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Ternary Generalized Jordan Ring Homomorphisms on Ternary Non-Archimedean Banach Algebras

Ismail Nikoufar[1]* and Hossein Rahimpoor[2]

Abstract. In this paper, we introduce the notion of the ternary generalized Jordan ring homomorphism on ternary non-Archimedean Banach algebras. Utilizing alternative fixed point methods, we establish the generalized Hyers-Ulam stability of ternary generalized Jordan ring homomorphisms on ternary non-Archimedean Banach algebras associated with the generalized additive functions in several variables.

1. Introduction

The first stability problem concerning group homomorphisms was raised by Ulam [30] and an answer to this problem has been given affirmatively in Banach spaces by Hyers [12] in 1941. Since then, many researchers were interested in Ulam-type stability. A generalization of Hyers' problem with unbounded Cauchy differences has been considered by Rassias [27], Bourgin [2] and Găvruta [11]. Moreover, Rassias [28] considered the Cauchy difference controlled by a product of different powers of norm. For the history and various aspects of stability theory we refer to [16, 23].

Bourgin [2, 3] is the first mathematician dealing with stability of the (ring) homomorphism. The stability of the approximate homomorphism, the approximate generalized homomorphism, and the derivation on some suitable Banach spaces was studied by a number of mathematician. For more insights, refer to [1, 8, 9, 20, 24, 25] and references

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therein. Stability of Jordan homomorphisms on Banach algebras was studied by Miura [26]. The concept of approximate n-Jordan homomorphisms from a normed algebra to a Banach algebra was established in [7, 14, 16]. Further properties of these Jordan homomorphisms, as well as their automatic continuity, are discussed in [6, 10].

A linear space A over a scalar field $\mathbb{K}$ with a nontrivial non-Archimedean absolute value $|\cdot|$ is called a non-Archimedean space [21, 29]. Note that by induction $|n| \leq 1$ for every integer n. A non-Archimedean space is a complete non-Archimedean space whenever every Cauchy sequence is convergent. Many applications of this theory can be found in quantum physics, p-adic strings and superstrings [15, 19].

A linear space A over a scalar field $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$ with a trilinear mapping or ternary product $[\cdot,\cdot,\cdot] : A \times A \times A \to A$ is called a ternary algebra whenever it is associative in the sense that

$$[[a,b,c],d,e] = [a,[b,c,d],e] = [a,b,[c,d,e]], \quad \forall a,b,c,d,e \in A.$$

We call a complete ternary non-Archimedean algebra A a ternary non-Archimedean Banach algebra whenever the norm satisfies the following property

$$\|[a,b,c]\| \leq \|a\| \, \|b\| \, \|c\|, \quad \forall a,b,c \in A.$$

Let $(A,[\cdot,\cdot,\cdot])$ and $(B,[\cdot,\cdot,\cdot])$ be two ternary non-Archimedean Banach algebras. A function $f : A \to B$ is a ternary Jordan ring homomorphism or a ternary Jordan additive homomorphism if f is additive and satisfies the following property

$$f([x,x,x]) = [f(x), f(x), f(x)], \quad \forall x \in A.$$

We now introduce the notion of a ternary generalized Jordan ring homomorphism between two ternary non-Archimedean Banach algebras A and B as follows. We say that a function $g : A \to B$ is a ternary generalized Jordan ring homomorphism if g is additive and there exits a ternary Jordan ring homomorphism $f : A \to B$ satisfying

$$\begin{aligned} g([x,x,x]) = {} & \frac{1}{3}[g(x), g(x), f(x)] \\ & + \frac{1}{3}[g(x), f(x), g(x)] + \frac{1}{3}[f(x), g(x), g(x)], \quad \forall x \in A. \end{aligned}$$

It is clear that every ternary Jordan ring homomorphism g is a ternary generalized Jordan ring homomorphism by taking $f = g$ but the converse is not true in general. So, our results can recover the stability of the ternary Jordan ring homomorphism.

Throughout this paper we suppose that $(A,[\cdot,\cdot,\cdot])$ and $(B,[\cdot,\cdot,\cdot])$ represent two ternary non-Archimedean Banach algebras. For convenience and for two given functions $f_1 : A \to B$ and $f_2 : A \to B$ we use the following abbreviations

$$\begin{aligned}
&\Gamma_f(x_1, \ldots, x_m) \\
&\quad := \sum_{k=2}^{m} \left(\sum_{i_1=2}^{k} \sum_{i_2=i_1+1}^{k+1} \cdots \sum_{i_{n-k+1}=i_{m-k}+1}^{m} \right) \\
&\quad \times f\left(\sum_{i=1, i \neq i_1, \ldots, i_{m-k+1}}^{m} x_i - \sum_{r=1}^{m-k+1} x_{i_r} \right) \\
&\quad + f\left(\sum_{i=1}^{m} x_i \right) - 2^{m-1} f(x_1)
\end{aligned}$$

and

$$\begin{aligned}
D_{f_1}(x) &= f_1([x, x, x]) - [f_1(x), f_1(x), f_1(x)], \\
D_{f_1, f_2}(x) &= f_2([x, x, x]) - \frac{1}{3}[f_2(x), f_2(x), f_1(x)] \\
&\quad - \frac{1}{3}[f_2(x), f_1(x), f_2(x)] \\
&\quad - \frac{1}{3}[f_1(x), f_2(x), f_2(x)], \quad \forall x, x_1, \ldots, x_m \in A, m \geq 2.
\end{aligned}$$

In this paper, using fixed point methods, we prove the generalized Hyers-Ulam stability of ternary generalized Jordan ring homomorphisms on ternary non-Archimedean Banach algebras associated with the generalized additive functional equation in several variables

$$\Gamma_f(x_1, \ldots, x_m) = 0,$$

which was introduced by Khodaei et al. [13], see also [22] for this matter. Note that each function satisfying this generalized multi-variate functional equation is additive [13].

2. Nearly Ternary Generalized Jordan Ring Homomorphisms

In this section, our focus lies on the generalized Hyers-Ulam stability of ternary generalized Jordan ring homomorphisms on ternary non-Archimedean Banach algebras.

Consider a generalized metric space (X, d), where the range of the generalized metric is allowed to include infinity Now, let's recall a fixed point theory result by Diaz and Margolis, which we will apply in our main results.

Theorem 2.1 ([5, 17, 18])**.** *Let (X,d) be a complete generalized metric space and let $H : X \to X$ be a strictly contractive mapping with Lipschitz constant L. Then for each $x \in X$, either*

$$d\left(H^m x, H^{m+1}x\right) = \infty, \quad \forall m \geq 0$$

or there exists a natural number k_0 such that

1. $d\left(H^m x, H^{m+1}x\right) < \infty$ *for all* $m \geq k_0$,
2. $\{H^m x\}$ *is convergent to a fixed point* a^* *of* H,
3. a^* *is the unique fixed point of* H *in the set*
$$\Lambda = \left\{a \in X : d\left(H^{k_0}x, a\right) < \infty\right\},$$
4. $d\left(a, a^*\right) \leq \dfrac{1}{1-L} d\left(a, Ha\right)$ *for all* $a \in \Lambda$.

In the following two theorems we give the conditions which imply the stability of ternary generalized Jordan ring homomorphisms on ternary non-Archimedean Banach algebras.

Theorem 2.2. *Let $f_1 : A \to B$ and $f_2 : A \to B$ be two functions for which there exist some functions $\varphi : A^m \to [0,\infty)$ and $\psi : A \to [0,\infty)$ such that*

$$\max\left\{\left\|\Gamma_{f_1}\left(x_1,\ldots,x_m\right)\right\|, \left\|\Gamma_{f_2}\left(x_1,\ldots,x_m\right)\right\|\right\} \leq \varphi\left(x_1,\ldots,x_m\right), \quad \forall x_1,\ldots,x_m \in A, \tag{2.1}$$

$$\max\left\{\left\|D_{f_1}(x)\right\|, \left\|D_{f_1,f_2}(x)\right\|\right\} \leq \psi(x), \quad \forall x \in A. \tag{2.2}$$

If there exists a constant $0 < L < 1$ such that

$$\varphi\left(\frac{x_1}{2},\ldots,\frac{x_m}{2}\right) \leq \frac{L}{|2|}\varphi\left(x_1,\ldots,x_m\right), \quad \forall x_1,\ldots,x_m \in A, \tag{2.3}$$

$$\psi\left(\frac{x}{2}\right) \leq \frac{L}{|2|}\psi(x), \quad \forall x \in A, \tag{2.4}$$

then there exist a unique ternary Jordan ring homomorphism $g_1 : A \to B$ and a unique ternary generalized Jordan ring homomorphism $g_2 : A \to B$ such that

$$\left\|f_1(x) - g_1(x)\right\| \leq \frac{1}{|2|^{m-1}}\frac{L}{1-L}\varphi\left(x,x,0,\ldots,0\right), \tag{2.5}$$

$$\left\|f_2(x) - g_2(x)\right\| \leq \frac{1}{|2|^{m-1}}\frac{L}{1-L}\varphi\left(x,x,0,\ldots,0\right), \quad \forall x \in A. \tag{2.6}$$

Proof. Put $x_1 = x_2 = x$ and $x_3 = x_4 = \ldots = x_n = 0$ in (2.1) to reach

$$\left\|\frac{\alpha}{2}f_i(2x) - \alpha f_i(x)\right\| \leq \varphi\left(x,x,0,\ldots,0\right), \quad \forall x \in A, \tag{2.7}$$

where $\alpha = 2^{m-1}$, $m \geq 2$. By substituting $\frac{x}{2}$ with x and then multiplying both sides of the resulting inequalities by $|2|$ we get

$$\left\| 2f_i\left(\frac{x}{2}\right) - f_i(x) \right\| \leq \frac{|2|}{|\alpha|}\varphi\left(\frac{x}{2}, \frac{x}{2}, 0, \ldots, 0\right). \tag{2.8}$$

Using (2.3) and (2.8) one has

$$\left\| 2f_i\left(\frac{x}{2}\right) - f_i(x) \right\| \leq \frac{L}{|\alpha|}\varphi(x, x, 0, \ldots, 0). \tag{2.9}$$

Consider $\Omega = \{g : g : A \to B,\ g \text{ is a map}\}$ and define the generalized metric d on Ω for $g, h \in \Omega$ as follows

$$d(g,h) = \inf\{t \in (0,\infty) :\ \|g(x) - h(x)\| \leq t\varphi(x, x, 0, \ldots, 0),\ x \in A\}.$$

The space (Ω, d) is a generalized complete metric space [4]. Let the function $F : \Omega \to \Omega$ defined by $(Fh)(x) = 2h\left(\frac{x}{2}\right)$ for all $x \in A$ and all $h \in \Omega$. Use [5, Lemma 1.3] to see that F is a strictly contractive mapping with the Lipschitz constant L. It follows from (2.9) that $d(Ff_i, f_i) \leq \frac{L}{|\alpha|}$. Therefore, according to Theorem 2.1, the sequence $\{F^n f_i\}$ converges to a fixed point g_i such that

$$g_i(x) = \lim_{n\to\infty} 2^n f_i\left(\frac{x}{2^n}\right), \quad \forall x \in A. \tag{2.10}$$

Note that g_i is the unique fixed point of F in the set $\Delta_i = \{g \in \Omega : d(f_i, g) < \infty\}$ for each $i \in \{1, 2\}$ and

$$d(g_i, f_i) \leq \frac{1}{1-L} d(Ff_i, f_i) \leq \frac{L}{|\alpha|(1-L)}.$$

This entails that

$$\|f_i(x) - g_i(x)\| \leq \frac{1}{|2|^{m-1}} \frac{L}{1-L}\varphi(x, x, 0, \ldots, 0), \quad \forall x \in A.$$

This implies that the inequalities (2.5) and (2.6) hold for all $x \in A$. On the other hand, it follows from (2.1) and (2.3) that

$$\begin{aligned}
\|\Gamma_{g_i}(x_1, \ldots, x_m)\| &= \lim_{n\to\infty} |2|^n \left\| \Gamma_{f_i}\left(\frac{x_1}{2^n}, \ldots, \frac{x_m}{2^n}\right) \right\| \\
&\leq \lim_{n\to\infty} |2|^n \varphi\left(\frac{x_1}{2^n}, \ldots, \frac{x_m}{2^n}\right) \\
&\leq \lim_{n\to\infty} |2|^n \frac{L^n}{|2|^n}\varphi(x_1, \ldots, x_m) \\
&= 0.
\end{aligned}$$

Hence, $\Gamma_{g_i}(x_1, \ldots, x_m) = 0$ for all $x_1, \ldots, x_m \in A$. This indicates that each g_i is additive. By using (2.2) and (2.4) we deduce

$$\begin{aligned}
\|D_{g_1}(x)\| &= \|g_1([x,x,x]) - [g_1(x), g_1(x), g_1(x)]\| \\
&= \lim_{n\to\infty} |2|^{3n} \left\| f_1\left(\left[\frac{x}{2^n}, \frac{x}{2^n}, \frac{x}{2^n}\right]\right) - \left[f_1\left(\frac{x}{2^n}\right), f_1\left(\frac{x}{2^n}\right), f_1\left(\frac{x}{2^n}\right)\right] \right\| \\
&= \lim_{n\to\infty} |2|^{3n} \left\| D_{f_1}\left(\frac{x}{2^n}\right) \right\| \\
&\leq \lim_{n\to\infty} |2|^{3n} \psi\left(\frac{x}{2^n}\right) \\
&\leq \lim_{n\to\infty} |2|^{3n} \frac{L^n}{|2|^n} \psi(x) \\
&= 0, \quad \forall x \in A.
\end{aligned}$$

Therefore, g_1 is a ternary Jordan ring homomorphism. It follows from (2.2) and (2.4) that

$$\begin{aligned}
\|D_{g_1,g_2}(x)\| = &\left\| g_2([x,x,x]) - \frac{1}{3}[g_2(x), g_2(x), g_1(x)] \right. \\
&\left. - \frac{1}{3}[g_2(x), g_1(x), g_2(x)] - \frac{1}{3}[g_1(x), g_2(x), g_2(x)] \right\| \\
= &\lim_{n\to\infty} |2|^{3n} \left\| f_2\left(\left[\frac{x}{2^n}, \frac{x}{2^n}, \frac{x}{2^n}\right]\right) \right. \\
&- \frac{1}{3}\left[f_2\left(\frac{x}{2^n}\right), f_2\left(\frac{x}{2^n}\right), f_1\left(\frac{x}{2^n}\right)\right] \\
&- \frac{1}{3}\left[f_2\left(\frac{x}{2^n}\right), f_1\left(\frac{x}{2^n}\right), f_2\left(\frac{x}{2^n}\right)\right] \\
&\left. - \frac{1}{3}\left[f_1\left(\frac{x}{2^n}\right), f_2\left(\frac{x}{2^n}\right), f_2\left(\frac{x}{2^n}\right)\right] \right\| \\
= &\lim_{n\to\infty} |2|^{3n} \left\| D_{f_1,f_2}\left(\frac{x}{2^n}\right) \right\| \\
\leq &\lim_{n\to\infty} |2|^{3n} \psi\left(\frac{x}{2^n}\right) \\
\leq &\lim_{n\to\infty} |2|^{3n} \frac{L^n}{|2|^n} \psi(x) \\
= &\ 0, \quad \forall x \in A.
\end{aligned}$$

This entails that g_2 is a ternary generalized Jordan ring homomorphism. This completes the proof. $\square$

Remark 2.3. We remark that in the proof of Theorem 2.2 the equality (2.10) implies that

$$g_i([x,x,x]) = \lim_{n\to\infty} 2^{3n} f_i\left(\left[\frac{x}{2^n}, \frac{x}{2^n}, \frac{x}{2^n}\right]\right), \quad \forall x \in A.$$

Since $[x,x,x] \in A$ and the ternary product $[\cdot,\cdot,\cdot]$ is trilinear we find that

$$\begin{aligned} g_i\left([x,x,x]\right) &= \lim_{k\to\infty} 2^k f_i\left(\frac{[x,x,x]}{2^k}\right) \\ &= \lim_{k\to\infty} 2^k f_i\left(\left[\frac{x}{2^{k/3}},\frac{x}{2^{k/3}},\frac{x}{2^{k/3}}\right]\right) \\ &= \lim_{n\to\infty} 2^{3n} f_i\left(\left[\frac{x}{2^n},\frac{x}{2^n},\frac{x}{2^n}\right]\right). \end{aligned}$$

Theorem 2.4. *Let $f_1 : A \to B$ and $f_2 : A \to B$ be two functions for which there exist some functions $\varphi : A^m \to [0,\infty)$ and $\psi : A \to [0,\infty)$ satisfying* (2.1) *and* (2.2)*. If there exists a constant $0 < L < 1$ such that*

$$\varphi\left(2x_1,\ldots,2x_m\right) \leq |2|L\varphi\left(x_1,\ldots,x_m\right), \tag{2.11}$$

$$\psi\left(2x\right) \leq |2|^3 L\psi\left(x\right), \quad \forall x_1,\ldots,x_m \in A,\ i \in \{1,2\}, \tag{2.12}$$

then there exist a unique ternary Jordan ring homomorphism $g_1 : A \to B$ and a unique ternary generalized Jordan ring homomorphism $g_2 : A \to B$ such that

$$\|f_i\left(x\right) - g_i\left(x\right)\| \leq \frac{1}{|2|^{m-1}}\frac{L}{1-L}\varphi\left(x,x,0,\ldots,0\right), \quad \forall x \in A.$$

Proof. Dividing both sides of (2.7) by $|\alpha|$ one has

$$\left\|\frac{1}{2}f_i\left(2x\right) - f_i\left(x\right)\right\| \leq \frac{1}{|\alpha|}\varphi\left(x,x,0,\ldots,0\right).$$

Using (2.11) and since $|2| < 1$ we deduce

$$\begin{aligned} \left\|\frac{1}{2}f_i\left(2x\right) - f_i\left(x\right)\right\| &\leq \frac{|2|L}{|\alpha|}\varphi\left(\frac{x}{2},\frac{x}{2},0,\ldots,0\right) \\ &\leq \frac{L}{|\alpha|}\varphi\left(\frac{x}{2},\frac{x}{2},0,\ldots,0\right). \end{aligned} \tag{2.13}$$

Define the generalized metric d on Ω, the set of all mappings $g,h : A \to B$, as follows

$$d\left(g,h\right) = \inf\left\{t \in (0,\infty) : \|g\left(x\right) - h\left(x\right)\| \leq t\varphi\left(\frac{x}{2},\frac{x}{2},0,\ldots,0\right),\ x \in A\right\}.$$

This space is a generalized complete metric space [4]. Let the function $G : \Omega \to \Omega$ defined by $\left(Gh\right)\left(x\right) = \frac{1}{2}h\left(2x\right)$ for all $x \in A$ and all $h \in \Omega$. The function G is a strictly contractive mapping with the Lipschitz constant L by [5, Lemma 1.2]. It follows from (2.13) that $d\left(Gf_i, f_i\right) \leq \frac{L}{|\alpha|}$. Therefore, according to Theorem 2.1, the sequence $\{G^n f_i\}$ converges to a fixed point g_i such that

$$g_i\left(x\right) = \lim_{n\to\infty}\frac{1}{2^n}f_i\left(2^n x\right).$$

The function g_i is the unique fixed point of G in the set $\Delta_i = \{g \in \Omega : d(f_i, g) < \infty\}$ for each $i \in \{1,2\}$ and

$$d(g_i, f_i) \leq \frac{1}{1-L} d(Gf_i, f_i) \leq \frac{L}{|\alpha|(1-L)}.$$

This means that

$$\|f_i(x) - g_i(x)\| \leq \frac{1}{|2|^{m-1}} \frac{L}{1-L} \varphi\left(\frac{x}{2}, \frac{x}{2}, 0, \ldots, 0\right), \quad \forall x \in A$$

and hence the desired results follow. Now we show that each g_i is additive and g_2 is a ternary generalized Jordan ring homomorphism. Using (2.1) and (2.11) one can deduce

$$\begin{aligned}
\|\Gamma_{g_i}(x_1, \ldots, x_m)\| &= \lim_{n\to\infty} \frac{1}{|2|^n} \|\Gamma_{f_i}(2^n x_1, \ldots, 2^n x_m)\| \\
&\leq \lim_{n\to\infty} \frac{1}{|2|^n} |2|^n L^n \varphi(x_1, \ldots, x_m) \\
&= 0.
\end{aligned}$$

We observe that g_i is additive. By applying (2.2) and (2.12) we get

$$\begin{aligned}
\|D_{g_1}(x)\| &= \|g_1([x,x,x]) - [g_1(x), g_1(x), g_1(x)]\| \\
&= \lim_{n\to\infty} \frac{1}{|2|^{3n}} \|f_1([2^n x, 2^n x, 2^n x]) - [f_1(2^n x), f_1(2^n x), f_1(2^n x)]\| \\
&= \lim_{n\to\infty} \frac{1}{|2|^{3n}} \|D_{f_1}(2^n x)\| \\
&\leq \lim_{n\to\infty} \frac{1}{|2|^{3n}} \psi(2^n x) \\
&\leq \lim_{n\to\infty} \frac{1}{|2|^{3n}} |2|^{3n} L^n \psi(x) \\
&= 0, \quad \forall x \in A.
\end{aligned}$$

Therefore, g_1 is a ternary Jordan ring homomorphism. By using (2.2) and (2.12) we have

$$\begin{aligned}
\|D_{g_1,g_2}(x)\| = \bigg\| & g_2([x,x,x]) - \frac{1}{3}[g_2(x), g_2(x), g_1(x)] \\
& - \frac{1}{3}[g_2(x), g_1(x), g_2(x)] - \frac{1}{3}[g_1(x), g_2(x), g_2(x)] \bigg\| \\
= \lim_{n\to\infty} \frac{1}{|2|^{3n}} \bigg\| & f_2([2^n x, 2^n x, 2^n x]) \\
& - \frac{1}{3}[f_2(2^n x), f_2(2^n x), f_1(2^n x)]
\end{aligned}$$

$$
\begin{aligned}
&- \frac{1}{3}\left[f_2\left(2^n x\right), f_1\left(2^n x\right), f_2\left(2^n x\right)\right] \\
&- \frac{1}{3}\left[f_1\left(2^n x\right), f_2\left(2^n x\right), f_2\left(2^n x\right)\right] \Bigg\| \\
&= \lim_{n\to\infty} \frac{1}{|2|^{3n}} \left\|D_{f_1,f_2}\left(2^n x\right)\right\| \\
&\leq \lim_{n\to\infty} \frac{1}{|2|^{3n}} \psi\left(2^n x\right) \\
&\leq \lim_{n\to\infty} \frac{1}{|2|^{3n}} |2|^{3n} L^n \psi(x) \\
&= 0, \quad \forall x \in A.
\end{aligned}
$$

This entails that g_2 is a ternary generalized Jordan ring homomorphism. This completes the proof. □

We now verify the Hyers-Ulam-Rassias stability of the ternary generalized Jordan ring homomorphisms on ternary non-Archimedean Banach algebras.

Corollary 2.5. *Let p, q and s be non-negative real numbers with $s < p < 1$. Let $f_1 : A \to B$ and $f_2 : A \to B$ be two functions, $\in \{1,2\}$, $m \geq 2$ and*

$$
\begin{aligned}
&\left\|\Gamma_{f_i}\left(x_1,\ldots,x_m\right)\right\| \leq q\left(\|x_1\|^p + \ldots + \|x_m\|^p\right), \quad \forall x_1,\ldots,x_m \in A, \\
&\max\{\left\|D_{f_1}(x)\right\|, \left\|D_{f_1,f_2}(x)\right\|\} \leq q\|x\|^s, \quad \forall x \in A.
\end{aligned}
$$

Then there exist a unique ternary ring homomorphism $g_1 : A \to B$ and a unique ternary generalized ring homomorphism $g_2 : A \to B$ such that

$$
\|f_i(x) - g_i(x)\| \leq \frac{2q|2|^{1-p}}{|2|^{m-1} - |2|^{m-p}} \|x\|^p, \quad \forall x \in A.
$$

Proof. Define

$$
\begin{aligned}
\varphi\left(x_1,\ldots,x_m\right) &:= q\left(\|x_1\|^p + \ldots + \|x_m\|^p\right), \\
\psi(x) &:= q\|x\|^s.
\end{aligned}
$$

It is enough to choose $L = |2|^{1-p}$ and apply Theorem 2.2. Note that in a non-Archimedean space and for a non-Archimedean absolute value by condition 3 we know that $|2| \leq 1$. □

Corollary 2.6. *Let p, q and s be non-negative real numbers with $1 < p < s - 2$. Let $f_1 : A \to B$ and $f_2 : A \to B$ be two functions, $i \in \{1,2\}$ and $m \geq 2$ such that*

$$
\begin{aligned}
&\left\|\Gamma_{f_i}\left(x_1,\ldots,x_m\right)\right\| \leq q\left(\|x_1\|^p + \ldots + \|x_m\|^p\right), \quad \forall x_1,\ldots,x_m \in A, \\
&\max\left\{\left\|D_{f_1}(x)\right\|, \left\|D_{f_1,f_2}(x)\right\|\right\} \leq q\|x\|^s, \quad \forall x \in A.
\end{aligned}
$$

Then there exist a unique ternary Jordan ring homomorphism $g_1 : A \to B$ and a unique ternary generalized Jordan ring homomorphism $g_2 : A \to B$ such that

$$\|f_i(x) - g_i(x)\| \leq \frac{2q}{|2|^{m-p} - |2|^{m-1}} \|x\|^p, \quad \forall x \in A.$$

Proof. Consider

$$\varphi(x_1, \ldots, x_m) := q\left(\|x_1\|^p + \ldots + \|x_m\|^p\right),$$
$$\psi(x) := q\|x\|^s.$$

It is sufficient to choose $L = |2|^{p-1}$ and then apply Theorem 2.4. Note that in a non-Archimedean space and for a non-Archimedean absolute value by condition 3 we know that $|2| \leq 1$. □

Conflict of Interest

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