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Error Function and Certain Subclasses of Analytic Univalent Functions

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Abstract. In the present investigation, our main aim is to introduce a certain subclass of analytic univalent functions related to the Error function. We discuss the implications of our main results, including the coefficient bound, extreme points, weighted mean, convolution, convexity, and radii properties, as well as any other related properties.

1. Introduction

One of the most widely applied functions studied in recent years is the error function which is used in partial differential equations physics as well as in probability science, statistics and applied mathematics. Properties and a series of inequalities related to this function can be seen in [3]. The error function can appears in most cases due to the normal curve. The inverse of this function was also introduced by Carlitz [4]. Also, because researchers have been able to define the Taylor series of this function as the series form of a normalized analytic function, they paved the way for this function based on analytic univalent functions. The motivation for introducing a certain subclass was based on the error function and its properties.

$$\mathrm{E_r\ f} = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt \tag{1.1}$$

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$$= \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{(2n+1)\, n!}$$
$$= \frac{2}{\sqrt{\pi}} \left(-\frac{z^3}{3} + \frac{z^5}{10} - \frac{z^7}{42} + \cdots \right),$$

was introduced in [1] and was studied in [3, 5] and [7](See also [2]).

After modification of (1.1), Ramachandran et al. [12] introduced the error function

$$\mathrm{E_r} f(z) = z + \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(2n-1)(n-1)!} z^n, \quad (z \in \mathbb{C} : |z| < 1). \tag{1.2}$$

The set of all analytic functions in the open unit disk that are equal to 0 at $z = 0$ and the derivative of these functions is equal to one at $z = 0$ (called this property is normalized) is as follows.

$$g(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad (z \in \mathbb{C} : |z| < 1). \tag{1.3}$$

Furthermore, $\mathcal{N}$ is a subclass of the above form by changing with negative coefficients and are of the type

$$g(z) = z - \sum_{n=2}^{\infty} a_n z^n, \qquad (a_n \geq 0). \tag{1.4}$$

The Hadamard product (convolution) $g * h$ for $h(z) = z + \sum\limits_{n=2}^{\infty} b_n z^n$ and $g(z)$ to form (1.3) $(g * h)(z) = z + \sum\limits_{n=2}^{\infty} a_n b_n z^n$ for more details see [6].

Now, we consider the functions

$$\begin{aligned} Q(z) &= [(2z - \mathrm{E_r} f) * (2z - \mathrm{E_r} f)](z) \\ &= z - \sum_{n=2}^{\infty} \frac{1}{[(2n-1)(n-1)!]^2} z^n. \end{aligned} \tag{1.5}$$

and

$$\begin{aligned} L_g(z) &= (Q * g)(z) \\ &= z - \sum_{n=2}^{\infty} \frac{1}{[(2n-1)(n-1)!]^2} a_n z^n. \end{aligned} \tag{1.6}$$

where $\mathrm{E_r} f(z)$ is (1.2) and $g(z)$ to form (1.4).

Definition 1.1. A function $g(z)$ of the form (1.4) is in the class $\mathcal{E}_D(v, u)$ if it satisfies the condition

(1.7)
$$\operatorname{Re}\left\{\frac{vz^2\left(L_g(z)\right)''+z\left(L_g(z)\right)'+L_g(z)-z}{vz\left(L_g(z)\right)'+(1-u)L_g(z)}\right\}<D,\quad (0\leq D<1).$$

where $0\leq v,u\leq 1$, $v<u$ and L_g are given by (1.6). Such as this class of univalent functions is studied in [10, 11].

2. Main Results

In the following theorems, we show that the coefficients related to class $\mathcal{E}_D(v,u)$ have a bound and this limit is sharp for the function $G(z)$ defined below, and this class is a convex set.

Theorem 2.1. *$g(z)\in\mathcal{N}$ belong to the class $\mathcal{E}_D(v,u)$ if and only if*
(2.1)
$$\sum_{n=2}^{\infty}\left[\frac{n\left(v(n-1)+1\right)-D(1-u)+1}{\left((n-1)!\,(2n-1)\right)^2}\right]a_n\leq 1-D+D(u-v),\quad (0\leq D<1).$$

Proof. By using (1.6) and (1.7), we conclude

$$\left\{\frac{z-\sum\limits_{n=2}^{\infty}\left[\frac{vn^2+n(1-v)+1}{\left((n-1)!\,(2n-1)\right)^2}\right]a_nz^n}{z(1+v-u)-\sum\limits_{n=2}^{\infty}\left[\frac{1+v-u}{\left((n-1)!\,(2n-1)\right)^2}\right]a_nz^n}\right\}<D,\quad (0\leq D<1).$$

Now, if we consider z as a real number and let $z\to 1^-$, we have

$$[1-D+D(u-v)]-\sum_{n=2}^{\infty}\left[\frac{n\left(v(n-1)+1\right)-D(1-u)+1}{\left((n-1)!\,(2n-1)\right)^2}\right]a_n\geq 0,$$

or

$$\sum_{n=2}^{\infty}\left[\frac{n\left(v(n-1)+1\right)-D(1-u)+1}{\left((n-1)!\,(2n-1)\right)^2}\right]a_n\leq 1-D+D(u-v).$$

Conversely, we suppose that (2.1) holds. We will show that (1.7) is satisfied and so $g\in\mathcal{E}_D(v,u)$.

Using the fact that $|A-(1+D)|<|A+(1-D)|$ if and only if $\operatorname{Re}(A)<D$ for $D\in[0,1)$, it is enough to show that

$$\begin{aligned}X&=\left|\frac{vz^2\left(L_g(z)\right)''+z\left(L_g(z)\right)'+L_g(z)-z}{vz\left(L_g(z)\right)'+(1-u)L_g(z)}-1-D\right|\\&<\left|\frac{vz^2\left(L_g(z)\right)''+z\left(L_g(z)\right)'+L_g(z)-z}{vz\left(L_g(z)\right)'+(1-u)L_g(z)}+1-D\right|\\&=Y\end{aligned}$$

But if $B = vz\left(L_g(z)\right)' + (1-u)\,L_g(z)$, we have

$$X = \frac{1}{|B|}\left|vz^2\left(L_g(z)\right)'' + z\left(L_g(z)\right)' + L_g(z) - z - (1+D)\,B\right|$$

By (1.6) and using triangular inequality we obtain

$$\begin{aligned} X &= \frac{1}{|B|}\Bigg| z\left(1-(1+D)(1+v-u)\right) \\ &\quad - \sum_{n=2}^{\infty} \frac{vn^2 + n(1-v) - (1+v-u)(1+D)}{\left((n-1)!\,(2n-1)\right)^2} a_n z^n \Bigg| \\ &< \frac{|z|}{|B|}\Bigg[1-(1+D)(1+v-D) \\ &\quad + \sum_{n=2}^{\infty} \frac{vn^2 + n(1-v) - (1+v-u)(1+D)}{\left((n-1)!\,(2n-1)\right)^2} a_n |z|^{n-1}\Bigg] \end{aligned}$$

and

$$\begin{aligned} Y &= \frac{1}{|B|}\Bigg| z\left(1+(1-D)(1+v-u)\right) \\ &\quad - \sum_{n=2}^{\infty} \frac{vn^2 + n(1-v) + 1 - (1+v-u)(1-D)}{\left((n-1)!\,(2n-1)\right)^2} a_n z^n \Bigg| \\ &\geq \frac{|z|}{|B|}\Bigg[1+(1-D)(1+v-u) \\ &\quad - \sum_{n=2}^{\infty} \frac{vn^2 + n(1-v) + 1 - (1+v-u)(1-D)}{\left((n-1)!\,(2n-1)\right)^2} a_n |z|^{n-1}\Bigg] \end{aligned}$$

Conversely when $z \in \partial\mathbb{D} = \{z \in \mathbb{C} : \ |z| = 1\}$, if (2.1) is established, $Y - X > 0$ is not unexpected due to the above inequalities, so the proof is complete. □

Remark 2.2. For $G(z)$, which is defined as follows, an equation occurs in (2.1).

$$G(z) = z - \frac{9\left(1-D+D(u-v)\right)}{2v+u-D+3} z^2.$$

Theorem 2.3. *$\mathcal{E}_D(v,u)$ is a convex set.*

Proof. Let $g_j(z)$ $(1 \leq j \leq m)$ belong to the class $\mathcal{E}_D(v,u)$, then $F(z) = \sum\limits_{j=1}^{m} \mu_j g_j(z)$ is also in $\mathcal{E}_D(v,u)$, where $\mu_j \geq 0$ and $\sum\limits_{j=1}^{m} \mu_j = 1$.

(We will prove this). But by the definition of $F(z)$, we obtain

$$F(z) = \sum_{j=1}^{m} \mu_j \left(z - \sum_{n=2}^{\infty} a_{n,j} z^n \right)$$
$$= z - \sum_{n=2}^{\infty} \left(\sum_{j=1}^{m} \mu_j a_{n,j} z^n \right).$$

But from Theorem 2.1, we have

$$\sum_{n=2}^{\infty} \left[\frac{n(v(n-1)+1) - D(1-u) + 1}{((n-1)!(2n-1))^2} \right] \left(\sum_{j=1}^{m} \mu_j a_{n,j} \right)$$
$$= \sum_{j=1}^{m} \sum_{n=2}^{\infty} \left\{ \frac{n(v(n-1)+1) - D(1-u) + 1}{((n-1)!(2n-1))^2} \mu_j a_{n,j} \right\}$$
$$\leq \sum_{j=1}^{m} \mu_j (1 - D + D(u-v)) = 1 - D + D(u-v),$$

which completes the proof. □

3. Investigation of Geometric Properties $\mathcal{E}_D(v,u)$

The class $\mathcal{E}_D(v,u)$ is closed under convolution with respect to v and u. This property is discussed and proved in the following theorem. In this section, the extreme points, weighted mean and radii properties are examined.

Theorem 3.1. *Suppose $g_k(z)$ for $(k=1,2)$ is as $g_k(z) = z - \sum_{n=2}^{\infty} a_{n,k} z^n$. If functions $g_k(z)$ belong to the class $\mathcal{E}_D(v,u)$, then convolutions g_1 and g_2 belong to the classes defined as follows:*

(i) *$(g_1 * g_2)(z)$ belongs to $\mathcal{E}_D(v,u_0)$ where*

$$u_0 \leq \frac{1 - D - \gamma(vn(n-1) + 1 + n) + D(1-v)}{D(\gamma - 1)},$$

$$\gamma = \left[\frac{vn(n-1) + n - D(1-u) + 1}{(n-1)!(2n-1)(1 - D + D(u-v))} \right]^2. \tag{3.1}$$

(ii) *$(g_1 * g_2)(z)$ belongs to $\mathcal{E}_D(v_0,u)$, where*

$$v_0 \leq \frac{\gamma(1 + n - D(1-u)) + D(1-u) - 1}{D + n(n-1)\gamma},$$

and γ is given by (3.1).

Proof. (i) Consider (2.1) and apply the Cauchy-Schwarz inequality to it

$$\sum_{n=2}^{\infty}\frac{vn\,(n-1)+n-D\,(1-u)+1}{\left((n-1)!\,(2n-1)\right)^{2}\left(1-D+D\,(u-v)\right)}\sqrt{a_{n,1}a_{n,2}}\leq 1.$$

Hence, we find the largest u_0 such that

$$\begin{aligned}\sum_{n=2}^{\infty}&\frac{vn\,(n-1)+n-D\,(1-u_0)+1}{\left((n-1)!\,(2n-1)\right)^{2}\left(1-D+D\,(u_0-v)\right)}a_{n,1}a_{n,2}\\&\leq\sum_{n=2}^{\infty}\frac{vn\,(n-1)+n-D\,(1-u)+1}{\left((n-1)!\,(2n-1)\right)^{2}\left(1-D+D\,(u-v)\right)}\sqrt{a_{n,1}a_{n,2}}\\&\leq 1,\end{aligned}$$

or equivalently

$$\sqrt{a_{n,1}a_{n,2}}\leq\frac{\left(1-D+D\,(u_0-v)\right)\left(vn\,(n-1)+n-D\,(1-u)+1\right)}{\left(1-D+D\,(u-v)\right)\left(vn\,(n-1)+n-D\,(1-u_0)+1\right)},\quad n\geq 2.$$

This inequality holds if

$$\begin{aligned}&\frac{\left((n-1)!\,(2n-1)\right)^{2}\left(1-D+D\,(u-v)\right)}{vn\,(n-1)+n-D\,(1-u)+1}\\&\quad\leq\frac{\left(1-D+D\,(u_0-v)\right)\left(vn\,(n-1)+n-D\,(1-u)+1\right)}{\left(1-D+D\,(u-v)\right)\left(vn\,(n-1)+n-D\,(1-u_0)+1\right)},\end{aligned}$$

or equivalently

$$u_0\leq\frac{1-D-\gamma\,(vn\,(n-1)+n+1)+D\,(1-v)}{D\,(\gamma-1)},$$

where γ is given by (3.1), so the proof of (i) is complete.

(ii) Proof (ii) is easily obtained in the same way as proof (i) and with a slight change of details.

□

Theorem 3.2. *Suppose $g_1(z)=z$ and*

$$g_n(z)=z-\frac{\left[(n-1)!\,(2n-1)\right]^{2}\left(1-D+D\,(u-v)\right)}{vn\,(n-1)+n-D\,(1-u)+1},\quad (n=2,3,\cdots)$$

Then $g\in\mathcal{E}_D(v,u)$ if and only if it can be expressed in the form $g(z)=\sum_{n=1}^{\infty}\delta_n g_n(z)$, where $\delta_n\geq 0$ and $\sum_{n=1}^{\infty}\delta_n=1$.

Proof. Let $g(z) = \sum_{n=1}^{\infty} \delta_n g_n(z)$, we will show that g belongs to the class $\mathcal{E}_D(v,u)$:

$$\begin{aligned} g(z) &= \sum_{n=1}^{\infty} \delta_n g_n(z) \\ &= \delta_1 g_1(z) + \sum_{n=2}^{\infty} \delta_n g_n(z) \\ &= z - \sum_{n=2}^{\infty} \frac{[(n-1)!\,(2n-1)]^2\,(1-D+D(u-v))}{vn(n-1)+n-D(1-u)+1} \delta_n z^n. \end{aligned}$$

Since

$$\begin{aligned} &\sum_{n=2}^{\infty} \left(\frac{vn(n-1)+n-D(1-u)+1}{[(n-1)!\,(2n-1)]^2\,(1-D+D(u-v))} \right. \\ &\qquad \left. \times \left[\frac{((n-1)!\,(2n-1))^2\,(1-D+D(u-v))}{vn(n-1)+n-D(1-u)+1} \delta_n \right] \right) \\ &= \sum_{n=2}^{\infty} \delta_n \end{aligned}$$

Because $\sum_{n=2}^{\infty} \delta_n = 1 - \delta_1$ so $\sum_{n=2}^{\infty} \delta_n < 1$, and this was stated in Theorem 2.1. So $g(z)$ belongs to the class $\mathcal{E}_D(v,u)$. One side proved to be complete.

Conversely, suppose that $g \in \mathcal{E}_D(v,u)$. Refer to the theorem 2.1,

$$a_n \le \frac{[(n-1)!\,(2n-1)]^2\,(1-D+D(u-v))}{vn(n-1)+n-D(1-u)+1}, \quad (n = 2,3,\ldots),$$

therefore, by letting

$$\delta_n = \frac{vn(n-1)+n-D(1-u)+1}{[(n-1)!\,(2n-1)]^2\,(1-D+D(u-v))} a_n, \quad (n = 2,3,\ldots),$$

and $\delta_1 = 1 - \sum_{n=2}^{\infty} \delta_n$, we conclude the required result. □

In the following theorem, we express another geometric property of class $\mathcal{E}_D(v,u)$, namely the weight mean.

Theorem 3.3. *If $g_1(z) = z - \sum_{n=2}^{\infty} a_n z^n$ and $g_2(z) = z - \sum_{n=2}^{\infty} b_n z^n$ belongs to the class $\mathcal{E}_D(v,u)$, then the weighted mean of g_1 and g_2 are also in $\mathcal{E}_D(v,u)$.*

Proof. By defining $G_j(z)$ as follows and indicating that it belongs to the class $\mathcal{E}_D(v,u)$, the sentence is obtained.

$$G_j(z) = \frac{1}{2}\left[(1-j)\, g_1(z) + (1+j)\, g_2(z)\right].$$

But, we have

$$G_j(z) = z - \sum_{n=2}^{\infty} \frac{1}{2}\left[(1-j)\, a_n + (1+j)\, b_n\right] z^n.$$

Since g_1 and g_2 are in the class $\mathcal{E}_D(v,u)$, so by Theorem 2.1, we have

$$\begin{aligned}
&\sum_{n=2}^{\infty} \frac{vn(n-1)+n-D(1-u)+1}{((n-1)!\,(2n-1))^2}\left[\frac{1}{2}\left[(1-j)\, a_n + (1+j)\, b_n\right]\right] \\
&\quad = \frac{1}{2}\sum_{n=2}^{\infty} \frac{vn(n-1)+n-D(1-u)+1}{((n-1)!\,(2n-1))^2}(1-j)\, a_n \\
&\qquad + \sum_{n=2}^{\infty} \frac{vn(n-1)+n-D(1-u)+1}{((n-1)!\,(2n-1))^2}(1+j)\, b_n \\
&\quad \le \frac{1}{2}\left[(1-j)(1-D+D(u-v)) + (1+j)(1-D+D(u-v))\right] \\
&\quad = 1-D+D(u-v).
\end{aligned}$$

Hence by Theorem 2.1, $G_j(z) \in \mathcal{E}_D(v,u)$. □

Finally, we show that class $\mathcal{E}_D(v,u)$ satisfies the property of starlike, convexity and also close-to-convex, of certain orders.

Theorem 3.4. *Let $g \in \mathcal{E}_D(v,u)$. Then*

(i) *$g(z)$ is starlike of order γ_1 in $|z| < R_1$, where $\gamma_1 \in [0,1)$ and:*

$$R_1 = \inf_m \left\{ \frac{vm(m-1)+m-D(1-u)+1}{[(m-1)!\,(2m-1)]^2(1-D+D(u-v))}\left(\frac{1-\gamma_1}{m-\gamma_1}\right)\right\}^{\frac{1}{m-1}}.$$

(ii) *$g(z)$ is convex of order γ_2 in $|z| < R_2$, where $\gamma_2 \in [0,1)$ and:*

$$R_2 = \inf_m \left\{ \frac{vm(m-1)+m-D(1-u)+1}{[(m-1)!\,(2m-1)]^2(1-D+D(u-v))}\left(\frac{1-\gamma_2}{m(m-\gamma_2)}\right)\right\}^{\frac{1}{m-1}}.$$

(iii) *(z) is close-to-convex of order γ_3 in $|z| < R_3$, where $\gamma_3 \in [0,1)$ and:*

$$R_3 = \inf_m \left\{ \frac{vm(m-1)+m-D(1-u)+1}{[(m-1)!\,(2m-1)]^2(1-D+D(u-v))}\left(\frac{1-\gamma_3}{m}\right)\right\}^{\frac{1}{m-1}}.$$

Proof. (i) For $0 \leq \gamma_1 < 1$ it must be shown that

$$\left|\frac{zg'}{g} - 1\right| < 1 - \gamma_1.$$

In other words, the equivalent is as follows

$$\left|\frac{zg'(z)}{g(z)} - 1\right| = \left|\frac{\sum\limits_{m=2}^{\infty}(m-1)\,a_m z^{m-1}}{1 - \sum\limits_{m=2}^{\infty} a_m z^{m-1}}\right| \leq \frac{\sum\limits_{m=2}^{\infty}(m-1)\,a_m |z|^{m-1}}{1 - \sum\limits_{m=2}^{\infty} a_m |z|^{m-1}} < 1 - \gamma_1,$$

or

$$\sum_{m=2}^{\infty}\left(\frac{m-\gamma_1}{1-\gamma_1}\right) a_m |z|^{m-1} < 1.$$

By (2.1) (note that instead of n in all relations m is substituted), it is easy to see that the above inequality holds if

$$|z|^{m-1} < \frac{vm(m-1) + m - D(1-u) + 1}{[(m-1)!\,(2m-1)]^2\,(1 - D + D(u-v))}\left(\frac{1-\gamma_1}{m-\gamma_1}\right).$$

This completes the proof of part (i).

(ii) Since g is convex if and only if zg' is starlike, we get the required result (ii).

(iii) We must show that $|g'(z) - 1| \leq 1 - \gamma_3$. But

$$|f'(z) - 1| = \left|\sum_{m=2}^{\infty} m a_m z^{m-1}\right| \leq \sum_{m=2}^{\infty} m a_m |z|^{m-1}.$$

Thus $|f'(z) - 1| < 1 - \gamma_3$ if $\sum\limits_{m=2}^{\infty} \frac{m}{1-\gamma_3} a_m |z|^{m-1} < 1$. But by Theorem 2.1, the last inequality holds if

$$|z|^{m-1} \leq \frac{vm(m-1) + m - D(1-u) + 1}{[(m-1)!\,(2m-1)]^2\,(1 - D + D(u-v))}\left(\frac{1-\gamma_3}{m}\right).$$

Here, the proof ends. □

Based on the results obtained for the class defined in this article, other geometric properties can also be considered. Such as Hankel's second-order determinant, called Fekete-Szegö problems [8, 9]. Which is beyond the scope of this article.

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