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Score-Valued Decision Making Models Based on Inverse Soft Matrices

Seda Erdinç[1*] and Emin Aygün[2]

Abstract. In this work, we introduce some useful concepts in soft set theory, such as $\overline{\alpha}$-inverse intersection and $\overline{\alpha}$-inverse union of inverse soft sets, together with the type soft $\overline{\alpha}$-upper, $\overline{\alpha}$-lower, $\overline{\alpha}$-intersection and $\overline{\alpha}$-union of inverse soft matrices. Our main contribution is of proposing a new decision-making method associated with inverse soft sets and inverse soft matrices.

1. Introduction

We encounter many problems in our daily lives containing uncertainty or unknown data. Often, these problems are difficult to solve with existing mathematical models. In 1999, Molodtsov [4] introduced soft set theory as a mathematical tool to solve problems involving uncertainty concepts. Later, Biswas et al.[13] gave deeper results on the operations of soft sets. Moreover, general results have been extended to soft groups by Ali et al. [8], Aktaş and Çağman [9] and by Atagün and Aygün [2, 3] to soft substructures of rings, fields and modules, while Maji et al.[13] investigated some applications of soft set theory and developed some algorithms dealing with decision-making problems in daily life.

Recently, soft set theory has been used as a new mathematical tool to solve different decision-making problems, we may refer to Maji et al. [12, 13], where they gave representations in the form of binary information table of soft sets and Çağman and Enginoğlu [10] whose defined some new operations of the soft to introduce more recent decision-making

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method. At the same time, Çağman and Enginoğlu [11] studied soft matrices and decision- making methods in the setting of fuzzy sets.

In 2016, Çetkin et al. [15] introduced the concept of inverse soft theory in order to solve different decision-making problems. Khalil and Hassan [1] in 2019 improved some results for inverse soft sets on the same parameter set, while Kamacı et al. [5] in 2018 introduced the concept of cardinality inverse soft matrix and gave more developed results.

In this research paper, which is based on inverse soft sets, we introduce a new group decision-making method using a matrix representation of the inverse soft set in order to find the optimum object and the ranking order of objects in the decision problems involving one or more parameter sets. Other sections of this paper are arranged as follows. Section 2 is devoted to giving some basic principles of soft set theory and inverse soft set theory. Section 3, we introduces decision-making score-valued inverse soft sets models with applications. The last section summarizes our main results and and gives some concluding remarks.

2. Preliminaries

In this section, we review some basic concepts for future sections.

Definition 2.1 ([4])**.** Let V be a set of objects, S be a set of parameters and $\mathbb{P}(V)$ be the power set of V. A soft set on the V is defined as the ordered pairs

$$\delta = \{(s_i, F(s_i)) : s_i \in S, F(s_i) \in \mathbb{P}(V)\}$$

where F is the mapping given by $F : S \longrightarrow \mathbb{P}(V)$.

Definition 2.2 ([6])**.** Let V be a set of objects, S_t be a set of parameters and $\mathbb{P}(S_t)$ be the power set of S_t. An inverse soft set on the V is defined as the ordered pairs

$$\overline{F}^{V}_{S_t} = \left\{\left(v_j, \overline{f}(v_j)\right) : v_j \in V, \overline{f}(v_j) \in \mathbb{P}(S_t)\right\}$$

where $\overline{F}$ is the mapping given by $\overline{F} : V \longrightarrow \mathbb{P}(S_t)$. The set of all inverse soft set that have over the same universal set V is denoted by $ISS(V)$.

Definition 2.3 ([6])**.** Let V_r be a set of objects, S be a set of parameters and $\mathbb{P}(S)$ be the power set of S. An inverse soft set on the S is defined as the ordered pairs

$$\overline{F}^{S}_{V_r} = \left\{\left(v^r_j, \overline{f}(v^r_j)\right) : v_j \in V_r, \overline{f}(v^r_j) \in \mathbb{P}(S)\right\}$$

where $\overline{F}$ is the mapping given by $\overline{F} : V_r \longrightarrow \mathbb{P}(S)$. The set of all inverse soft sets that have over the same parameter set S is denoted by $ISS(S)$.

Example 2.4. Let $V = \{v_1, v_2, v_3, v_4, v_5\}$ be an universal set, $S_1 = \{s_1^1, s_2^1, s_3^1, s_4^1\}$ and $S_2 = \{s_1^2, s_2^2, s_3^2, s_4^2, s_5^2\}$ be two parameter set;

$$\begin{aligned}\overline{F} = \{&\left(v_1, \{s_1^1, s_2^1\}\right), \left(v_2, \{s_2^1, s_3^1, s_4^1\}\right), \left(v_3, \{s_3^1, s_4^1\}\right),\\ &\left(v_4, \{s_1^1, s_2^1, s_3^1\}\right), \left(v_5, \{s_1^1, s_4^1\}\right)\}\\ \overline{G} = \{&\left(v_1, \{s_3^2, s_4^2, s_5^2\}\right), \left(v_2, \{s_4^2, s_5^2\}\right), \left(v_3, \{s_2^2, s_3^2\}\right),\\ &\left(v_4, \{s_1^2, s_4^2, s_5^2\}\right), \left(v_5, \{s_2^2, s_3^2, s_4^2\}\right)\}\end{aligned}$$

then $\overline{F}$ and $\overline{G}$ are inverse soft sets that have a common universal set. $\overline{F}, \overline{G} \in ISS(V)$.

Example 2.5. Let $V_1 = \{v_1^1, v_2^1, v_3^1, v_4^1\}$, $V_2 = \{v_1^2, v_2^2, v_3^2, v_4^2, v_5^2\}$ be two universal sets and $S = \{s_1, s_2, s_3, s_4, s_5\}$ be a set of parameters.

$$\begin{aligned}\overline{F} = \{&\left(v_1^1, \{s_1, s_2, s_3\}\right), \left(v_2^1, \{s_4, s_5\}\right), \left(v_3^1, \{s_2, s_3, e_4\}\right),\\ &\left(v_4^1, \{s_2, s_3, s_4, s_5\}\right)\}\\ \overline{G} = \{&\left(v_1^2, \{s_1, s_3\}\right), \left(v_2^2, \{s_1, s_3, s_5\}\right), \left(v_3^2, \{s_1, s_2, s_3\}\right),\\ &\left(v_4^2, \{s_1, s_2, s_5\}\right), \left(v_5^2, \{s_4, s_5\}\right)\}\end{aligned}$$

then $\overline{F}$ and $\overline{G}$ are inverse soft sets that have a common parameter set. $\overline{F}, \overline{G} \in ISS(S)$.

Definition 2.6 ([15])**.** Let $\overline{F}$, $\overline{G} \in ISS(V)$. The inverse soft union of $\overline{F}$ and $\overline{G}$ is an inverse soft set, denoted by $\overline{F} \widetilde{\cup} \overline{G}$ is defined as $\left\{\left(v, \overline{F}(v) \cup \overline{G}(v)\right) \mid v \in V, \overline{F}(v), \overline{G}(v) \in \mathbb{P}(S)\right\}$.

Definition 2.7 ([15])**.** Let $\overline{F}$, $\overline{G} \in ISS(V)$. The inverse soft intersection of $\overline{F}$ and $\overline{G}$ is an inverse soft set, denoted by $\overline{F} \widetilde{\cap} \overline{G}$ is defined as $\left\{\left(v, \overline{F}(v) \cap \overline{G}(v)\right) \mid v \in V, \overline{F}(v), \overline{G}(v) \in \mathbb{P}(S)\right\}$.

Definition 2.8 ([1])**.** Let $\overline{F}$, $\overline{G} \in ISS(S)$, where $\overline{F} : V_m \longrightarrow \mathbb{P}(S)$, $\overline{G} : V_n \longrightarrow \mathbb{P}(S)$, then the inverse soft set "$\overline{F}$ AND $\overline{G}$" is defined as $(\overline{F}$ AND $\overline{G}) = (\overline{H}, Vm \times V_n)$ where $\overline{H}(v_i, v_j) = \overline{F}(v_i) \cap \overline{G}(v_j)$ for all $(v_i, v_j) \in V_m \times V_n$.

Definition 2.9 ([1])**.** Let $\overline{F}$, $\overline{G} \in ISS(S)$, where $\overline{F} : V_m \longrightarrow \mathbb{P}(S)$, $\overline{G} : V_n \longrightarrow \mathbb{P}(S)$, then the inverse soft set "$\overline{F}$ OR $\overline{G}$" is defined as ($\overline{F}$ OR $\overline{G}$)=$(\overline{H}, V_m \times V_n)$ where $\overline{H}(v_i, v_j) = \overline{F}(v_i) \cup \overline{G}(v_j)$ for all $v_i, v_j) \in V_m \times V_n$.

Definition 2.10 ([6])**.** Let $V_r = \{v_j^r, j \in J\}$, $S_t = \{s_i^t, i \in I\}$ where I, J are index sets then the inverse soft matrix for inverse soft set $\left(\overline{F}_{S_t}^{V_r} or \overline{F}_{V_r}^{S_t}\right)$ is $[a_{ij}]^{(t,r)}$, where for all i and j

$$[a_{ij}]^{(t,r)} = \begin{cases} 1, & \text{if } s_i^t \in \overline{F}(v_j^r),\\ 0, & \text{if } s_i^t \notin \overline{F}(v_j^r),\end{cases} \tag{2.1}$$

Therefore the inverse soft matrix corresponds to both the $ISS(V)$ described in Definition 2.2 and $ISS(S)$ described in Definition 2.3. The set of all inverse soft matrices over the same universal set V is denoted by $ISM(V)$. The set of all inverse soft matrices over the same parameter set S is denoted by $ISM(S)$.

Example 2.11. Let $V_1 = \{v_1^1, v_2^1, v_3^1, v_4^1\}$, be an universal set and $S_1 = \{s_1^1, s_2^1, s_3^1, s_4^1\}$, $S_2 = \{s_1^2, s_2^2, s_3^2, s_4^2, s_5^2\}$ be two set of parameters set.For the inverse soft sets

$$\overline{F}_{S_1}^{V_1} = \left\{\left(v_1^1, \left\{s_1^1, s_3^1, s_4^1\right\}\right), \left(v_2^1, \left\{s_2^1, s_3^1, s_4^1\right\}\right), \left(v_3^1, \left\{s_3^1, s_4^1\right\}\right), \left(v_4^1, \left\{s_1^1, s_2^1\right\}\right)\right\}$$

$$\overline{F}_{S_2}^{V_1} = \left\{\left(v_1^1, \left\{s_2^2, s_4^2\right\}\right), \left(v_2^1, \left\{s_3^2, s_5^2\right\}\right), \left(v_3^1, \left\{s_2^2, s_3^2, s_4^2\right\}\right), \left(v_4^1, \left\{s_1^2, s_3^2\right\}\right)\right\}$$

and

$$[a_{ij}]^{(1,1)} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{pmatrix}, \qquad [a_{kj}]^{(2,1)} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

are inverse soft matrices corresponding to $\overline{F}_{S_1}^{V_1}$ and $\overline{F}_{S_2}^{V_1}$, respectively.

Definition 2.12 ([6])**.** Let $[a_{pj}]^{(t_1,r)} \in ISM_{m_1\times r}, [b_{qj}]^{(t_2,r)} \in ISM_{m_2\times r}$. Then, the inverse soft matrix $[c_{sj}]^{(t_1\times t_2,r)}$ is called "And" row-product of $[a_{pj}]^{(t_1,r)}$ and $[b_{qj}]^{(t_2,r)}$, denoted $[a_{pj}]^{(t_1,r)} \wedge_r [b_{qj}]^{(t_2,r)}$, if

$$c_{sj} = \min\left\{{a_{pj}}^{(t_1,r)}, {b_{qj}}^{(t_2,r)}\right\}$$

where $s = (p-1)m_2 + q$.

Definition 2.13 ([6])**.** Let $[a_{pj}]^{(t_1,r)} \in ISM_{m_1\times r}, [b_{qj}]^{(t_2,r)} \in ISM_{m_2\times r}$. Then, the inverse soft matrix $[c_{sj}]^{(t_1\times t_2,r)}$ is called "Or" row-product of $[a_{pj}]^{(t_1,r)}$ and $[b_{qj}]^{(t_2,r)}$, denoted $[a_{pj}]^{(t_1,r)} \vee_r [b_{qj}]^{(t_2,r)}$, if

$$c_{sj} = \max\left\{{a_{pj}}^{(t_1,r)}, {b_{qj}}^{(t_2,r)}\right\}$$

where $s = (p-1)m_2 + q$.

Definition 2.14 ([6])**.** Let $[a_{pj}]^{(t_1,r)} \in ISM_{m_1\times r}, [b_{qj}]^{(t_2,r)} \in ISM_{m_2\times r}$. Then, the inverse soft matrix $[c_{sj}]^{(t_1\times t_2,r)}$ is called "And-Not" row-product of $[a_{pj}]^{(t_1,r)}$ and $[b_{qj}]^{(t_2,r)}$, denoted $[a_{pj}]^{(t_1,r)}\overline{\wedge}_r[b_{qj}]^{(t_2,r)}$, if $c_{sj} = \min\{{a_{pj}}^{(t_1,r)}, 1 - {b_{qj}}^{(t_2,r)}\}$ where $s = (p-1)m_2 + q$.

Definition 2.15 ([6])**.** Let $[a_{pj}]^{(t_1,r)} \in ISM_{m_1\times r}, [b_{qj}]^{(t_2,r)} \in ISM_{m_2\times r}$. Then, the inverse soft matrix $[c_{sj}]^{(t_1\times t_2,r)}$ is called "Or-Not" row-product

of $[a_{pj}]^{(t_1,r)}$ and $[b_{qj}]^{(t_2,r)}$, denoted $[a_{pj}]^{(t_1,r)}\overline{\bigtriangledown}_r[b_{qj}]^{(t_2,r)}$, if

$$c_{sj} = \max\left\{{a_{pj}}^{(t_1,r)}, 1 - {b_{qj}}^{(t_2,r)}\right\}$$

where $s = (p-1)m_2 + q$.

Example 2.16. Let's performe the operations $\wedge_r$ and $\vee_r$ operations of the inverse soft matrices given below.

$$[a_{ij}] = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{pmatrix}, \qquad [b_{kj}] = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

and

$$[a_{ij}] \wedge_r [b_{kj}] = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}, \qquad [a_{ij}] \vee_r [b_{kj}] = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

3. Decision Making With Score-Valued Inverse Soft Sets

Throughout this section,$V = \{v_1, v_2, \ldots, v_n\}$ is the universal set, $S = \{s_1, s_2, \ldots, s_m\}$ is the parameter set and $[a_{ij}] \in ISM(V)$, $ISM_{(m\times n)}$ denotes an the set of $m \times n$ inverse soft matrices over V.

Notations:

- $|a_{ij}|_j$ denotes j. column of the inverse soft matrix $[a_{ij}]$,
- $|0|_j$ means that the j. column of $[a_{ij}]$ consist of zeros,
- $|1|_j$ means that the j. column of $[a_{ij}]$ consist of 1.

So, we can express an inverse soft matrix $[a_{ij}]$ as $\bigcup_{j=1}^{m}|a_{ij}|_j$ by using their columns.

Definition 3.1. Let $\overline{F} \in ISS(V)$ and $\emptyset \neq \overline{\alpha} \subseteq S$.Then

(i) upper $\overline{\alpha}$-inverse inclusion of $\overline{F}$ is defined as

$$(\overline{F}, V)^{\supseteq\overline{\alpha}} = \{x \in V \mid \overline{F}(x) \supseteq \overline{\alpha}\}$$

(ii) lower $\overline{\alpha}$-inverse inclusion of $\overline{F}$ is defined as

$$(\overline{F}, V)^{\subseteq\overline{\alpha}} = \{x \in V \mid \overline{F}(x) \subseteq \overline{\alpha}\}.$$

Definition 3.2. Let $\overline{F} \in ISS(V)$ and $\emptyset \neq \overline{\alpha} \subseteq S$.Then

(i) $\overline{\alpha}$-inverse intersection of $\overline{F}$ is defined as

$$(\overline{F}, V)^{\cap\overline{\alpha}} = \{x \in V \mid \overline{F}(x) \cap \overline{\alpha} \neq \emptyset\}$$

(ii) $\overline{\alpha}$-inverse union of $\overline{F}$ is defined as

$$(\overline{F}, V)^{\cup\overline{\alpha}} = \{x \in V \mid \overline{F}(x) \cup \overline{\alpha} = S\}.$$

Definition 3.3. Let $[a_{ij}] \in ISM_{(m\times n)}$ and $\overline{\alpha} = \{s_i \mid i \in I \subseteq S\}$ where $I = \{1, 2, \ldots, m\}$. Then,

(i) $\overline{\alpha}$-upper inverse soft matrix of $[a_{ij}]$ denoted by $[a_{ij}]^{\overline{\alpha}}$ is defined as,

$$[a_{ij}]^{\overline{\alpha}} = \bigcup_{j=1}^{n} \begin{cases} |a_{ij}|_j, & \text{if } \forall i \in I, [a_{ij}] = 1 \\ |0|_j, & \text{otherwise} \end{cases} \tag{3.1}$$

(ii) $\overline{\alpha}$-lower inverse soft matrix of $[a_{ij}]$ denoted by $[a_{ij}]_{\overline{\alpha}}$ is defined as,

$$[a_{ij}]_{\overline{\alpha}} = \bigcup_{j=1}^{n} \begin{cases} |0|_j, & \text{if } \forall i \in \{1, 2, \ldots, m\}\backslash I, [a_{ij}] = 1 \\ |a_{ij}|_j, & \text{otherwise} \end{cases} \tag{3.2}$$

Definition 3.4. Let $[a_{ij}] \in ISM_{(m\times n)}$ and $\overline{\alpha} = \{s_i \mid i \in I \subseteq S\}$ where $I = \{1, 2, \ldots, m\}$. Then,

(i) $\overline{\alpha}$-intersection inverse soft matrix of $[a_{ij}]$ denoted by $[a_{ij}]^{\cap\overline{\alpha}}$ is defined as,

$$[a_{ij}]^{\cap\overline{\alpha}} = \bigcup_{j=1}^{n} \begin{cases} |a_{ij}|_j, & \text{if } \exists i \in I, [a_{ij}] = 1 \\ |0|_j, & \text{otherwise} \end{cases} \tag{3.3}$$

(ii) $\overline{\alpha}$-union inverse soft matrix of $[a_{ij}]$ denoted by $[a_{ij}]^{\cup\overline{\alpha}}$ is defined as,

$$[a_{ij}]^{\cup\overline{\alpha}} = \bigcup_{j=1}^{n} \begin{cases} |a_{ij}|_j, & \text{if } \forall i \in \{1, 2, \ldots, m\}\backslash I, [a_{ij}] = 1 \\ |0|_j, & \text{otherwise} \end{cases} \tag{3.4}$$

Definition 3.5. Let $[a_{ij}] \in ISM_{(m\times n)}$ The aggregate-column function, denoted by ς_j is defined by

$$\varsigma_j : ISM_{(m\times n)} \longrightarrow \mathbb{Z}, \quad \varsigma_j([a_{ij}]) = \sum_{i=1}^{m} [a_{ij}] \quad for j \in \{1, 2, \ldots, n\} \tag{3.5}$$

for brevity, $\varsigma_j([a_{ij}])$ value is shown as ς_j. Score-value for each column is defined as follows.

$$D_k = \sum_{j=1}^{n} \varsigma_{kj} = \sum_{j=1}^{n} \varsigma_k - \varsigma_j$$

Input: Algorithm 1
- Inverse soft sets.

Output: Optimum choice
Step 1: Decision maker choose feasible subsets of the parameter set and then create inverse soft sets.

Step 2: Construct the inverse soft matrix.
Step 3: Find the aggregate-column function (ς_j) and score-value (D_k) for each column.
for each $j \in \{1, 2, \ldots, n\}$ **do**
$\varsigma_j([a_{ij}]) = \sum_{i=1}^{m} [a_{ij}]$
end for
if $(D_j) = \max_k(D_k)$ **then**
v_j is optimum choice.
else
there is more than one j any v_j can selected.
end if
Step 4: output optimum choice

The step by step procedure of Algorithm 1 is illustrated in Figure 1.

Example 3.6. Suppose that a student determine some parameters for the profession student will decide. Let $V = \{v_1, v_2, v_3, v_4\}$ be the set of profession student and parameter set $S = \{s_1 = \text{future significance}, s_2 = \text{economic value}, s_3 = \text{area of interest}, s_4 = \text{ability}\}$

(i) $\overline{F} = \{(v_1, \{s_1, s_4\}), (v_2, \{s_1, s_2, s_3\}), (v_3, \{s_2\}), (v_4, \{s_1, s_2\})\}$

(ii) $[a_{ij}] = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

(iii) $\varsigma_j([a_{ij}]) = \sum_{i=1}^{m} [a_{ij}]$
$\varsigma_1 = a_{11} + a_{21} + a_{31} + a_{41} = 1 + 0 + 0 + 1 = 2,$
$\varsigma_2 = a_{12} + a_{22} + a_{32} + a_{42} = 1 + 1 + 1 + 0 = 3,$
$\varsigma_3 = a_{13} + a_{23} + a_{33} + a_{43} = 0 + 1 + 0 + 0 = 1,$
$\varsigma_4 = a_{14} + a_{24} + a_{34} + a_{44} = 1 + 1 + 0 + 0 = 2$

(iv) $D_k = \sum_{j=1}^{n} \varsigma_k - \varsigma_j \quad D_1 = 0, D_2 = 4, D_3 = -4, D_4 = 0$

(v) v_2 is optimum choice since $(D_j) = max_k(D_k) = D_2$

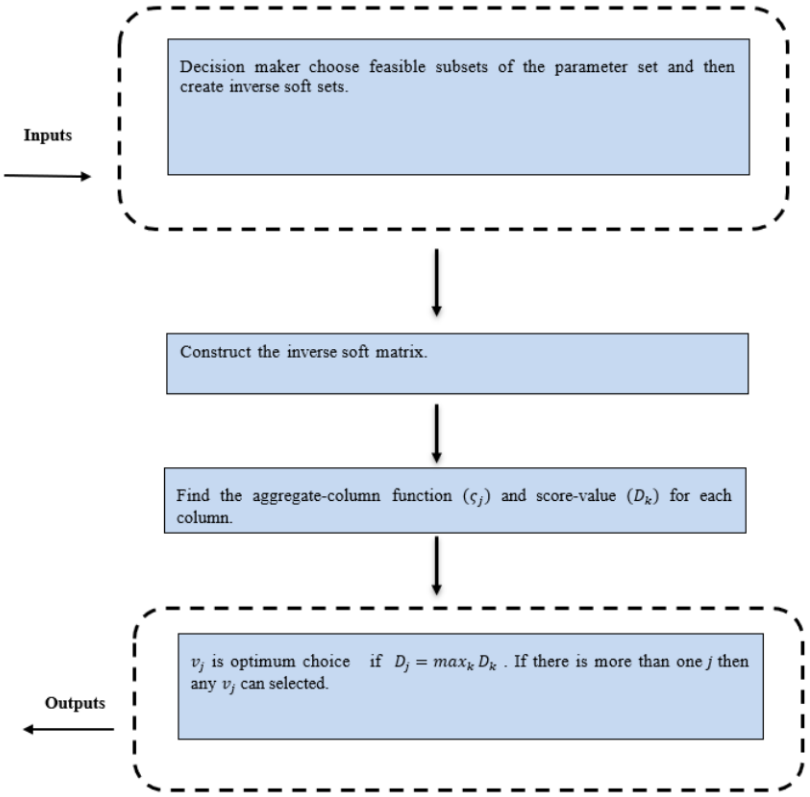


FIGURE 1. The framework of Algorithm 1

Input: Algorithm 2

- Inverse soft sets.

Output: Optimum choice

Step 1: Decision maker choose feasible subsets of the parameter set and then create inverse soft sets.

Step 2: Construct the inverse soft matrix.

Step 3: Find $D(v_j)(s_i) = R(v_j)(s_i) - C(v_j)(s_i)$.

for each $j \in \{1, 2, \ldots, n\}$ **do**

$C(v_j)(s_i) = |\{v_k \in V \mid a_{ij} \geq a_{ik}\}|$, $R(v_j)(s_i) = |\{v_k \in V \mid a_{ik} \geq a_{ij}]\}|$.

end for

if $(D_j) = \max_k(D_k)$ **then**

$D_j = \sum_{j=1}^{n} D(v_j)(s_i)$, v_j is optimum choice.

else

there is more than one j any v_j can selected.

end if

Step 4: output optimum choice

The step by step procedure of Algorithm 2 is illustrated in Figure 2.

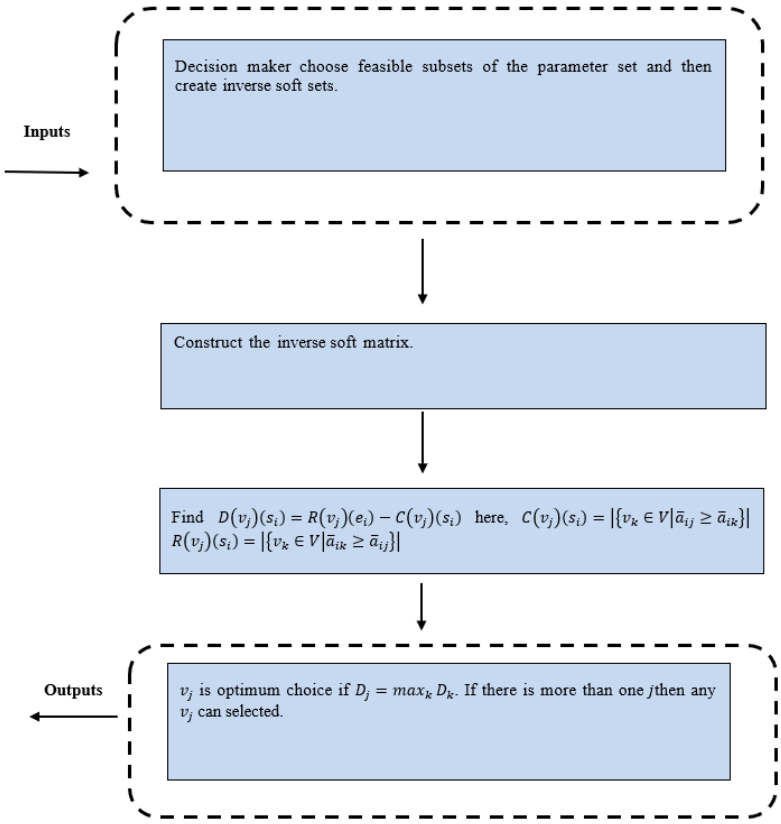


FIGURE 2. The framework of Algorithm 2

Example 3.7. Let us consider the decision making problem given in Example 3.6.

(i) $\overline{F} = \{(v_1, \{s_1, s_4\}), (v_2, \{s_1, s_2, s_3\}), (v_3, \{s_2\}), (v_4, \{s_1, s_2\})\}$

(ii) $[a_{ij}] = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

(iii) For $[a_{ij}]$ matrix, since $a_{11} \geq a_{11}$, $a_{12} \geq a_{11}$, $a_{14} \geq a_{11}$, $C(v_1)(s_1) = |\{v_1, v_2, v_4\}| = 3$ and $a_{11} \geq a_{11}$, $a_{11} \geq a_{12}$, $a_{11} \geq a_{13}$, $a_{11} \geq a_{14}$, $R(v_1)(s_1) = |\{v_1, v_2, v_3, v_4\}| = 4$, $C(v_1)(s_2) = |\{v_1, v_2, v_3, v_4\}| = 4R(v_1)(s_2) = |\{v_1\}| = 1$

(iv) $R(v_j)(s_i), C(v_j)(s_i)$ and $D(v_j)(s_i)$ values for each $i \in \{1, 2, 3, 4\} j \in \{1, 2, 3, 4\}$ are given in table below.

TABLE 1. Score-Value

	v_1	v_2	v_3	v_4
s_1	$R=4$ $C=3$ $D=1$	$R=4$ $C=3$ $D=1$	$R=1$ $C=4$ $D=-3$	$R=4$ $C=1$ $D=3$
s_2	$R=1$ $C=4$ $D=-3$	$R=4$ $C=3$ $D=1$	$R=4$ $C=3$ $D=1$	$R=4$ $C=3$ $D=1$
s_3	$R=3$ $C=4$ $D=-1$	$R=4$ $C=1$ $D=3$	$R=3$ $C=4$ $D=-1$	$R=3$ $C=4$ $D=-1$
s_4	$R=4$ $C=3$ $D=1$	$R=3$ $C=3$ $D=0$	$R=3$ $C=4$ $D=-1$	$R=3$ $C=4$ $D=-1$
D_j	$D_1=-2$	$D_2=5$	$D_3=-4$	$D_4=0$

(v) v_2 is optimum choice since $(D_j)=\max_k(D_k)=D_2$

Input: Algorithm 3 (Multi Participant Decision Making)

- Inverse soft sets.

Output: Optimum choice

Step 1: Decision maker choose feasible subsets of the parameter set and then create inverse soft sets.

Step 2: Construct the inverse soft matrix.

Step 3: Obtain matrices$[\vartheta_{pj}]=\wedge_r{}_{t=1}^{n}[a_{ij}^t]$.

for each $j\in\{1,2,\ldots,n\}$ **do**

aggregate-column function $\varsigma_j([\vartheta_{pj}])$ and score-value D_k for each column

end for

if $(D_j)=\max_k(D_k)$ **then**

$D_j=\sum_{j=1}^{n}D(v_j)(s_i)$, v_j is optimum choice.

else

there is more than one j any v_j can selected.

end if

Step 4: output optimum choice

The step by step procedure of Algorithm 3 is illustrated in Figure 3.

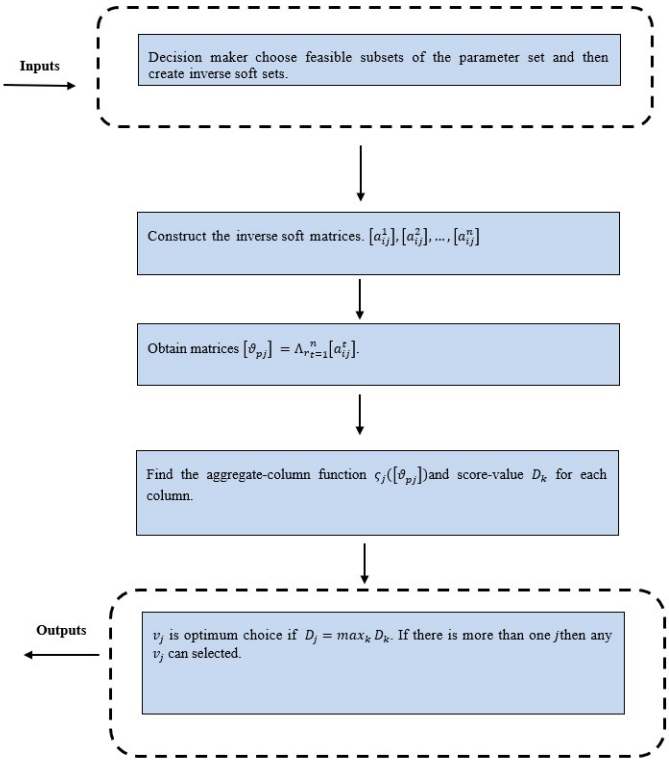


FIGURE 3. The framework of Algorithm 3

Note: In Step 4 of this algorithm, it can be taken $\vee_r, \overline{\wedge}_r, \overline{\vee}_r$ instead of $\wedge_r$ according to type of the problem.

Example 3.8. Business partners A and B want to buy a piece of land. Person A will evaluate the land economically and Person B will set the parameters related to the use of the land. $V = \{v_1, v_2, v_3, v_4\}$ is the set of lands they intend to buy.

$S_A = \{s_1^1 = \text{reasonable price}, s_2^1 = \text{transport}, s_3^1 = \text{location}\}$

$S_B = \{s_1^2 = \text{soil type}, s_2^2 = \text{proximity to water source}, s_3^2 = \text{the size of land}\}$

By determining the lands that correspond for each of their criteria, they created the following inverse soft sets as follows, respectively:

(i) $\overline{F}_A = \left\{\left(v_1, \{s_1^1, s_2^1\}\right), \left(v_2, \{s_1^1, s_3^1\}\right), \left(v_3, \{s_2^1\}\right), (v_4, \{s_1^1, s_2^1, s_3^1\})\right\}$

$\overline{F}_B = \left\{\left(v_1, \{s_1^2, s_2^2\}\right), \left(v_2, \{s_2^2, s_3^2\}\right), \left(v_3, \{s_1^2\}\right), \left(v_4, \{s_1^2, s_3^2\}\right)\right\}$

(ii) The inverse soft matrices of soft sets $\overline{F}_A$ and $\overline{F}_B$ are respectively:

$$[a_{ij}] = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \qquad [b_{ij}] = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

They obtain inverse soft matrix:

(iii) $[\vartheta_{pj}] = [a_{ij}] \wedge_r [b_{ij}] = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$

(iv) $[\varsigma_j([\vartheta_{pj}]) = \sum_{p=1}^{9} \vartheta_{pj},\ \varsigma_1 = 4,\ \varsigma_2 = 4,\ \varsigma_3 = 1,\ \varsigma_4 = 6,\ D_k = \sum_{j=1}^{n} \varsigma_{kj} = \varsigma_k - \varsigma_j D_1 = 1,\ D_2 = 1,\ D_3 = -11,\ D_4 = 9$

(v) v_4 is optimum choice since $(D_j) = \max_k(D_k) = D_4$
Consequently, v_4 is the optimal land to buy.

Input: Algorithm 4 (Set-Restricted Inverse SoftDecision Making)

- Inverse soft sets.

Output: Optimum choice

Step 1: Decision maker choose feasible subsets of the parameter set and then create inverse soft sets.

Step 2: Construct the inverse soft matrix.

Step 3: According to the $\overline{\alpha}$ set, create the $\overline{\alpha}-$ intersection inverse soft matrix $\overline{\alpha}-$ intersection inverse soft matrix $[a_{ij}^1]^{\cap\overline{\alpha}_1}, [a_{ij}^2]^{\cap\overline{\alpha}_2}, \ldots, [a_{ij}^n]^{\cap\overline{\alpha}_n}$. Obtain matrices $[\vartheta_{pj}] = \wedge_r {}_{t=1}^{n} [a^t ij]^{\cap\overline{\alpha}_t}$

for each $j \in \{1, 2, \ldots, n\}$ **do**
aggregate-column function $\varsigma_j([\vartheta_{pj}])$ and score-value D_k for each column
end for
if $(D_j) = \max_k(D_k)$ **then**
v_j is optimum choice.
else
there is more than one j any v_j can selected.

end if

Step 4: output optimum choice

The step by step procedure of Algorithm 4 is illustrated in Figure 4.

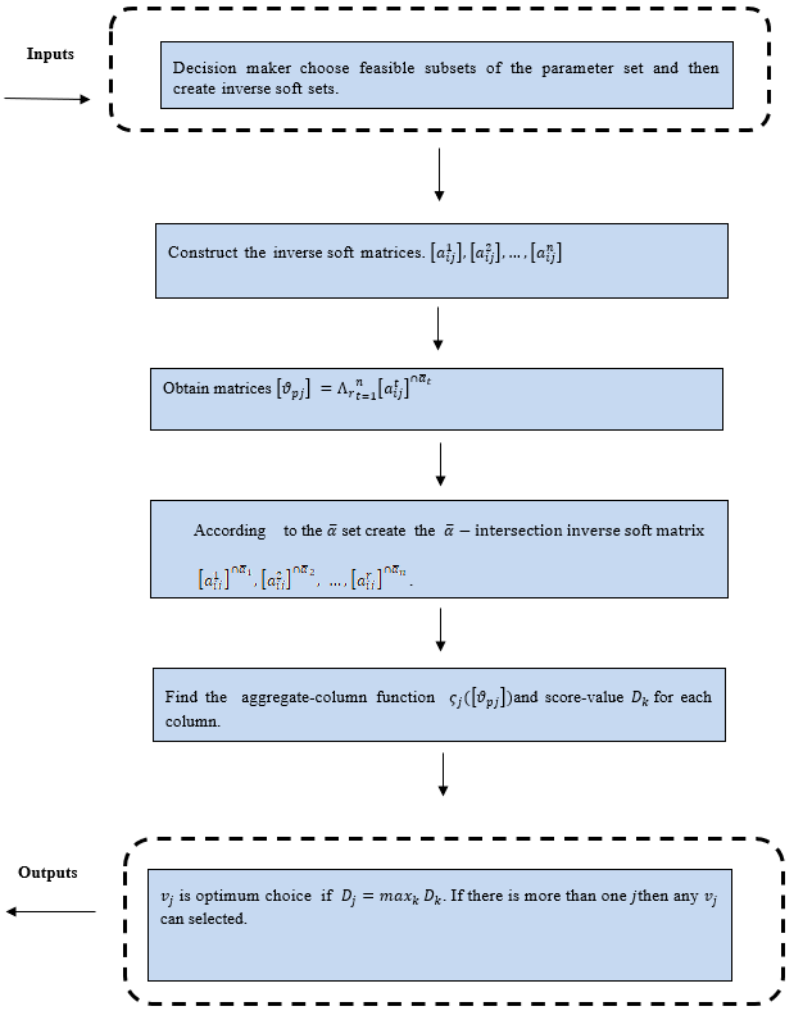


FIGURE 4. The framework of Algorithm 4

Note: In Step 3 of this algorithm, it can be taken $\overline{\alpha}$-union inverse soft matrix instead of $\overline{\alpha}$-intersection inverse soft matrix according to type of the problem. Furthermore in Step 4 of this algorithm, it can be taken $\vee_r, \overline{\wedge}_r, \overline{\vee}_r$ instead of $\wedge_r$ according to type of the problem.

Example 3.9. Let us consider the decision making problem given in Example 3.8. Let choose the sets $\overline{\alpha}_1 = \{s_2^1\}$ for Person A, $\overline{\alpha}_2 = \{s_3^2\}$ for Person B. By determining the lands that correspond for each of their

criteria, they created the following inverse soft sets as follows, respectively:

(i) $\overline{F}_A = \left\{\left(v_1, \{s_1^1, s_2^1\}\right), \left(v_2, \{s_1^1, s_3^1\}\right), \left(v_3, \{s_2^1\}\right), \left(v_4, \{s_1^1, s_2^1, s_3^1\}\right)\right\}$
$\overline{F}_B = \left\{\left(v_1, \{s_1^2, s_2^2\}\right), \left|\left(v_2, \{s_2^2, s_3^2\}\right), \left(v_3, \{s_1^2\}\right), \left(v_4, \{s_1^2, s_3^2\}\right)\right\}\right.$

(ii) The inverse soft matrices of soft sets $\overline{F}_A$ and $\overline{F}_B$ are respectively:

$$[a_{ij}] = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \qquad [b_{ij}] = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

Then, they obtain $\overline{\alpha}_t-$ intersection inverse soft matrices for $t = \{1, 2\}$ as follows:

(iii)

$$[a_{ij}]^{\cap\overline{\alpha}_1} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad [b_{ij}]^{\cap\overline{\alpha}_2} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

(iv) $[\vartheta_{pj}] = [a_{ij}]^{\cap\overline{\alpha}_1} \wedge_r [b_{ij}]^{\cap\overline{\alpha}_2} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(v) $(\varsigma_j([\vartheta_{pj}]) = \sum_{p=1}^{9} \vartheta_{pj}$, $\varsigma_1 = 0$, $\varsigma_2 = 0$, $\varsigma_3 = 0$, $\varsigma_4 = 6$, $D_k = \sum_{j=1}^{n} \varsigma_{kj} = \varsigma_k - \varsigma_j$, $D_1 = -6$, $D_2 = -6$, $D_3 = -6$, $D_4 = 18$

(vi) v_4 is optimum choice since $(D_j) = \max_k(D_k) = D_4$
Consequently, v_4 is the optimal land to buy.

Corollary 3.10. *In this research, we introduced some new useful concepts such as the $\overline{\alpha}-$ inverse intersection and $\overline{\alpha}-$ inverse union of inverse soft sets and the types of $\overline{\alpha}-$ upper, $\overline{\alpha}-$ lower, $\overline{\alpha}-$ intersection and $\overline{\alpha}-$ union of inverse soft matrices. Furthermore, we tried to develop new decision-making methods based on inverse soft sets and inverse soft matrices. The proposed decision-making approaches would be beneficial for applications in new research areas. These studies are expected to lead to new studies in inverse soft sets and inverse soft matrices. In addition to these, this paper can be made for the fuzzy and rough sets.*

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