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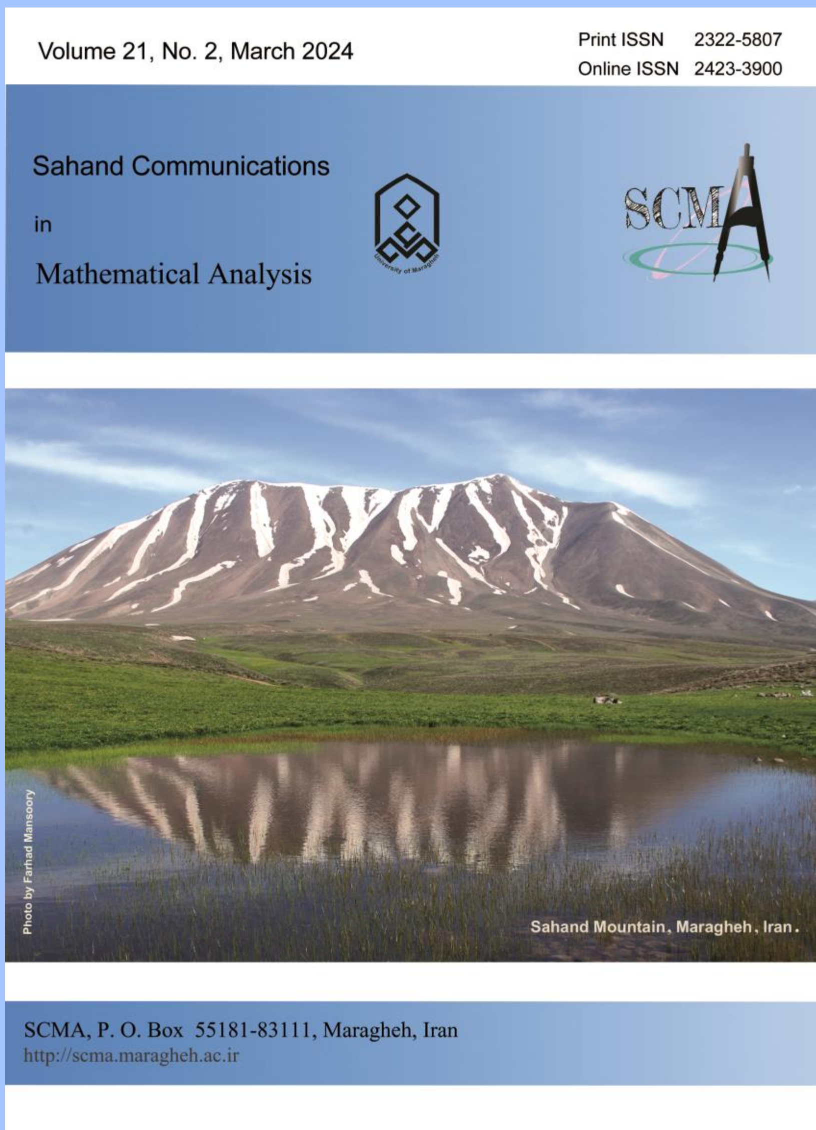

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Some New Results for the $\mathscr{M}$-Transform Involving the Incomplete I-Functions

Sanjay Bhatter[1], Nishant[2] and Sunil Dutt Purohit[3*]

Abstract. Integral transformations are crucial for solving a variety of actual issues. The right choice of integral transforms aids in simplifying both integral and differential problems into a solution-friendly algebraic equation. In this paper, $\mathscr{M}$-transform is applied to establish the image formula for the multiplication of a family of polynomials and incomplete I-functions. Additionally, we discovered image formulations for a few significant and valuable cases of incomplete I-functions. Numerous previously unknown and novel conclusions can be reached by assigning specific values to the parameters involved in the primary conclusions drawn in this study.

1. Introduction and Preliminaries

For many decades, integral transforms [1, 11] have played a precious role in solving many differential and integral equations. Using an appropriate integral transform helps to reduce differential and integral operator, from a considered domain into multiplication operators in another domain.

The classical integral transforms used in solving differential equations, integral equations, in analysis and the theory of functions are the Laplace transform, the Fourier integral transform and the Mellin transform [10, 15, 20]. Recently, Akel [17] introduced the following $\mathscr{M}$-transform:

$$\mathscr{M}_{\lambda,\mu}[g(x)](\mathfrak{p},\mathfrak{q},\mathfrak{r}) = \int_0^{\infty} \frac{e^{-\mathfrak{p}x-\frac{\mathfrak{q}}{x}}}{(x^{\mu}+\mathfrak{r}^{\mu})^{\lambda}}\, g(\mathfrak{r}x)\, dx, \tag{1.1}$$

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with $\lambda \in \mathbb{C}$, $\Re(\lambda) > 0, \mu \in \mathbb{N}$, $\mathfrak{p}, \mathfrak{q} \in \mathbb{C}$, and $\mathfrak{r} \in \mathbb{R}^+$ are the transform variables.

The $\mathscr{M}$-transform is closely linked to the well-known integral transforms Laplace, natural, Sumudu, classical Stieltjes and Srivastava-Luo-Raina M-transform as stated in (1.1) equation.

Put $\lambda = \mathfrak{q} = 0$ and $\mathfrak{r} = 1$ in (1.1), we obtain the Laplace transform [13] and it is defined by:

$$L[g(x)](\mathfrak{p}) = \int_0^\infty e^{-\mathfrak{p}x} g(x)\, dx, \quad \Re(\mathfrak{p}) > 0. \tag{1.2}$$

From equation (1.1) and (1.2), we have the following Laplace-$\mathscr{M}_{\lambda,\mu}[g(x)](\mathfrak{p}, \mathfrak{q}, \mathfrak{r})$ transfrom duality relation (see [17]):

$$L[g(x)](\mathfrak{p}) = \mathscr{M}_{0,\mu}[g(x)](\mathfrak{p}, 0, 1), \quad \Re(\mathfrak{p}) > 0, \tag{1.3}$$

and

$$\begin{aligned}\mathscr{M}_{\lambda,\mu}[g(x)](\mathfrak{p}, \mathfrak{q}, \mathfrak{r}) &= L\left[\frac{e^{-\mathfrak{q}/x} g(\mathfrak{r}x)}{(x^\mu + \mathfrak{r}^\mu)^\lambda}\right](\mathfrak{p}) \\ &= \frac{1}{\mathfrak{r}} L\left[\frac{e^{-\mathfrak{q}\mathfrak{r}/x} g(x)}{\left(\frac{x^\mu}{\mathfrak{r}^\mu} + \mathfrak{r}^\mu\right)^\lambda}\right]\left(\frac{\mathfrak{p}}{\mathfrak{r}}\right), \quad \mathfrak{p}, \mathfrak{q}, \mathfrak{r} > 0.\end{aligned}$$

Put $\lambda = \mathfrak{q} = 0$, we obtain the Natural transform [4, 6] and it is described as:

$$N[g(x)](\mathfrak{p}, \mathfrak{q}) = \int_0^\infty e^{-\mathfrak{p}x} g(\mathfrak{q}x) dx, \quad \mathfrak{p}, \mathfrak{q} > 0. \tag{1.4}$$

From equations (1.1) and (1.4), we have the following Natural- $\mathscr{M}_{\lambda,\mu}[g(x)](\mathfrak{p}, \mathfrak{q}, \mathfrak{r})$ transform duality relation (see [17]):

$$N[g(x)](\mathfrak{p}, \mathfrak{r}) = \mathscr{M}_{0,\mu}[g(x)](\mathfrak{p}, 0, \mathfrak{r}), \quad \mathfrak{p}, \mathfrak{r} > 0, \tag{1.5}$$

and

$$\mathscr{M}_{\lambda,\mu}[g(x)](\mathfrak{p}, \mathfrak{q}, \mathfrak{r}) = N\left[\frac{e^{-\mathfrak{q}\mathfrak{r}/x} g(x)}{\left(\frac{x^\mu}{\mathfrak{r}^\mu} + \mathfrak{r}^\mu\right)^\lambda}\right](\mathfrak{p}, \mathfrak{r}), \quad \mathfrak{p}, \mathfrak{q}, \mathfrak{r} > 0.$$

Sumudu transform [5] is described by:

$$S[g(x)](\mathfrak{q}) = \int_0^\infty e^{-x} g(\mathfrak{q}x) dx, \quad \mathfrak{q} > 0. \tag{1.6}$$

From equations (1.1) and (1.6), we have the following Sumudu- $\mathscr{M}_{\lambda,\mu}[g(x)](\mathfrak{p}, \mathfrak{q}, \mathfrak{r})$ transform duality relation (see [17]):

$$S[g(x)](\mathfrak{r}) = \mathscr{M}_{0,\mu}[g(x)](1, 0, \mathfrak{r}), \quad \mathfrak{r} > 0, \tag{1.7}$$

and

$$\mathscr{M}_{\lambda,\mu}[g(x)](\mathfrak{p},\mathfrak{q},\mathfrak{r}) = \frac{1}{\mathfrak{p}} S\left[\frac{e^{-\mathfrak{q}\mathfrak{r}/x} g(x)}{\left(\frac{x^{\mu}}{\mathfrak{r}^{\mu}} + \mathfrak{r}^{\mu}\right)^{\lambda}}\right]\left(\frac{\mathfrak{r}}{\mathfrak{p}}\right), \quad \mathfrak{p},\mathfrak{q},\mathfrak{r} > 0.$$

Srivsatava-Luo-Raina M-transform [8] is defined by:

$$M_{\lambda,\mu}[g(x)](\mathfrak{p},\mathfrak{r}) = \int_0^{\infty} \frac{e^{-\mathfrak{p}x} g(\mathfrak{r}x)}{(x^{\mu} + \mathfrak{r}^{\mu})^{\lambda}} dx, \quad \lambda \in \mathbb{C},\ \Re(\lambda) \geq 0,\ \mu \in \mathbb{N}. \tag{1.8}$$

From equations (1.1) and (1.8), we have the following Srivsatava-Luo-Raina M-$\mathscr{M}_{\lambda,\mu}[g(x)](\mathfrak{p},\mathfrak{q},\mathfrak{r})$ transform duality relation (see [17]):

$$\mathscr{M}_{\lambda,\mu}[g(x)](\mathfrak{p},0,\mathfrak{r}) = M_{\lambda,\mu}[g(x)](\mathfrak{p},\mathfrak{r}), \tag{1.9}$$

with $\lambda \in \mathbb{C}$, $\Re(\lambda) > 0$, $\mu \in \mathbb{N}, \mathfrak{p} \in \mathbb{C}$, and $\mathfrak{r} \in \mathbb{R}^+$.

Another special case of the integral transform (1.1), when $\mathfrak{p}$, $\mathfrak{q} = 0$, is a generalization of the Stieltjes transform, which was studied in (see [9, 12]).

$$St_{\lambda}[g(x)](\mathfrak{r}) = \int_0^{\infty} \frac{g(x)}{(x+\mathfrak{r})^{\lambda}} dx, \tag{1.10}$$

with $\lambda \in \mathbb{C}, \Re(\lambda) > 0$ and $\mathfrak{r} \in \mathbb{C}/(-\infty,0)$.

Lemma 1.1. *For $\lambda,\mathfrak{p},\mathfrak{q} \in \mathbb{C}; \mathfrak{r} \in \mathbb{R}^+, \Re(\lambda) \geq 0, \mu \in \mathbb{N}$ and $a,\nu > 0$, then we have the following assertion:*

(1.11)

$$\mathscr{M}_{\lambda,\mu}\left[t^{v-1}\right](\mathfrak{p},\mathfrak{q},\mathfrak{r}) = \frac{\mathfrak{r}^{v-\mu\lambda-1}\,\mathfrak{p}^{-v}}{\mu\,\Gamma(\lambda)} H^{2,\,1}_{1,\,2}\left[\mathfrak{p}\mathfrak{r}\ \middle|\ \begin{matrix} \left(1,\ \frac{1}{\mu}\right) \\ (v,1)_{\mathfrak{p}\mathfrak{q}}, \left(\lambda, \frac{1}{\mu}\right)\end{matrix}\right],$$

$$\begin{aligned} &\mathscr{M}_{\lambda,\mu}\left[e^{-a\,t}\right](\mathfrak{p},\mathfrak{q},\mathfrak{r}) \\ &= \frac{\mathfrak{r}^{-\mu\lambda}}{\mu\,(\mathfrak{p}+a\mathfrak{r})\,\Gamma(\lambda)} H^{2,\,1}_{1,\,2}\left[\mathfrak{r}\,(\mathfrak{p}+a\mathfrak{r})\ \middle|\ \begin{matrix} \left(1,\ \frac{1}{\mu}\right) \\ (1,1)_{(\mathfrak{p}+a\mathfrak{r})\mathfrak{q}}, \left(\lambda, \frac{1}{\mu}\right)\end{matrix}\right], \end{aligned} \tag{1.12}$$

and

$$\begin{aligned} &\mathscr{M}_{\lambda,\mu}[t^{v-1}\,e^{-a\,t}](\mathfrak{p},\mathfrak{q},\mathfrak{r}) \\ &= \frac{\mathfrak{r}^{\mathfrak{v}-\mu\lambda-1}}{\mu\,(\mathfrak{p}+a\mathfrak{r})^{v}\,\Gamma(\lambda)} H^{2,\,1}_{1,\,2}\left[\mathfrak{r}\,(\mathfrak{p}+a\mathfrak{r})\ \middle|\ \begin{matrix} \left(1,\ \frac{1}{\mu}\right) \\ (v,1)_{\mathfrak{p}\mathfrak{q}}, \left(\lambda, \frac{1}{\mu}\right)\end{matrix}\right], \end{aligned} \tag{1.13}$$

where $H^{2,\,1}_{1,\,2}[z]$ is the extended H-function described in the following way:

$$H\,^{2,\,1}_{1,\,2}(\,z;\,g) = H^{2,\,1}_{1,\,2}\left[z;\,g\ \middle|\ \begin{matrix} (b,\ \varrho) \\ (c_1,\varphi_1)_e, (c_2,\varphi_2)\end{matrix}\right]$$

$$= \frac{1}{2\pi i}\int_{d-i\infty}^{d+i\infty}\Gamma(1-b-\varrho t)\Gamma_g(c_1+\varphi_1 t)\Gamma(c_2+\varphi_2 t)z^{-t}dt.$$

$\Gamma_g(z)$ is the extended Gamma function[16] described in the following way:

$$\Gamma_g(z)=\int_0^{\infty} t^{z-1}e^{-t-\frac{g}{t}}dt, \quad \Re(z)>0,\, \Re(g)>0,$$

and $\Gamma(z)$ is the classical gamma function.

The following are new categories of incomplete I-functions proposed by Jangid et al. [14] (see [2, 13, 21, 23]):

(1.14)

$$\begin{aligned}{}^{\gamma}I_{p,q}^{m,n}(z) &= {}^{\gamma}I_{p,\,q}^{m,\,n}\left[z \,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_2,\mathcal{U}_2;\varpi_2),\cdots,(\mathrm{u}_p,\mathcal{U}_p;\varpi_p)\\ (\mathrm{e}_1,\mathcal{E}_1;\kappa_1),\cdots,(\mathrm{e}_q,\mathcal{E}_q;\kappa_q)\end{matrix}\right]\\ &= {}^{\gamma}I_{p,\,q}^{m,\,n}\left[z \,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\\ &= \frac{1}{2\pi i}\int_{\pounds}\psi(l,t)\,z^l\,dl,\end{aligned}$$

and

(1.15)

$$\begin{aligned}{}^{\Gamma}I_{p,\,q}^{m,n}(z) &= {}^{\Gamma}I_{p,\,q}^{m,n}\left[z \,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_2,\mathcal{U}_2;\varpi_2),\cdots,(\mathrm{u}_p,\mathcal{U}_p;\varpi_p)\\ (\mathrm{e}_1,\mathcal{E}_1;\kappa_1),\cdots,(\mathrm{e}_q,\mathcal{E}_q;\kappa_q)\end{matrix}\right]\\ &= {}^{\Gamma}I_{p,\,q}^{m,\,n}\left[z \,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\\ &= \frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\,z^l\,dl,\end{aligned}$$

for all $z \neq 0$, where

(1.16)

$$\psi(l,t)=\frac{\{\gamma\,(1-\mathrm{u}_1+\mathcal{U}_1 l,t)\}^{\varpi_1}\prod_{k=1}^{m}\{\Gamma(\mathrm{e}_k-\mathcal{E}_k l)\}^{\kappa_k}\prod_{k=2}^{n}\{\Gamma(1-\mathrm{u}_k+\mathcal{U}_k l)\}^{\varpi_k}}{\prod_{k=n+1}^{p}\{\Gamma(\mathrm{u}_k-\mathcal{U}_k l)\}^{\varpi_k}\prod_{k=m+1}^{q}\{\Gamma(1-\mathrm{e}_k+\mathcal{E}_k l)\}^{\kappa_k}},$$

and

(1.17)

$$\Psi(l,t)=\frac{\{\Gamma(1-\mathrm{u}_1+\mathcal{U}_1 l,t)\}^{\varpi_1}\prod_{k=1}^{m}\{\Gamma(\mathrm{e}_k-\mathcal{E}_k l)\}^{\kappa_k}\prod_{k=2}^{n}\{\Gamma(1-\mathrm{u}_k+\mathcal{U}_k l)\}^{\varpi_k}}{\prod_{k=n+1}^{p}\{\Gamma(\mathrm{u}_k-\mathcal{U}_k l)\}^{\varpi_k}\prod_{k=m+1}^{q}\{\Gamma(1-\mathrm{e}_k+\mathcal{E}_k l)\}^{\kappa_k}}.$$

The incomplete I-functions [21] ${}^{\gamma}I_{p,\,q}^{m,\,n}(z)$ and ${}^{\Gamma}I_{p,\,q}^{m,\,n}(z)$ exist for all $t \geq 0$ under the same contour and conditions as stated in Rathie [3].
The incomplete Gamma functions $\gamma(w, y)$ and $\Gamma(w, y)$ are defined as follows:

$$\gamma(w, y) = \int_0^y v^{w-1} e^{-v} dv, \quad (\Re(w) > 0;\, y \geq 0)\,,$$

and

$$\Gamma(w, y) = \int_y^\infty v^{w-1} e^{-v} dv, \quad (\Re(w) > 0;\, y \geq 0)\,,$$

acknowledged as the lower and upper Gamma functions respectively.
This is significant to note that for $t = 0$ the incomplete I-function modified to the I-function studied in [3].

Additionally, if $\varpi_1 = 1$ the incomplete I-function holds the decomposition formula given below:

$${}^{\gamma}I_{p,\,q}^{m,\,n}(z) +{}^{\Gamma} I_{p,\,q}^{m,\,n}(z) = I_{p,\,q}^{m,\,n}(z), \quad (\varpi_1 = 1)\,.$$

A general class of polynomials was studied by Srivastava [7, 21], described in the following way:

$$S_V^U[t] = \sum_{\mathfrak{O}=0}^{[V/U]} \frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!} A_{V,\mathfrak{O}}\; t^{\mathfrak{O}}, \tag{1.18}$$

where $U \in \mathbb{Z}^+$ and $A_{V,\,\mathfrak{O}}$ are arbitrary constants, real or complex. The notations $[k]$ indicate the floor function and $(\kappa)_\mu$ indicate the Pochhammer symbol described by:

$$(\kappa)_0 = 1 \quad \text{and} \quad (\kappa)_\mu = \frac{\Gamma(\kappa+\mu)}{\Gamma(\kappa)}, \quad (\mu \in \mathbb{C}),$$

in the form of the Gamma function.

2. Main Results

In this section, we establish the $\mathscr{M}$-transform associated with the multiplication of the family of polynomials and the incomplete I-function.

Theorem 2.1. *For* $\lambda, \mathfrak{p}, \mathfrak{q} \in \mathbb{C}; \mathfrak{r} \in \mathbb{R}^+, \Re(\lambda), \xi_1 \geq 0, \mu \in \mathbb{N}$ *and using the condition presented in* (1.15)*, then the result is obtained as:*

(2.1)

$$\mathscr{M}_{\lambda,\mu}\left[{}^{\Gamma}I_{p,\,q}^{m,\,n}\left[zx^{\xi_1}\;\middle|\; \begin{matrix} (\mathrm{u}_1, \mathcal{U}_1; \varpi_1 : t)\,, (\mathrm{u}_j, \mathcal{U}_j; \varpi_j)_{2,\,p} \\ (\mathrm{e}_j, \mathcal{E}_j; \kappa_j)_{1,\,q} \end{matrix}\right]\right](\mathfrak{p}, \mathfrak{q}, \mathfrak{r})$$

$$= \frac{\mathfrak{r}^{-\mu\,\lambda}}{\mathfrak{p}^2\mathfrak{q}\mu} (\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}} \times \frac{1}{2\pi i} \int_\Delta B\left(-\frac{s}{\mu}, \lambda + \frac{s}{\mu}\right) \left(\frac{\mathfrak{p}\mathfrak{r}}{(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}\right)^{-s}$$

$$\times\, {}^{\Gamma}I^{m,\,n+1}_{p+1,\,q}\left[z\left(\frac{\mathfrak{r}\,(\mathfrak{pq})^{\frac{1}{\mathfrak{pq}}}}{\mathfrak{p}}\right)^{\xi_1}\,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t)\,,\left(1-\left(\frac{1+s}{\mathfrak{pq}}\right),\frac{\xi_1}{\mathfrak{pq}},1\right)\\ (\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]ds.$$

Proof. The LHS of equation (2.1) is:

$$T_1=\mathscr{M}_{\lambda,\mu}\left[{}^{\Gamma}I^{m,\,n}_{p,\,q}\left[zx^{\xi_1}\,\middle|\,\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t)\,,(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right](\mathfrak{p},\mathfrak{q},\mathfrak{r}).$$

The incomplete I-function is replaced by (1.15) and the $\mathscr{M}$-transform is defined in (1.1).

$$T_1=\int_0^{\infty}\frac{e^{-\mathfrak{p}x-\frac{\mathfrak{q}}{x}}}{(x^{\mu}+\mathfrak{r}^{\mu})^{\lambda}}\left[\frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\,\left(\mathfrak{r}zx^{\xi_1}\right)^{l}\,dl\right]dx.$$

Change the integration order and with the help of (1.11) of Lemma (1.1), we obtain:

$$\begin{aligned}T_1&=\frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\,(z)^{l}\,\frac{\mathfrak{r}^{\xi_1 l+1-\mu\lambda-1}\,\mathfrak{p}^{-\xi_1 l-1}}{\mu\Gamma(\lambda)}\\ &\quad\times H^{2,\,1}_{1,\,2}\left[\mathfrak{pr}\,\middle|\,\begin{matrix}\left(1,\frac{1}{\mu}\right)\\ (\xi_1 l+1,1)_{\mathfrak{pq}},\left(\lambda,\frac{1}{\mu}\right)\end{matrix}\right]dl\\ &=\frac{\mathfrak{r}^{-\mu\lambda}}{\mathfrak{p}\mu}\frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\,\left[z\left(\frac{\mathfrak{r}}{\mathfrak{p}}\right)^{\xi_1}\right]^{l}\\ &\quad\times\frac{1}{2\pi i}\int_{\Delta}\frac{\Gamma_{\mathfrak{pq}}(\xi_1 l+1+s)\Gamma\left(\lambda+\frac{s}{\mu}\right)\Gamma\left(1-1-\frac{s}{\mu}\right)}{\Gamma(\lambda)}(\mathfrak{pr})^{-s}ds\,dl.\end{aligned}$$

Change the order of integration, use the property of the Gamma and Beta function [19] and after some adjustments of terms, we achieve the intended outcomes. □

Theorem 2.2. *For* $\lambda,\mathfrak{p},\mathfrak{q}\in\mathbb{C};\mathfrak{r}\in\mathbb{R}^{+},\Re(\lambda),\xi_1\geq 0,\mu\in\mathbb{N}$ *and using the condition presented in* (1.15)*, the result is obtained as:*

$$\begin{aligned}&\mathscr{M}_{\lambda,\mu}\left[{}^{\gamma}I^{m,\,n}_{p,\,q}\left[zx^{\xi_1}\,\middle|\,\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t)\,,(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right](\mathfrak{p},\mathfrak{q},\mathfrak{r})\\ &=\frac{\mathfrak{r}^{-\mu\,\lambda}}{\mathfrak{p}^2\mathfrak{q}\mu}\,(\mathfrak{pq})^{\frac{1}{\mathfrak{pq}}}\times\frac{1}{2\pi i}\int_{\Delta}B\left(-\frac{s}{\mu},\lambda+\frac{s}{\mu}\right)\left(\frac{\mathfrak{pr}}{(\mathfrak{pq})^{\frac{1}{\mathfrak{pq}}}}\right)^{-s}\end{aligned}$$

$$\times\, {}^{\gamma}I^{m,\,n+1}_{p+1,\,q}\left[z\left(\frac{\mathfrak{r}\,(\mathfrak{pq})^{\frac{1}{\mathfrak{pq}}}}{\mathfrak{p}}\right)^{\xi_1}\,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t)\,,\left(1-\left(\frac{1+s}{\mathfrak{pq}}\right),\frac{\xi_1}{\mathfrak{pq}},1\right),\\ (\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]ds.$$

Theorem 2.2 is proved in the same way as Theorem 2.1 under the same conditions.

Theorem 2.3. *For* $\lambda,\mathfrak{p},\mathfrak{q}\in\mathbb{C};\mathfrak{r}\in\mathbb{R}^+,\Re(\lambda),\xi_1,\xi_2\geq 0,\mu\in\mathbb{N}$, $U\in\mathbb{Z}^+$, $A_{V,\mathfrak{O}}$ *are arbitrary constants, real or complex and using the condition presented in* (1.15)*, the result is obtained as:*

(2.2)

$$\mathscr{M}_{\lambda,\mu}\left[{}^{\Gamma}I^{m,\,n}_{p,\,q}\left[zx^{\xi_1}\,\middle|\,\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t)\,,(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]S^U_V\left[x^{\xi_2}t\right]\right](\mathfrak{p},\mathfrak{q},\mathfrak{r})$$

$$=\frac{\mathfrak{r}^{-\mu\lambda}}{\mathfrak{p}^2\mathfrak{q}\mu}(\mathfrak{pq})^{\frac{1}{\mathfrak{pq}}}\sum_{\mathfrak{O}=0}^{[V/U]}\frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!}A_{V,\mathfrak{O}}\left[t\left(\frac{\mathfrak{r}\,(\mathfrak{pq})^{\frac{1}{\mathfrak{pq}}}}{\mathfrak{p}}\right)^{\xi_2}\right]^{\mathfrak{O}}$$

$$\times\frac{1}{2\pi i}\int_{\Delta}B\left(-\frac{s}{\mu},\lambda+\frac{s}{\mu}\right)\left(\frac{\mathfrak{pr}}{(\mathfrak{pq})^{\frac{1}{\mathfrak{pq}}}}\right)^{-s}{}^{\Gamma}I^{m,\,n+1}_{p+1,\,q}\left[z\left(\frac{\mathfrak{r}\,(\mathfrak{pq})^{\frac{1}{\mathfrak{pq}}}}{\mathfrak{p}}\right)^{\xi_1}\,\middle|\right.$$

$$\left.\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t)\,,\left(1-\left(\frac{1+\xi_2\mathfrak{O}+s}{\mathfrak{pq}}\right),\frac{\xi_1}{\mathfrak{pq}},1\right),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]ds.$$

Proof. The LHS of equation (2.2) is:

$$T_2=\mathscr{M}_{\lambda,\mu}\left[{}^{\Gamma}I^{m,\,n}_{p,\,q}\left[zx^{\xi_1}\,\middle|\,\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t)\,,(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right.$$

$$\left.\times\, S^U_V\left[x^{\xi_2}t\right]\right](\mathfrak{p},\mathfrak{q},\mathfrak{r}).$$

Replace the incomplete I-function and Srivastava polynomial by (1.15) and (1.18) and using $\mathscr{M}$-transform definition in (1.1), we get:

$$T_2=\int_0^{\infty}\frac{e^{-\mathfrak{p}x-\frac{\mathfrak{q}}{x}}}{(x^{\mu}+\mathfrak{r}^{\mu})^{\lambda}}\left[\frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\,\left(\mathfrak{r}zx^{\xi_1}\right)^{l}\,dl\right.$$

$$\left.\times\sum_{\mathfrak{O}=0}^{[V/U]}\frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!}A_{V,\mathfrak{O}}\,\left(tx^{\xi_2}\mathfrak{r}\right)^{\mathfrak{O}}\right]dx.$$

Change the integration order and with the help of (1.11) of Lemma (1.1), we obtain:

$$\sum_{\mathfrak{O}=0}^{[V/U]} \frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!} A_{V,\mathfrak{O}} \, (t)^{\mathfrak{O}} \, \frac{1}{2\pi i} \int_{\pounds} \; \Psi(l,t) \, (z)^l \; \frac{\mathfrak{r}^{\xi_1 l + \xi_2 \mathfrak{O} + 1 - \mu\lambda - 1}}{\mu}$$
$$\times \frac{\mathfrak{p}^{-\xi_1 l - \xi_2 \mathfrak{O} - 1}}{\Gamma(\lambda)} H^{2,\,1}_{1,\,2} \left[\mathfrak{p}\mathfrak{r} \,\middle|\, \begin{matrix} \left(1, \frac{1}{\mu}\right), \\ (\xi_1 l + \xi_2 R + 1, 1)_{\mathfrak{p}\mathfrak{q}}, \left(\lambda, \frac{1}{\mu}\right) \end{matrix} \right] dl$$
$$= \frac{\mathfrak{r}^{-\mu\lambda}}{\mathfrak{p}\mu} \sum_{\mathfrak{O}=0}^{[V/U]} \frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!} A_{V,\mathfrak{O}} \left[t \left(\frac{\mathfrak{r}}{\mathfrak{p}}\right)^{\xi_2} \right]^{\mathfrak{O}} \frac{1}{2\pi i} \int_{\pounds} \; \Psi(l,t) \; \left[z \left(\frac{\mathfrak{r}}{\mathfrak{p}}\right)^{\xi_1} \right]^l$$
$$\times \frac{1}{2\pi i} \int_{\Delta} \frac{\Gamma_{\mathfrak{p}\mathfrak{q}}(\xi_1 l + \xi_2 \mathfrak{O} + 1 + s)\Gamma\left(\lambda + \frac{s}{\mu}\right)\Gamma\left(-\frac{s}{\mu}\right)}{\Gamma(\lambda)} (\mathfrak{p}\mathfrak{r})^{-s} ds \; dl.$$

Change the order of integration, use the property of the Gamma and Beta function [19] and after some adjustments of terms, we achieve the intended outcomes. ☐

Theorem 2.4. *For* $\lambda, \mathfrak{p}, \mathfrak{q} \in \mathbb{C}; \mathfrak{r} \in \mathbb{R}^+, \Re(\lambda), \xi_1, \xi_2 \geq 0, \mu \in \mathbb{N}$*,* $U \in \mathbb{Z}^+$*,*$A_{V,\,\mathfrak{O}}$ *are arbitrary constants, real or complex and using the condition presented in* (1.15)*, then the result is obtained as:*

$$\mathscr{M}_{\lambda,\mu} \left[{}^{\gamma}I^{m,\,n}_{p,\,q} \left[z x^{\xi_1} \,\middle|\, \begin{matrix} (\mathrm{u}_1, \mathcal{U}_1; \varpi_1 : t), (\mathrm{u}_j, \mathcal{U}_j; \varpi_j)_{2,\,p} \\ (\mathrm{e}_j, \mathcal{E}_j; \kappa_j)_{1,\,q} \end{matrix} \right] S^U_V \left[x^{\xi_2} t \right] \right] (\mathfrak{p}, \mathfrak{q}, \mathfrak{r})$$
$$= \frac{\mathfrak{r}^{-\mu\,\lambda}}{\mathfrak{p}^2 \mathfrak{q} \mu} (\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}} \sum_{\mathfrak{O}=0}^{[V/U]} \frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!} A_{V,\mathfrak{O}} \left[t \left(\frac{\mathfrak{r}\,(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}} \right)^{\xi_2} \right]^{\mathfrak{O}}$$
$$\times \frac{1}{2\pi i} \int_{\Delta} B\left(-\frac{s}{\mu}, \lambda + \frac{s}{\mu}\right) \left(\frac{\mathfrak{p}\mathfrak{r}}{(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}} \right)^{-s} {}^{\gamma}I^{m,\,n+1}_{p+1,\,q} \left[z \left(\frac{\mathfrak{r}(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}} \right)^{\xi_1} \,\middle|\, \begin{matrix} (\mathrm{u}_1, \mathcal{U}_1; \varpi_1 : t), \left(1 - \left(\frac{1+\xi_2\mathfrak{O}+s}{\mathfrak{p}\mathfrak{q}}\right), \frac{\xi_1}{\mathfrak{p}\mathfrak{q}}, 1\right), (\mathrm{u}_j, \mathcal{U}_j; \varpi_j)_{2,\,p} \\ (\mathrm{e}_j, \mathcal{E}_j; \kappa_j)_{1,\,q} \end{matrix} \right] ds.$$

Theorem 2.4 is proved in the same way as Theorem 2.3 under the same conditions.

Theorem 2.5. *For* $\lambda, \mathfrak{p}, \mathfrak{q} \in \mathbb{C}; \mathfrak{r} \in \mathbb{R}^+, \Re(\lambda), a \geq 0, \mu \in \mathbb{N}$ *and using the condition presented in* (1.15)*, then the result is obtained as:*

$$\mathscr{M}_{\lambda,\mu} \left[{}^{\Gamma}I^{m,\,n}_{p,\,q} \left[z e^{\frac{-ax}{l}} \,\middle|\, \begin{matrix} (\mathrm{u}_1, \mathcal{U}_1; \varpi_1 : t), (\mathrm{u}_j, \mathcal{U}_j; \varpi_j)_{2,\,p} \\ (\mathrm{e}_j, \mathcal{E}_j; \kappa_j)_{1,\,q} \end{matrix} \right] \right] (\mathfrak{p}, \mathfrak{q}, \mathfrak{r}) \tag{2.3}$$

$$= \frac{\mathfrak{r}^{-\mu\lambda}}{\mu\mathfrak{q}\left(\mathfrak{p}+a\mathfrak{r}\right)^2} \times \left[\left(\mathfrak{p}+a\mathfrak{r}\right)\mathfrak{q}\right]^{\frac{1}{(\mathfrak{p}+a\mathfrak{r})\mathfrak{q}}} \frac{1}{2\pi i}\int_{\Delta} B\left(-\frac{s}{\mu}, \lambda+\frac{s}{\mu}\right)$$

$$\times \left[\frac{\mathfrak{r}(\mathfrak{p}+\mathfrak{a}\mathfrak{r})}{\left((\mathfrak{p}+a\mathfrak{r})\,\mathfrak{q}\right)^{\frac{1}{((\mathfrak{p}+a\mathfrak{r})\mathfrak{q})}}}\right]^{-s} \left({}^{\Gamma}I^{m,\,n+1}_{p+1,\,q}\left[z \,\middle|\, \begin{matrix} (\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t)\,,\left(1-\left(\frac{1+s}{(\mathfrak{p}+a\mathfrak{r})q}\right),0,1\right),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p} \\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right) ds.$$

Proof. The LHS of equation (2.3) is:

$$T_3 = \mathscr{M}_{\lambda,\mu}\left[{}^{\Gamma}I^{m,\,n}_{p,\,q}\left[ze^{\frac{-ax}{l}} \,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t)\,,(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right](\mathfrak{p},\mathfrak{q},\mathfrak{r}).$$

Replace the incomplete I-function by (1.15) and $\mathscr{M}$-transform definition in (1.1), we get:

$$T_3 = \int_0^\infty \frac{e^{-\mathfrak{p}x-\frac{\mathfrak{q}}{x}}}{\left(x^\mu+\mathfrak{r}^\mu\right)^\lambda}\left[\frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\,\left(ze^{\frac{-a\mathfrak{r}x}{l}}\right)^l\,dl\right]dx.$$

Change the integration order and with the help of (1.12) of Lemma (1.1), we obtain:

$$T_3 = \frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\,(z)^l\,\frac{\mathfrak{r}^{-\mu\lambda}}{\mu\left(\mathfrak{p}+a\mathfrak{r}\right)\Gamma(\lambda)}H^{2,\,1}_{1,\,2}\left[\mathfrak{r}\left(\mathfrak{p}+a\mathfrak{r}\right)\,\middle|\,\begin{matrix}\left(1,\frac{1}{\mu}\right),\\ (1,1)_{(\mathfrak{p}+a\mathfrak{r})\mathfrak{q}},\left(\lambda,\frac{1}{\mu}\right)\end{matrix}\right]dl$$

$$= \frac{\mathfrak{r}^{-\mu\lambda}}{\mu\left(\mathfrak{p}+a\mathfrak{r}\right)}\times\frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\,(z)^l$$

$$\times\frac{1}{2\pi i}\int_{\Delta}\frac{\Gamma_{(\mathfrak{p}+a\mathfrak{r})\mathfrak{q}}(1+s)\Gamma\left(\lambda+\frac{s}{\mu}\right)\Gamma\left(1-1-\frac{s}{\mu}\right)}{\Gamma(\lambda)}\left(\mathfrak{r}\left(\mathfrak{p}+a\mathfrak{r}\right)\right)^{-s}ds\,dl.$$

Change the order of integration, use the property of the Gamma and Beta function [19] and after some adjustments of terms, we achieve the intended outcomes. □

Theorem 2.6. *For* $\lambda,\mathfrak{p},\mathfrak{q}\in\mathbb{C};\mathfrak{r}\in\mathbb{R}^+,\Re(\lambda),a\geq 0,\mu\in\mathbb{N}$ *and using the condition presented in* (1.15), *then the result is obtained as:*

$$\mathscr{M}_{\lambda,\mu}\left[{}^{\gamma}I^{m,\,n}_{p,\,q}\left[ze^{\frac{-ax}{l}} \,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t)\,,(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right](\mathfrak{p},\mathfrak{q},\mathfrak{r})$$

$$= \frac{\mathfrak{r}^{-\mu\lambda}}{\mu\mathfrak{q}\left(\mathfrak{p}+a\mathfrak{r}\right)^2} \times \left[\left(\mathfrak{p}+a\mathfrak{r}\right)\mathfrak{q}\right]^{\frac{1}{(\mathfrak{p}+a\mathfrak{r})\mathfrak{q}}} \frac{1}{2\pi i}\int_{\Delta} B\left(-\frac{s}{\mu}, \lambda+\frac{s}{\mu}\right)$$

$$\times \left[\frac{\mathfrak{r}(\mathfrak{p}+\mathfrak{a}\mathfrak{r})}{((\mathfrak{p}+a\mathfrak{r})\,\mathfrak{q})^{\frac{1}{((\mathfrak{p}+a\mathfrak{r})\mathfrak{q})}}}\right]^{-s} \left({}^{\gamma}I^{m,\,n+1}_{p+1,\,q}\left[z \,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t)\,,\left(1-\left(\frac{1+s}{(\mathfrak{p}+a\mathfrak{r})q}\right),0,1\right),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right)ds.$$

Theorem 2.6 is proved in the same way as Theorem 2.5 under the same conditions.

Theorem 2.7. *For* $\lambda,\mathfrak{p},\mathfrak{q}\in\mathbb{C};\mathfrak{r}\in\mathbb{R}^+,\Re(\lambda),a,b\geq 0,\mu\in\mathbb{N}$, $U\in\mathbb{Z}^+$, $A_{V,\mathfrak{O}}$ *be arbitrary constants, real or complex and using the condition presented in* (1.15)*, then the result is obtained as:*

(2.4)

$$\mathscr{M}_{\lambda,\mu}\left[{}^{\Gamma}I^{m,\,n}_{p,\,q}\left[ze^{\frac{-ax}{l}} \,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t)\,,(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right] S^U_V\left[t\,e^{\frac{-bx}{R}}\right]\right](\mathfrak{p},\mathfrak{q},\mathfrak{r})$$

$$= \frac{\mathfrak{r}^{-\mu\lambda}}{\mu\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{r})^2}[(\mathfrak{p}+(a+b)\mathfrak{r})\mathfrak{q}]^{\frac{1}{(\mathfrak{p}+(a+b)\mathfrak{r})\mathfrak{q}}}$$

$$\times \sum_{\mathfrak{O}=0}^{[V/U]}\frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!}A_{V,\mathfrak{O}}\,(t)^{\mathfrak{O}}\frac{1}{2\pi i}\int_{\Delta}B\left(-\frac{s}{\mu},\lambda+\frac{s}{\mu}\right)$$

$$\times \left[\frac{\mathfrak{r}(\mathfrak{p}+(\mathfrak{a}+\mathfrak{b})\mathfrak{r})}{[(\mathfrak{p}+(\mathfrak{a}+\mathfrak{b})\mathfrak{r})\mathfrak{q}]^{\frac{1}{[(\mathfrak{p}+(\mathfrak{a}+\mathfrak{b})\mathfrak{r})\mathfrak{q}]}}}\right]^{-s}\left({}^{\Gamma}I^{m,\,n+1}_{p+1,\,q}\left[z \,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t)\,,\left(1-\left(\frac{1+s}{(\mathfrak{p}+(a+b)\mathfrak{r})q}\right),0,1\right),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right)ds.$$

Proof. The LHS of equation (2.4) is:

$$T_4=\mathscr{M}_{\lambda,\mu}\left[{}^{\Gamma}I^{m,\,n}_{p,\,q}\left[ze^{\frac{-ax}{l}} \,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t)\,,(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\times S^U_V\left[t\,e^{\frac{-bx}{R}}\right]\right](\mathfrak{p},\mathfrak{q},\mathfrak{r}).$$

Replace the incomplete I-function and Srivastava polynomial by (1.15) and (1.18) and $\mathscr{M}$-transform definition in (1.1), we get:

$$T_4=\int_0^\infty \frac{e^{-\mathfrak{p}x-\frac{\mathfrak{q}}{x}}}{(x^\mu+\mathfrak{r}^\mu)^\lambda}\left[\frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\,\left(ze^{\frac{-a\mathfrak{r}x}{l}}\right)^l\,dl\right.$$

$$\left.\times\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}\,\left(t\,e^{\frac{-b\mathfrak{q}x}{R}}\right)^R\right]dx.$$

Change the integration order and with the help of (1.12) of Lemma (1.1), we obtain:

$$T_4 = \sum_{\mathfrak{O}=0}^{[V/U]} \frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!} A_{V,\mathfrak{O}}\ (t)^{\mathfrak{O}} \frac{1}{2\pi i} \int_{\pounds} \Psi(l,t)\,(z)^l \frac{\mathfrak{r}^{-\mu\lambda}}{\mu(\mathfrak{p}+(a+b)\mathfrak{r})\Gamma(\lambda)}$$
$$\times H_{1,\,2}^{2,\,1}\left[\mathfrak{r}(\mathfrak{p}+(a+b)\mathfrak{r}) \,\middle|\, \begin{matrix} \left(1, \frac{1}{\mu}\right), \\ (1,1)_{(\mathfrak{p}+(a+b)\mathfrak{r})\mathfrak{q}}, \left(\lambda, \frac{1}{\mu}\right) \end{matrix}\right] dl$$
$$= \frac{\mathfrak{r}^{-\mu\lambda}}{\mu(\mathfrak{p}+(a+b)\mathfrak{r})} \sum_{\mathfrak{O}=0}^{[V/U]} \frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!} A_{V,\mathfrak{O}}\ (t)^{\mathfrak{O}} \times \frac{1}{2\pi i} \int_{\pounds} \Psi(l,t)\ (z)^l$$
$$\times \frac{1}{2\pi i} \int_{\Delta} \frac{\Gamma_{(\mathfrak{p}+(a+b)\mathfrak{r})\mathfrak{q}}(1+s)\Gamma\left(\lambda+\frac{s}{\mu}\right)\Gamma\left(1-1-\frac{s}{\mu}\right)}{\Gamma(\lambda)}$$
$$\times (\mathfrak{r}(\mathfrak{p}+(a+b)\mathfrak{r}))^{-s} ds\, dl.$$

Change the order of integration, use the property of the Gamma and Beta function [19] and after some adjustments of terms, we achieve the intended outcomes. □

Theorem 2.8. *For $\lambda, \mathfrak{p}, \mathfrak{q} \in \mathbb{C}; \mathfrak{r} \in \mathbb{R}^+, \Re(\lambda), a, b \geq 0, \mu \in \mathbb{N}$, $U \in \mathbb{Z}^+$, $A_{V,\mathfrak{O}}$ be arbitrary constants, real or complex and using the condition presented in* (1.15)*, then the result is obtained as:*

$$\mathscr{M}_{\lambda,\mu}\left[{}^{\gamma}I_{p,\,q}^{m,\,n}\left[ze^{\frac{-ax}{l}} \,\middle|\, \begin{matrix} (\mathrm{u}_1, \mathcal{U}_1; \varpi_1 : t), (\mathrm{u}_j, \mathcal{U}_j; \varpi_j)_{2,\,p} \\ (\mathrm{e}_j, \mathcal{E}_j; \kappa_j)_{1,\,q} \end{matrix}\right] S_V^U\left[t\, e^{\frac{-bx}{R}}\right]\right](\mathfrak{p}, \mathfrak{q}, \mathfrak{r})$$
$$= \frac{\mathfrak{r}^{-\mu\lambda}}{\mu\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{r})^2}[(\mathfrak{p}+(a+b)\mathfrak{r})\mathfrak{q}]^{\frac{1}{(\mathfrak{p}+(a+b)\mathfrak{r})\mathfrak{q}}}$$
$$\times \sum_{\mathfrak{O}=0}^{[V/U]} \frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!} A_{V,\mathfrak{O}}\ (t)^{\mathfrak{O}} \frac{1}{2\pi i} \int_{\Delta} B\left(-\frac{s}{\mu}, \lambda+\frac{s}{\mu}\right)$$
$$\times \left[\frac{\mathfrak{r}(\mathfrak{p}+(\mathfrak{a}+\mathfrak{b})\mathfrak{r})}{[(\mathfrak{p}+(\mathfrak{a}+\mathfrak{b})\mathfrak{r})\mathfrak{q}]^{\frac{1}{[(\mathfrak{p}+(\mathfrak{a}+\mathfrak{b})\mathfrak{r})\mathfrak{q}]}}}\right]^{-s} \left({}^{\gamma}I_{p+1,\,q}^{m,\,n+1}\left[z \,\middle|\, \begin{matrix} (\mathrm{u}_1, \mathcal{U}_1; \varpi_1 : t), \left(1-\left(\frac{1+s}{(\mathfrak{p}+(a+b)\mathfrak{r})q}\right), 0, 1\right), (\mathrm{u}_j, \mathcal{U}_j; \varpi_j)_{2,\,p} \\ (\mathrm{e}_j, \mathcal{E}_j; \kappa_j)_{1,\,q} \end{matrix}\right]\right) ds.$$

Theorem 2.8 is proved in the same way as Theorem 2.7 under the same conditions.

Theorem 2.9. *For* $\lambda, \mathfrak{p}, \mathfrak{q} \in \mathbb{C}; \mathfrak{r} \in \mathbb{R}^+, \Re(\lambda), \xi_1, a \geq 0, \mu \in \mathbb{N}$ *and using the condition presented in* (1.15)*, then the result is obtained as:*

$$\begin{aligned}
&\mathscr{M}_{\lambda,\mu}\left[{}^{\Gamma}I^{m,\,n}_{p,\,q}\left[zx^{\xi_1}e^{\frac{-ax}{l}}\;\middle|\;\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right](\mathfrak{p},\mathfrak{q},\mathfrak{r})\\
&=\frac{\mathfrak{r}^{-\mu\lambda}}{\mathfrak{p}\mu\mathfrak{q}\,(\mathfrak{p}+a\mathfrak{r})}(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}\;\frac{1}{2\pi i}\int_{\Delta}B\left(-\frac{s}{\mu},\lambda+\frac{s}{\mu}\right)\left[\frac{\mathfrak{r}(\mathfrak{p}+\mathfrak{a}\mathfrak{r})}{(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}\right]^{-s}\\
&\times{}^{\Gamma}I^{m,\,n+1}_{p+1,\,q}\left[z\left(\frac{\mathfrak{r}\,(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}+a\mathfrak{r}}\right)^{\xi_1}\;\middle|\;\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),\left(1-\left(\frac{1+s}{\mathfrak{p}\mathfrak{q}}\right),\frac{\xi_1}{\mathfrak{p}\mathfrak{q}},1\right),\\(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]ds.
\end{aligned}\tag{2.5}$$

Proof. The LHS of equation (2.5) is:

$$T_5=\mathscr{M}_{\lambda,\mu}\left[{}^{\Gamma}I^{m,n}_{p,q}\left[zx^{\xi_1}e^{\frac{-ax}{l}}\;\middle|\;\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,p}\\(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,q}\end{matrix}\right]\right](\mathfrak{p},\mathfrak{q},\mathfrak{r}).$$

Replace the incomplete I-function by (1.15) and using $\mathscr{M}$-transform definition in (1.1), we get:

$$T_5=\int_0^\infty\frac{e^{-\mathfrak{p}x-\frac{\mathfrak{q}}{x}}}{(x^\mu+\mathfrak{r}^\mu)^\lambda}\left[\frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\left(z\mathfrak{r}x^{\xi_1}e^{\frac{-a\mathfrak{r}x}{l}}\right)^l dl\right]dx.$$

Change the integration order and with the help of (1.13) of Lemma (1.1), we obtain:

$$\begin{aligned}
T_5&=\frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\,(z)^l\,\frac{\mathfrak{r}^{\xi_1 l+1-\mu\lambda-1}}{\mu\,(\mathfrak{p}+a\mathfrak{r})^{\xi_1 l+1}\,\Gamma(\lambda)}\\
&\quad\times H^{2,\,1}_{1,\,2}\left[\mathfrak{r}\,(\mathfrak{p}+a\mathfrak{r})\;\middle|\;\begin{matrix}\left(1,\frac{1}{\mu}\right),\\(\xi_1 l+1,1)_{(\mathfrak{p}\mathfrak{q}},\left(\lambda,\frac{1}{\mu}\right)\end{matrix}\right]dl\\
&=\frac{\mathfrak{r}^{-\mu\lambda}}{\mu\,(\mathfrak{p}+a\mathfrak{r})}\frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\left[z\left(\frac{\mathfrak{r}}{\mathfrak{p}+a\mathfrak{r}}\right)^{\xi_1}\right]^l\\
&\quad\times\frac{1}{2\pi i}\int_{\Delta}\frac{\Gamma_{(\mathfrak{p}\mathfrak{q}}(\xi_1 l+1+s)\Gamma\left(\lambda+\frac{s}{\mu}\right)\Gamma\left(-\frac{s}{\mu}\right)}{\Gamma(\lambda)}(\mathfrak{r}\,(\mathfrak{p}+a\mathfrak{r}))^{-s}ds\,dl.
\end{aligned}$$

Change the order of integration, use the property of the Gamma and Beta function [19] and after some adjustments of terms, we achieve the intended outcomes. □

Theorem 2.10. *For $\lambda, \mathfrak{p}, \mathfrak{q} \in \mathbb{C}; \mathfrak{r} \in \mathbb{R}^+, \Re(\lambda), \xi_1, a \geq 0, \mu \in \mathbb{N}$ and using the condition presented in* (1.15)*, then the result is obtained as:*

$$\mathscr{M}_{\lambda,\mu}\left[{}^{\gamma}I^{m,\,n}_{p,\,q}\left[zx^{\xi_1}e^{\frac{-ax}{l}}\;\middle|\;\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right](\mathfrak{p},\mathfrak{q},\mathfrak{r})$$
$$=\frac{\mathfrak{r}^{-\mu\lambda}}{\mathfrak{p}\mu\mathfrak{q}\,(\mathfrak{p}+a\mathfrak{r})}(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}\;\frac{1}{2\pi i}\int_{\Delta}B\left(-\frac{s}{\mu},\lambda+\frac{s}{\mu}\right)\left[\frac{\mathfrak{r}(\mathfrak{p}+\mathfrak{a}\mathfrak{r})}{(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}\right]^{-s}$$
$$\times\,{}^{\gamma}I^{m,\,n+1}_{p+1,\,q}\left[z\left(\frac{\mathfrak{r}\,(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}+a\mathfrak{r}}\right)^{\xi_1}\;\middle|\;\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),\left(1-\left(\frac{1+s}{\mathfrak{p}\mathfrak{q}}\right),\frac{\xi_1}{\mathfrak{p}\mathfrak{q}},1\right),\\(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]ds.$$

Theorem 2.10 is proved in the same way as Theorem 2.9 under the same conditions.

Theorem 2.11. *For $\lambda, \mathfrak{p}, \mathfrak{q} \in \mathbb{C}; \mathfrak{r} \in \mathbb{R}^+, \Re(\lambda), a, b, \xi_1, \xi_2 \geq 0, \mu \in \mathbb{N}$, $U \in \mathbb{Z}^+$, $A_{V,\mathfrak{D}}$ be arbitrary constants, real or complex and using the condition presented in* (1.15)*, then the result is obtained as:*

(2.6)
$$\mathscr{M}_{\lambda,\mu}\left[{}^{\Gamma}I^{m,\,n}_{p,\,q}\left[zx^{\xi_1}e^{\frac{-ax}{l}}\;\middle|\;\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\times S^U_V\left[tx^{\xi_2}e^{\frac{-bx}{\mathfrak{D}}}\right]\right](\mathfrak{p},\mathfrak{q},\mathfrak{r})$$
$$=\frac{\mathfrak{r}^{-\mu\lambda}(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}\mu\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{r})}\sum_{\mathfrak{D}=0}^{[V/U]}\frac{(-V)_{U\mathfrak{D}}}{\mathfrak{D}!}A_{V,\mathfrak{D}}\left[t\left(\frac{\mathfrak{r}\,(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}+(a+b)\mathfrak{r}}\right)^{\xi_2}\right]^{\mathfrak{D}}$$
$$\times\frac{1}{2\pi i}\int_{\Delta}B\left(-\frac{s}{\mu},\lambda+\frac{s}{\mu}\right)\left[\frac{\mathfrak{r}(\mathfrak{p}+(\mathfrak{a}+\mathfrak{b})\mathfrak{r})}{(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}\right]^{-s}$$
$$\times\,{}^{\Gamma}I^{m,\,n+1}_{p+1,\,q}\left[z\left(\frac{\mathfrak{r}\,(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}+(\mathfrak{a}+\mathfrak{b})\mathfrak{r}}\right)^{\xi_1}\;\middle|\;\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),\\\left(1-\left(\frac{1+\xi_2\mathfrak{D}+s}{\mathfrak{p}\mathfrak{q}}\right),\frac{\xi_1}{\mathfrak{p}\mathfrak{q}}\right),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]ds.$$

Proof. The LHS of equation (2.6) is:

$$T_6 = \mathscr{M}_{\lambda,\mu}\Bigg[{}^{\Gamma} I_{p,\,q}^{m,\,n}\left[zx^{\xi_1}e^{\frac{-ax}{l}}\;\middle|\;\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right] \times S_V^U\left[tx^{\xi_2}e^{\frac{-bx}{\mathfrak{O}}}\right]\Bigg](\mathfrak{p},\mathfrak{q},\mathfrak{r}).$$

Replace the incomplete I-function and Srivastava polynomial by (1.15) and (1.18) and using $\mathscr{M}$-transform definition in (1.1), we get:

$$T_6 = \int_0^\infty \frac{e^{-\mathfrak{p}x-\frac{\mathfrak{q}}{x}}}{(x^\mu+\mathfrak{r}^\mu)^\lambda}\left[\frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\left(z\mathfrak{r}x^{\xi_1}e^{\frac{-a\mathfrak{r}x}{l}}\right)^l dl \times \sum_{\mathfrak{O}=0}^{[V/U]}\frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!}A_{V,\mathfrak{O}}\left(t\,\mathfrak{r}x^{\xi_2}e^{\frac{-b\mathfrak{q}x}{R}}\right)^{\mathfrak{O}}\right]dx.$$

Change the integration order and with the help of (1.12) of Lemma (1.1), we obtain:

$$\begin{aligned}T_6 &= \sum_{\mathfrak{O}=0}^{[V/U]}\frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!}A_{V,\mathfrak{O}}\,(t)^{\mathfrak{O}}\frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\,(z)^l\frac{\mathfrak{r}^{\xi_1 l+\xi_2\mathfrak{O}+1-\mu\lambda-1}}{\mu(\mathfrak{p}+(a+b)\mathfrak{r})^{\xi_1 l+\xi_2\mathfrak{O}+1}}\\
&\quad\times\frac{1}{\Gamma(\lambda)}H_{1,\,2}^{2,\,1}\left[\mathfrak{r}(\mathfrak{p}+(a+b)\mathfrak{r})\;\middle|\;\begin{matrix}\left(1,\frac{1}{\mu}\right),\\ (\xi_1 l+\xi_2\mathfrak{O}+1,1)_{\mathfrak{pq}},\left(\lambda,\frac{1}{\mu}\right)\end{matrix}\right]dl\\
&= \frac{\mathfrak{r}^{-\mu\lambda}}{\mu(\mathfrak{p}+(a+b)\mathfrak{r})}\sum_{\mathfrak{O}=0}^{[V/U]}\frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!}A_{V,\mathfrak{O}}\left[t\left(\frac{\mathfrak{r}}{\mathfrak{p}+(\mathfrak{a}+\mathfrak{b})\mathfrak{r}}\right)^{\xi_2}\right]^{\mathfrak{O}}\\
&\quad\times\frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\left[z\left(\frac{\mathfrak{r}}{\mathfrak{p}+(\mathfrak{a}+\mathfrak{b})\mathfrak{r}}\right)^{\xi_1}\right]^l\\
&\quad\times\frac{1}{2\pi i}\int_{\Delta}\frac{\Gamma_{\mathfrak{pq}}(\xi+1l+\xi_2\mathfrak{O}+1+s)\Gamma\left(\lambda+\frac{s}{\mu}\right)\Gamma\left(1-1-\frac{s}{\mu}\right)}{\Gamma(\lambda)}\\
&\quad\times(\mathfrak{r}(\mathfrak{p}+(a+b)\mathfrak{r}))^{-s}ds\;dl.\end{aligned}$$

Change the order of integration, use the property of the Gamma and Beta function [19] and after some adjustments of terms, we achieve the intended outcomes. □

Theorem 2.12. *For* $\lambda,\mathfrak{p},\mathfrak{q}\in\mathbb{C};\mathfrak{r}\in\mathbb{R}^+,\Re(\lambda),a,b,\xi_1,\xi_2\geq 0,\mu\in\mathbb{N}$, $U\in\mathbb{Z}^+$, $A_{V,\mathfrak{O}}$ *be arbitrary constants, real or complex and using the* *condition presented in* (1.15)*, then the result is obtained as:*

(2.7)

$$\mathscr{M}_{\lambda,\mu}\left[{}^{\gamma}I_{p,\,q}^{m,\,n}\left[zx^{\xi_1}e^{\frac{-ax}{l}}\;\middle|\;\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right.$$

$$\left.\times S_V^U\left[tx^{\xi_2}e^{\frac{-bx}{\mathfrak{O}}}\right]\right](\mathfrak{p},\mathfrak{q},\mathfrak{r})$$

$$=\frac{\mathfrak{r}^{-\mu\lambda}\,(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}\mu\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{r})}\sum_{\mathfrak{O}=0}^{[V/U]}\frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!}A_{V,\mathfrak{O}}\left[t\left(\frac{\mathfrak{r}\,(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}+(a+b)\mathfrak{r}}\right)^{\xi_2}\right]^{\mathfrak{O}}$$

$$\times\frac{1}{2\pi i}\int_{\Delta}B\left(-\frac{s}{\mu},\lambda+\frac{s}{\mu}\right)\left[\frac{\mathfrak{r}(\mathfrak{p}+(\mathfrak{a}+\mathfrak{b})\mathfrak{r})}{(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}\right]^{-s}$$

$$\times{}^{\gamma}I_{p+1,\,q}^{m,\,n+1}\left[z\left(\frac{\mathfrak{r}\,(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}+(\mathfrak{a}+\mathfrak{b})\mathfrak{r}}\right)^{\xi_1}\;\middle|\;\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),\\ \left(1-\left(\frac{1+\xi_2\mathfrak{O}+s}{\mathfrak{p}\mathfrak{q}}\right),\frac{\xi_1}{\mathfrak{p}\mathfrak{q}}\right),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]ds.$$

Theorem 2.12 is proved in the same way as Theorem 2.11 under the same conditions.

Remark 2.13.

- If we set $U=1$, $A_{V,0}=1$ and $A_{V,\mathfrak{O}}=0\;\forall\;R\neq 0$ in Theorems 2.3, 2.4, 2.7, 2.8, 2.11 and 2.12, then the result is the same as that of Theorems 2.1, 2.2, 2.5, 2.6, 2.9 and 2.10, respectively.
- If we set $\xi_1,\xi_2=0$ in Theorems 2.11 and 2.12, then the result is the same as that of Theorems 2.7 and 2.8, respectively.
- If we set $a,b=0$ in Theorems 2.11 and 2.12, then the result is the same as that of Theorems 2.3 and 2.4, respectively.
- If we set $\varpi_j=1\forall j=1,2,\ldots,p$, $\kappa_j=1\forall j=1,2,\ldots,q$, $\xi_1=1$ and $\mathfrak{q}=0$ in Theorems 2.1 and 2.2, then the result is the same as Theorem 2 and Theorem 1, respectively mentioned in Bansal et al. [18].
- If we take the substitutions $z=-z$,$\varpi_j=1\;\forall\;1\leq j\leq p$, $\kappa_j=1\;\forall\;1\leq j\leq q$, $\mathrm{u}_j\to(1-\mathrm{u}_j)\forall\;1\leq j\leq p$, $\mathrm{e}_j\to(1-\mathrm{e}_j)\;\forall\;1\leq j\leq q$, $\xi_1=1$ and $\mathfrak{q}=0$ in Theorems 2.1 and 2.2, then the result is the same as corollary 1 mentioned in Bansal et al. [18].

3. Special Cases

In this section, as a particular instance of Theorem 2.11 and Theorem 2.12, we establish the $\mathscr{M}$-transform for the multiplication of the Srivastava polynomial with the incomplete $\bar{I}$-function and the incomplete $\bar{H}$-function. Further, some special values will be given to the Srivastava polynomial in order to get outcomes in the form of Hermite and Laguerre polynomials. If we provide the parameter of particular features, we get the following special cases to delineate the use of fundamental outcomes.

(i) Incomplete $\bar{I}$-function: If we set $\kappa_k = 1$ for $1 \leq k \leq m$ in (1.15) and making use of the connection, that is (see [13, 22]):

$$\begin{aligned} {}^{\Gamma}\overline{I}_{p,\,q}^{m,\,n}(z) &= {}^{\Gamma}\overline{I}_{p,\,q}^{m,\,n}\left[z \,\middle|\, \begin{array}{l} (\mathrm{u}_1,\mathcal{U}_1;\varpi_1 : t)\,,(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p} \\ (\mathrm{e}_j,\mathcal{E}_j;1)_{1,\,m},(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{m+1,\,q} \end{array}\right] \\ &= \frac{1}{2\pi i}\int_{\pounds} \overline{\Psi}(l,t)\, z^l\, dl, \end{aligned} \tag{3.1}$$

where,

$$\overline{\Psi}(l,t) = \frac{\{\Gamma(1-\mathrm{u}_1+\mathcal{U}_1 l,t)\}^{\varpi_1}\prod\limits_{k=1}^{m}\{\Gamma(\mathrm{e}_k-\mathcal{E}_k l)\}\prod\limits_{k=2}^{n}\{\Gamma(1-\mathrm{u}_k+\mathcal{U}_k l)\}^{\varpi_k}}{\prod\limits_{k=n+1}^{p}\{\Gamma(\mathrm{u}_k-\mathcal{U}_k l)\}^{\varpi_k}\prod\limits_{k=m+1}^{q}\{\Gamma(1-\mathrm{e}_k+\mathcal{E}_k l)\}^{\kappa_k}},$$

in (2.6) and (2.7), then we obtain the corollaries including the incomplete $\overline{I}$-function defined in equation (3.1) as:

Corollary 3.1. *If $\lambda, \mathfrak{p}, \mathfrak{q} \in \mathbb{C}; \Re(\lambda), \xi_1, \xi_2, a, b \geq 0; \mu \in \mathbb{N}$, $\mathfrak{r} \in \mathbb{R}^+, U \in \mathbb{Z}^+$, $A_{V,\mathfrak{O}}$ be arbitrary constants, real or complex and using the condition presented in* (1.15)*, then the result is obtained as:*

$$\begin{aligned} &\mathscr{M}_{\lambda,\mu}\left[{}^{\Gamma}\overline{I}_{p,\,q}^{m,\,n}\left[z\,x^{\xi_1}\,e^{-\frac{ax}{l}} \,\middle|\, \begin{array}{l} (\mathrm{u}_1,\mathcal{U}_1;\varpi_1 : t)\,,(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p} \\ (\mathrm{e}_j,\mathcal{E}_j;1)_{1,\,m},(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{m+1,\,q} \end{array}\right]\right. \\ &\quad \left.\times\, S_V^U\left[t\,x^{\xi_2}\,e^{\frac{-bx}{\mathfrak{O}}}\right]\right](\mathfrak{p},\mathfrak{q},\mathfrak{r}) \\ &= \frac{\mathfrak{r}^{-\mu\,\lambda}\,(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}\mu\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{q})}\sum_{\mathfrak{O}=0}^{[V/U]}\frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!}A_{V,\mathfrak{O}} \\ &\quad \times\left(t\left(\frac{\mathfrak{r}\,(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}+(a+b)\mathfrak{r}}\right)^{\xi_2}\right)^{\mathfrak{O}}\frac{1}{2\pi i}\int_{\Delta}B\left(\lambda+\frac{s}{\mu},-\frac{s}{\mu}\right)\left[\frac{\mathfrak{r}(\mathfrak{p}+(\mathfrak{a}+\mathfrak{b})\mathfrak{r})}{(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}\right]^{-s} \end{aligned}$$

$$\times\, {}^{\Gamma}\overline{I}^{m,\,n+1}_{p+1;q}\left[z\left(\frac{\mathfrak{r}\,(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}+(a+b)\mathfrak{r}}\right)^{\xi_1} \middle| \begin{array}{c} (\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t)\,,\left(1-\left(\frac{1+\xi_2\mathfrak{O}+s}{\mathfrak{p}\mathfrak{q}}\right),\frac{\xi_1}{\mathfrak{p}\mathfrak{q}},1\right) \\ (\mathrm{e}_j,\mathcal{E}_j;1)_{1,\,m}, \end{array} \right.$$
$$\left. \begin{array}{c} ,(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p} \\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{m+1,\,q} \end{array}\right] ds.$$

Corollary 3.2. *If* $\lambda,\mathfrak{p},\mathfrak{q}\in\mathbb{C};\Re(\lambda),\xi_1,\xi_2,a,b\geq 0;\mu\in\mathbb{N},\ \mathfrak{r}\in\mathbb{R}^+,U\in\mathbb{Z}^+$, $A_{V,\mathfrak{O}}$ *be arbitrary constants, real or complex and using the condition presented in* (1.15), *then the result is obtained as:*

$$\mathscr{M}_{\lambda,\mu}\left[{}^{\gamma}\overline{I}^{m,\,n}_{p,\,q}\left[z\,x^{\xi_1}\,e^{-\frac{ax}{l}} \middle| \begin{array}{c} (\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t)\,,(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p} \\ (\mathrm{e}_j,\mathcal{E}_j;1)_{1,\,m},(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{m+1,\,q} \end{array}\right]\right.$$
$$\left.\times\, S^U_V\left[t\,x^{\xi_2}\,e^{\frac{-bx}{\mathfrak{O}}}\right]\right](\mathfrak{p},\mathfrak{q},\mathfrak{r})$$
$$=\frac{\mathfrak{r}^{-\mu\,\lambda}\,(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}\mu\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{q})}\sum_{\mathfrak{O}=0}^{[V/U]}\frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!}A_{V,\mathfrak{O}}$$
$$\times\left(t\left(\frac{\mathfrak{r}\,(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}+(a+b)\mathfrak{r}}\right)^{\xi_2}\right)^{\mathfrak{O}}\frac{1}{2\pi i}\int_{\Delta}B\left(\lambda+\frac{s}{\mu},-\frac{s}{\mu}\right)\left[\frac{\mathfrak{r}(\mathfrak{p}+(\mathfrak{a}+\mathfrak{b})\mathfrak{r})}{(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}\right]^{-s}$$
$$\times\, {}^{\gamma}\overline{I}^{m,\,n+1}_{p+1;q}\left[z\left(\frac{\mathfrak{r}\,(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}+(a+b)\mathfrak{r}}\right)^{\xi_1} \middle| \begin{array}{c} (\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t)\,,\left(1-\left(\frac{1+\xi_2\mathfrak{O}+s}{\mathfrak{p}\mathfrak{q}}\right),\frac{\xi_1}{\mathfrak{p}\mathfrak{q}},1\right) \\ (\mathrm{e}_j,\mathcal{E}_j;1)_{1,\,m}, \end{array} \right.$$
$$\left. \begin{array}{c} ,(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p} \\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{m+1,\,q} \end{array}\right] ds.$$

(ii) Incomplete $\overline{H}$-function: If we set $\varpi_j=1$ for $n+1\leq j\leq p$ and $\kappa_j=1$ for $1\leq j\leq m$ in (1.15) and making use of the connection, that is (see [13, 22]):

(3.2)
$$\overline{\Gamma}^{m,\,n}_{p,\,q}(z)=\overline{\Gamma}^{m,\,n}_{p,\,q}\left[z\,\middle|\,\begin{array}{c} (\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t)\,,(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,n}\,,(\mathrm{u}_j,\mathcal{U}_j;1)_{n+1,\,p} \\ (\mathrm{e}_j,\mathcal{E}_j;1)_{1,\,m},(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{m+1,\,q} \end{array}\right]$$
$$=\overline{\Gamma}^{m,\,n}_{p,\,q}\left[z\,\middle|\,\begin{array}{c} (\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t)\,,(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,n}\,,(\mathrm{u}_j,\mathcal{U}_j)_{n+1,\,p} \\ (\mathrm{e}_j,\mathcal{E}_j)_{1,\,m},(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{m+1,\,q} \end{array}\right]$$
$$=\frac{1}{2\pi i}\int_{\pounds}\overline{\Psi}(l,t)\;z^l\;dl,$$

where,

$$\overline{\Psi}(l,t)=\frac{\{\Gamma(1-\mathrm{u}_1+\mathcal{U}_1 l,t)\}^{\varpi_1}\prod_{k=1}^{m}\Gamma(e_j-\mathcal{E}_j l)\prod_{k=2}^{n}\{\Gamma(1-\mathrm{u}_j+\mathcal{U}_j l)\}^{\varpi_j}}{\prod_{k=n+1}^{p}\Gamma(\mathrm{u}_j-\mathcal{U}_j l)\prod_{k=m+1}^{q}\{\Gamma(1-e_j+\mathcal{E}_j l)\}^{\kappa_j}},$$

in (2.6) and (2.7), then we obtain the corollaries including the incomplete $\overline{H}$-function defined in equation (3.2) as follows.

Corollary 3.3. *If* $\lambda,\mathfrak{p},\mathfrak{q}\in\mathbb{C};\Re(\lambda),\xi_1,\xi_2,a,b\geq 0;\mu\in\mathbb{N},\ \mathfrak{r}\in\mathbb{R}^+,U\in\mathbb{Z}^+$*,* $A_{V,\mathfrak{O}}$ *be arbitrary constants, real or complex and using the condition presented in* (1.15)*, then the result is obtained as:*

$$\mathscr{M}_{\lambda,\mu}\Bigg[\overline{\Gamma}^{m,n}_{p,q}\left[z\,x^{\xi_1}\,e^{-\frac{ax}{l}}\ \middle|\ \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,n},(\mathrm{u}_j,\mathcal{U}_j;1)_{n+1,p}\\(\mathrm{e}_j,\mathcal{E}_j;1)_{1,m},(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{m+1,q}\end{matrix}\right]$$
$$\times S_V^U\left[t\,x^{\xi_2}\,e^{\frac{-bx}{\mathfrak{O}}}\right](\mathfrak{p},\mathfrak{q},\mathfrak{r})\Bigg]$$
$$=\frac{\mathfrak{r}^{-\mu\lambda}\,(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}\mu\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{r})}\sum_{\mathfrak{O}=0}^{[V/U]}\frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!}A_{V,\mathfrak{O}}$$
$$\times\left(t\left(\frac{\mathfrak{r}\,(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}+(a+b)\mathfrak{r}}\right)^{\xi_2}\right)^{\mathfrak{O}}\frac{1}{2\pi i}\int_{\Delta}B\left(\lambda+\frac{s}{\mu},-\frac{s}{\mu}\right)\left[\frac{\mathfrak{r}(\mathfrak{p}+(\mathfrak{a}+\mathfrak{b})\mathfrak{r})}{(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}\right]^{-s}$$
$$\times\overline{\Gamma}^{m,\,n+1}_{p+1,\,q}\left[z\left(\frac{\mathfrak{r}\,(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}+(a+b)\mathfrak{r}}\right)^{\xi_1}\ \middle|\ \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),\left(1-\left(\frac{1+\xi_2\mathfrak{O}+s}{\mathfrak{p}\mathfrak{q}}\right),\frac{\xi_1}{\mathfrak{p}\mathfrak{q}},1\right)\\(\mathrm{e}_j,\mathcal{E}_j;1)_{1,m},\end{matrix}\right.$$
$$\left.\begin{matrix},(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,n},(\mathrm{u}_j,\mathcal{U}_j;1)_{n+1,p}\\(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{m+1,q}\end{matrix}\right]ds.$$

Corollary 3.4. *If* $\lambda,\ \mathfrak{p},\ \mathfrak{q}\in\mathbb{C};\Re(\lambda),\ \xi_1,\xi_2,a,\ b\geq 0;\mu\ \in\mathbb{N},\ \mathfrak{r}\in\mathbb{R}^+,U\in\mathbb{Z}^+$*,* $A_{V,\mathfrak{O}}$ *be arbitrary constants, real or complex and using the condition presented in* (1.15)*, then the result is obtained as:*

$$\mathscr{M}_{\lambda,\mu}\Bigg[\overline{\gamma}^{m,n}_{p,q}\left[z\,x^{\xi_1}\,e^{-\frac{ax}{l}}\ \middle|\ \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,n},(\mathrm{u}_j,\mathcal{U}_j;1)_{n+1,p}\\(\mathrm{e}_j,\mathcal{E}_j;1)_{1,m},(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{m+1,q}\end{matrix}\right]$$
$$\times S_V^U\left[t\,x^{\xi_2}\,e^{\frac{-bx}{\mathfrak{O}}}\right](\mathfrak{p},\mathfrak{q},\mathfrak{r})\Bigg]$$
$$=\frac{\mathfrak{r}^{-\mu\lambda}\,(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}\mu\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{r})}\sum_{\mathfrak{O}=0}^{[V/U]}\frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!}A_{V,\mathfrak{O}}$$

$$\times \left(t\left(\frac{\mathfrak{r}\,(\mathfrak{pq})^{\frac{1}{\mathfrak{pq}}}}{\mathfrak{p}+(a+b)\mathfrak{r}}\right)^{\xi_2}\right)^{\mathfrak{O}} \frac{1}{2\pi i}\int_{\Delta} B\left(\lambda+\frac{s}{\mu},-\frac{s}{\mu}\right)\left[\frac{\mathfrak{r}(\mathfrak{p}+(\mathfrak{a}+\mathfrak{b})\mathfrak{r})}{(\mathfrak{pq})^{\frac{1}{\mathfrak{pq}}}}\right]^{-s}$$

$$\times\ \overline{\gamma}^{m,\,n+1}_{p+1,\,q}\left[z\left(\frac{\mathfrak{r}\,(\mathfrak{pq})^{\frac{1}{\mathfrak{pq}}}}{\mathfrak{p}+(a+b)\mathfrak{r}}\right)^{\xi_1}\ \middle|\ \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),\left(1-\left(\frac{1+\xi_2\mathfrak{O}+s}{\mathfrak{pq}}\right),\frac{\xi_1}{\mathfrak{pq}},1\right),\\ (\mathrm{e}_j,\mathcal{E}_j;1)_{1,m},\end{matrix}\right.$$

$$\left.\begin{matrix}(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,n},(\mathrm{u}_j,\mathcal{U}_j;1)_{n+1,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{m+1,q}\end{matrix}\right]ds.$$

(iii) Laguerre Polynomial: If we set $A_{V,\mathfrak{O}} = \binom{V+\alpha}{V}\frac{1}{(\alpha+1)_{\mathfrak{O}}}$ and $U=1$ in (1.18) then $S^1_V[t] \to L^{(\alpha)}_V(t)$ and making use of the connection, that is (see [7]):

$$L^{\alpha}_V(t) = \sum_{\mathfrak{O}=0}^{V}\binom{V+\alpha}{V-\mathfrak{O}}\frac{(-t)^{\mathfrak{O}}}{\mathfrak{O}!}, \tag{3.3}$$

in (2.6) and (2.7), then we obtain the corollaries including the laguerre polynomial defined in equation (3.3) as follows.

Corollary 3.5. *If* $\lambda,\mathfrak{p},\mathfrak{q}\in\mathbb{C};\Re(\lambda),\xi_1,\xi_2,a,b\geq 0;\mu\in\mathbb{N},\ \mathfrak{r}\in\mathbb{R}^+$ *and using the condition presented in* (1.15)*, then the result is obtained as:*

$$\mathscr{M}_{\lambda,\mu}\left[{}^{\Gamma}I^{m,\,n}_{p,\,q}\left[z\,x^{\xi_1}\,e^{-\frac{ax}{l}}\ \middle|\ \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,q}\end{matrix}\right]\right.$$

$$\left.\times\, L^{\alpha}_V\left[t\,x^{\xi_2}\,e^{\frac{-bx}{\mathfrak{O}}}\right](\mathfrak{p},\mathfrak{q},\mathfrak{r})\right]$$

$$= \frac{\mathfrak{r}^{-\mu\lambda}\,(\mathfrak{pq})^{\frac{1}{\mathfrak{pq}}}}{\mathfrak{p}\mu\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{r})}$$

$$\times\sum_{\mathfrak{O}=0}^{[V]}\binom{V+\alpha}{V-\mathfrak{O}}\left(-t\left(\frac{\mathfrak{r}\,(\mathfrak{pq})^{\frac{1}{\mathfrak{pq}}}}{\mathfrak{p}+(a+b)\mathfrak{r}}\right)^{\xi_2}\right)^{\mathfrak{O}}\frac{1}{2\pi i}\int_{\Delta}B\left(\lambda+\frac{s}{\mu},-\frac{s}{\mu}\right)$$

$$\times\left[\frac{\mathfrak{r}(\mathfrak{p}+(\mathfrak{a}+\mathfrak{b})\mathfrak{r})}{(\mathfrak{pq})^{\frac{1}{\mathfrak{pq}}}}\right]^{-s}\left({}^{\Gamma}I^{m,\,n+1}_{p+1,\,q}\left[z\left(\frac{\mathfrak{r}\,(\mathfrak{pq})^{\frac{1}{\mathfrak{pq}}}}{\mathfrak{p}+(a+b)\mathfrak{r}}\right)^{\xi_1}\ \middle|\right.\right.$$

$$\left.\left.\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),\left(1-\left(\frac{1+\xi_2\mathfrak{O}+s}{\mathfrak{pq}}\right),\frac{\xi_1}{\mathfrak{pq}},1\right),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,q}\end{matrix}\right]\right)ds.$$

Corollary 3.6. *If* $\lambda, \mathfrak{p}, \mathfrak{q} \in \mathbb{C}; \Re(\lambda), \xi_1, \xi_2, a, b \geq 0; \mu \in \mathbb{N}$*,* $\mathfrak{r} \in \mathbb{R}^+$ *and using the condition presented in* (1.15)*, then the result is obtained as:*

$$\mathscr{M}_{\lambda,\mu}\left[{}^{\gamma}I_{p,\,q}^{m,\,n}\left[z\,x^{\xi_1}\,e^{-\frac{ax}{l}}\;\middle|\;\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t)\,,(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right.$$
$$\left.\times L_V^{\alpha}\left[t\,x^{\xi_2}\,e^{\frac{-bx}{\mathfrak{O}}}\right](\mathfrak{p},\mathfrak{q},\mathfrak{r})\right]$$
$$=\frac{\mathfrak{r}^{-\mu\,\lambda}\,(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}\mu\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{r})}$$
$$\times\sum_{\mathfrak{O}=0}^{[V]}\binom{V+\alpha}{V-\mathfrak{O}}\left(-t\left(\frac{\mathfrak{r}\,(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}+(a+b)\mathfrak{r}}\right)^{\xi_2}\right)^{\mathfrak{O}}\frac{1}{2\pi i}\int_{\Delta}B\left(\lambda+\frac{s}{\mu},-\frac{s}{\mu}\right)$$
$$\times\left[\frac{\mathfrak{r}(\mathfrak{p}+(\mathfrak{a}+\mathfrak{b})\mathfrak{r})}{(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}\right]^{-s}\left({}^{\gamma}I_{p+1,\,q}^{m,\,n+1}\left[z\left(\frac{\mathfrak{r}\,(\mathfrak{p}\mathfrak{q})^{\frac{1}{\mathfrak{p}\mathfrak{q}}}}{\mathfrak{p}+(a+b)\mathfrak{r}}\right)^{\xi_1}\;\middle|\right.\right.$$
$$\left.\left.\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t)\,,\left(1-\left(\frac{1+\xi_2\mathfrak{O}+s}{\mathfrak{p}\mathfrak{q}}\right),\frac{\xi_1}{\mathfrak{p}\mathfrak{q}},1\right),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right)ds.$$

Remark 3.7.
- If we set $U=1, A_{V,0}=1$ and $A_{V,\mathfrak{O}}=0\ \forall\ \mathfrak{O}\neq 0$ in Corollaries 3.1 and 3.2, then we get the special case (in terms of incomplete $\overline{I}$-function) of Theorems 2.9 and 2.10, respectively.
- If we set $U=1, A_{V,0}=1$ and $A_{V,\mathfrak{O}}=0\ \forall\ \mathfrak{O}\neq 0$ in Corollaries 3.3 and 3.4, then we get the special case (in terms of incomplete $\overline{H}$- function) of Theorems 2.9 and 2.10, respectively.

4. Concluding Remarks

In this article, we obtain the $\mathscr{M}$-transform for the incomplete I-function which is the extension of the I-function investigated by Jangid et al.[14] and we also study $\mathscr{M}$-transform for the product of incomplete I-function and the Srivastava Polynomial. As the incomplete I-function generalize a variety of incomplete functions like: I-function, Meijer G-function, hypergeometric function, H-function, $\bar{I}$-function, Mittag-Leffler function [20] and many other functions. Also, the Srivastava Polynomial generalize various other polynomial like: Hermite polynomial, Jacobi polynomial, Gegenbauer polynomial, Legendre polynomial, Tchebycheff polynomial, Gould-Hopper Polynomial and several other polynomials. Hence primary outcomes are significant and can help us to determine the number of different $\mathscr{M}$-transforms linked with numerous kinds of special functions and polynomials.

It is interesting to note that if we use the equation (1.3), (1.5), (1.7) and (1.9) one may easily obtain the well known transform like Laplace, natural, Sumudu and Srivastava-Luo-Raina M-transform involving the incomplete I-function and the family of polynomials as a special case of our main findings. Also, we can find the stieltjes transform defined in (1.10) by setting the value $\mathfrak{p} = \mathfrak{q} = 0$ in equation (1.1) as a special case of our main result.

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