table 6 are able to model the dependency of the mentioned crude oil prices. By results of Table 3, Table 4, Table 5 and Table 6 we summarize that convex combination of the selected copulas improves fitness of them to model dependence of the crude oil prices data.

References

1. M. Al-Harthy, S. Begg and B. Bratvold Reidar, *Copulas: A new technique to model dependence in petroleum decision making*, Journal of Petroleum Science and Engineering, 57 (2007), pp. 195-208.
2. T. Bacigál and M. Komorníková, *Fitting Archimedean copulas to bivariate geodetic data*, COMPSTAT, (2006), pp. 649-656.
3. H. Bal and V. Najjari, *Archimedean copulas family via hyperbolic generator*, GU J. Sci., 26 (2013), pp. 195-200.
4. P. Bertail, P. Doukhan and P. Soulier, *Dependence in Probability and Statistics*, Springer Science, LLC, (2006).
5. R.T. Clemen and T. Reilly, *Correlations and Copulas for Decision and Risk Analysis*, Management Science, 45 (1999), pp. 208-224.
6. S. Çelebioğlu, *Archimedean copulas And An Application*, Selcuk University Journal of Science, 22 (2003), pp. 43-52.
7. E.W. Frees and E.A. Valdez, *Understanding Relationships Using Copulas*, N. Am. Actuar. J., 2 (1998), pp. 1-25.
8. A. Friend and E. Rogge, *Correlation at First Sight, Economic Notes: Review of Banking*, Finance and Monetary Economics, (2004).
9. C. Genest and J. MacKay, *Copules archimédienneset familles de loisbi dimensionnelles dont les margessontdonnées*, Canad. J. Stat., 14 (1986a), pp. 145-159.
10. C. Genest and J. MacKay, *The joy of copula, Bivariate distributions with uniformmarginals*, Amer. Stat., 40 (1986b), pp. 280-285.
11. C. Genest, K. Ghoudi and L.P. Rivest, *A semiparametric estimation procedure of dependence parameters in multivariate families of distributions*, Biometrika, 82 (1995), pp. 543-552.
12. C. Genest and L.P. Rivest, *Statistical Inference Procedures for Bivariate Archimedean copulas*, J. Am. Stat. Assoc., 88:423 (1993), pp. 1034-1043.
13. C. Genest, B. Remillard and D. Beaudion, *Goodness-of-fit tests for copulas: A review and a power study*, Insurance: Mathematics and Economics, 44 (2009), pp. 199-213.
14. L. Hua and H. Joe, *Tail order and intermediate tail dependence of multivariate copulas*, J. Multivariate Anal., 102 (2011), pp. 1454-1471.
15. Z. Hui-Ming, L. Rong and L. Sufang, *Modelling dynamic dependence between crude oil prices and Asia-Pacific stock market returns*, International Review of Economics and Finance, 29 (2014),

pp. 208-223.

16. H. Joe, *Asymptotic efficiency of the two-stage estimation method for copula-based models*, J. Multivariate Anal., 94 (2005), pp. 401-419.
17. H. Joe, *Multivariate Models and Dependence Concepts*, Chapman-Hall, London, (1997).
18. I. Kojadinovic and J. Yan, *Tests of serial independence for continuous multivariate time series based on a Möbius decomposition of the independence empirical copula process*, Ann. Inst. Stat. Math., 63 (2011), pp. 347-373.
19. D.D. Mari and S. Kotz, *Correlation and Dependence*, Imperial College Press, London, U.K, (2001).
20. V. Najjari, T. Bacigál and H. Bal, *An Archimedean copula family with hyperbolic cotangent generator*, Int. J. Unc. Fuzz. Knowl. Based Syst., 22 (2014).
21. V. Najjari, H. Bal and S. Çelebioğlu, *Modeling of Dependence Structures in Meteorological Data Via Archimedean Copulas*, U.P.B. Sci. Bull., Series D, 75:3: (2013), pp. 131-138.
22. V. Najjari, H. Bal, F. Öztürk and İ. Alp, *Stochastic Frontier Models By Copulas and An Application*, U.P.B. Sci. Bull., Series A, 78 (2016).
23. R.B. Nelsen, *An introduction to copulas*, Springer, New York, (2006), (Second edition).
24. A. Pirmoradian and A. Hamzah, *Simulation of Tail Dependence in cot-copula*, Proceedings of the 58th WSC of the ISI, (2011).
25. B. Schweizer, *Thirty years of copulas. In: Dall Aglio G, Kotz S, Salinetti G (eds) Advances in Probability Distributions with Given Marginals*, Kluwer, Dordrecht, (1991), pp. 13-50.
26. P.X-K. Song, *Correlated Data Analysis: Modeling, Analytics, and Applications*, Springer Science, LLC, (2007).
27. H. Tsukahara, *Semiparametric estimation in copula models*, Canad. J. Stat., 33 (2005), pp. 357-375.

Young Researchers and Elite Club, Maragheh branch, Islamic Azad University, Maragheh, Iran.

Email address: vnajjari@iau-maragheh.ac.ir; vnajjari@gazi.edu.tr