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On μ^*-R_0 and μ^*-R_1 Spaces with Separation Axioms

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Abstract. In this paper, we define and investigate μ^*-R_0 and μ^*-R_1 spaces in a generalized topological space together with a topology. Independence of these spaces from the existing allied concepts is shown by examples, which motivates to explore them further. It is interesting to note that $(\mu X, \mu Y)$-continuous image of μ^*-R_0 space is neither μ^*-R_0 nor μ-R_0. Further, conditions under which the $(\mu X, \mu Y)$-continuous image of μ^*-R_0 space becomes μ^*-R_0 and μ-R_0 are established. Also, some new versions of separation axioms are defined and they are used as a tool to investigate μ^*-R_0 and μ^*-R_1 spaces. Further, the conditions under which these spaces coincide are obtained.

1. Introduction

The idea of generalized topology (in short, GT) and continuous type function between different generalized topological spaces (in short, GTS) was given by Á. Császár [3]. The author also studied some separation axioms in GTS [4]. Further, B. Roy studied characterizations of μ-R_0 and μ-R_1 spaces [6]. B. Roy and R. Sen used topology in GTS to introduce a new type of sets named as μ^* open (closed) sets [7, 8]. Again, R. K. Tiwari, J. K. Maitra and R. Vishwakarma explored more aspects of μ^*-open sets and the notion of μ^*-continuous mapping was initiated and studied [9]. Further, P. Chettri and S. Basnett studied more characteristics of μ^* open(closed) sets and μ^*-continuity [1]. Recently, the notion of contra (μ^*, μ)-continuity, μ^*-open mapping and the μ^*-kernel of a set have been initiated and explored [1]. To explore the notions of the above-mentioned spaces along with separation axioms in terms of

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μ^* sets and their interplay in a generalized topological space together with a topology, we introduce μ^*-R_0 and μ^*-R_1 spaces. The conditions under which the $(\mu X, \mu Y)$-continuous image of μ^*-R_0 space becomes μ^*-R_0 and μ-R_0 are established. Also, μ^*-R_0 and μ-R_0 spaces are studied via some separation axioms. It is observed that in a μ^*-R_1 space, the notions of μ^*-T_0, μ^*-T_1 and μ^*-T_2 coincide.

2. Preliminaries

We list some of the definitions and results used in this article. In a nonnull set X, $\mu X \subseteq P(X)$ is called a GT and $(X, \mu X)$ is called GTS if $\phi \in \mu X$ and arbitrary union of members of μX is in μX. Here, $A \in \mu X$ and $X - A$ are respectively called μ-open and μ-closed sets in X [3]. The generalized closure and interior of a set B on X, written as $c_\mu(B)$ and $i_\mu(B)$ respectively, are defined as $c_\mu(B) = \cap\{P \subseteq X : B \subseteq P, X - P \in \mu X\}$ and $i_\mu(B) = \cup\{Q \subseteq X : Q \subseteq B, Q \in \mu X\}$ [3, 5]. Also, for $P \subseteq X$, $c_\mu(X - P) = X - i_\mu(P)$ and $i_\mu(X - P) = X - c_\mu(P)$ [3]. Openness in topology and openness in generalized topology in X are independent [1].

By a space $(X, \mu X, \tau)$, we mean a GT μX and a topology τ on X. In a space $(X, \mu X, \tau)$, $P \subseteq X$ is termed as μ^*-open if $P \subseteq cl(i_\mu(P))$ and μ^*-closed if $i(c_\mu(P)) \subseteq P)$, where $i(B)$ and $cl(B)$ means usual interior and closure of B relative to τ [7]. In analogous to $c_\mu(B)$ and $i_\mu(B)$, the notions of $c_{\mu^*}(B)$ and $i_{\mu^*}(B)$ respectively are defined. In a space $(X, \mu X, \tau)$ for $A \subseteq X$ and $x \in X$, $x \in c_{\mu^*}(A)$ iff for any μ^*-open set P containing x, $A \cap P \neq \phi$ [2]. We denote the set of all μ^*-open sets and μ^*-closed sets by $\mu^*O(X_\tau)$ and $\mu^*C(X_\tau)$, respectively.

A mapping g from GTS $(X, \mu X)$ to $(Y, \mu Y)$ is termed as $(\mu X, \mu Y)$-continuous if $g^{-1}(Q) \in \mu X$ for any $Q \in \mu Y$ [3]. Here, g is named as μ^*-open [2] if $g(Q) \in \mu^*O(Y_\sigma)$ for any $Q \in \mu^*O(X_\tau)$. Also, g is called μ^*-closed [2] if $g(Q) \in \mu^*C(Y_\sigma)$ for any $Q \in \mu^*C(X_\tau)$.

In a space $(X, \mu X, \tau)$, for $P \subseteq X$, μ^*-kernel of P is written as μ^*-ker(P) and is defined as μ^*-ker$(P) = \bigcap\{S : P \subseteq S,\ S \in \mu^*O(X_\tau)\}$, if no such S exists then μ^*-ker$(P) = X$ [2]. A GTS $(X, \mu X)$ is termed as μ-R_0 if $\forall p \in P \in \mu X$, $c_\mu\{p\} \subseteq P$. It is called μ-R_1 if $\forall\ r, s \in X$ with $c_\mu\{r\} \neq c_\mu\{s\}$ $\exists\ P, Q \in \mu X$ such that $c_\mu\{r\} \subseteq P$ and $c_\mu\{s\} \subseteq Q$ and $P \cap Q = \phi$. Every μ-R_1 space is μ-R_0 but not conversely [6].

3. μ^*-R_0 Spaces

In this section we shall first define the concept of μ^*-R_0 space and show that this notion is independent of the existing notion of μ-R_0 space, which motivates us to study it further.

Definition 3.1. A space $(X, \mu X, \tau)$ is called μ^*-R_0 if for given any $u \in U \in \mu^*O(X_\tau)$, $c_{\mu^*}\{u\} \subseteq U$.

Let us see the following examples:

Example 3.2. Let us consider a space $(\mathbb{R},\ \mu\mathbb{R},\ \tau)$ where $\mathbb{R}$ is the set of real numbers,

$$\tau = \{\mathbb{R}, \phi, \mathbb{R} - \{3\}, \mathbb{R} - \{5\}, \mathbb{R} - \{3,5\}, \mathbb{R} - \{1,2,3,4,5\}\}$$

and

$$\mu\mathbb{R} = \{\mathbb{R}, \phi, \{1,2,3\}, \mathbb{R} - \{1,2,3\}\}.$$

Here,

$$\mu^*O(\mathbb{R}_\tau) = \{\mathbb{R}, \phi, \{1,2,3\}, \{1,2,3,4\}, \{1,2,3,5\}, \{1,2,3,4,5\}, \mathbb{R} - \{1,2,3\}\}$$

and

$$\mu^*C(X_\tau) = \{\phi, \mathbb{R}, \{1,2,3\}, \mathbb{R} - \{1,2,3\}, \mathbb{R} - \{1,2,3,4\}, \mathbb{R} - \{1,2,3,5\}, \mathbb{R} - \{1,2,3,4,5\}\}$$

Clearly, $(\mathbb{R}, \mu\mathbb{R}, \tau)$ is μ-R_0 space but not μ^*-R_0 because $4 \in \{1,2,3,4\} \in \mu^*O(\mathbb{R}_\tau)$ but $c_{\mu^*}(\{4\}) = \mathbb{R} - \{1,2,3\} \nsubseteq \{1,2,3,4\}$.

Example 3.3. Let us consider a space $(\mathbb{R},\ \mu\mathbb{R},\ \tau)$ where $\mathbb{R}$ is the set real numbers, $\tau = \{\mathbb{R}, \phi, \{1,\ 2\}\}$ and $\mu\mathbb{R} = \{\mathbb{R}, \phi, \{1\}, \{1,\ 2\}, \{2\}\}$. Here,

$$\mu^*O(\mathbb{R}_\tau) = \{\mathbb{R}, \phi, \{1\}, \{1,\ 2\}, \{2\}, \mathbb{R} - \{1\}, \mathbb{R} - \{2\}, \mathbb{R} - \{1,\ 2\}\}.$$

Clearly, $(\mathbb{R},\ \mu\mathbb{R},\ \tau)$ is μ^*-R_0 space but not μ-R_0 because $1 \in \{1,2\} \in \mu\mathbb{R}$ but $c_\mu(\{1\}) = \mathbb{R} - \{2,\} \nsubseteq \{1,2\}$.

So from the above examples, Example 3.2 and Example 3.3 we can conclude that μ-R_0 space and μ^*-R_0 space are independent of each other.

Now let's characterize μ^*-R_0 space.

Theorem 3.4. *In a space $(X, \mu X, \tau)$ the following conditions are equivalent.*

(1) *$(X, \mu X, \tau)$ is a μ^*-R_0 space.*
(2) *For each $S \in \mu^*C(X_\tau)$ with $u \notin S$ then $S \subseteq P$ and $u \notin P$ for some $P \in \mu^*O(X_\tau)$.*
(3) *For each $S \in \mu^*C(X_\tau)$ with $u \notin S$ then $S \cap c_{\mu^*}(\{u\}) = \phi$.*
(4) *For all $u, v(\neq u) \in X$ either $c_{\mu^*}(\{u\}) = c_{\mu^*}(\{v\})$ or $c_{\mu^*}(\{u\}) \cap c_{\mu^*}(\{v\}) = \phi$.*

Proof. (1) $\Rightarrow$ (2) Let $S \in \mu^*C(X_\tau)$ with $u \notin S$. So, $u \in X - S \in \mu^*O(X_\tau)$ and $c_{\mu^*}(\{u\}) \subseteq X - S$. Taking $P = X - c_{\mu^*}(\{u\})$, $P \in \mu^*O(X_\tau)$ with $S \subseteq P$ and $u \notin P$.

(2) $\Rightarrow$ (3) Let $S \in \mu^*C(X_\tau)$ with $u \notin S$, then there exists $P \in \mu^*O(X_\tau)$ such that $S \subseteq P$ and $u \notin P$. As $P \in \mu^*O(X_\tau)$, $P \cap c_{\mu^*}(\{u\}) = \phi$ and hence $S \cap c_{\mu^*}(\{u\}) = \phi$.

(3) $\Rightarrow$ (4) For all $u, v(\neq u) \in X$, either $c_{\mu^*}(\{u\}) = c_{\mu^*}(\{v\})$ or $c_{\mu^*}(\{u\}) \neq c_{\mu^*}(\{v\})$. If $c_{\mu^*}(\{u\}) = c_{\mu^*}(\{v\})$ then it is done. If not, then there exists $w \in c_{\mu^*}(\{u\})$ satisfying $w \notin c_{\mu^*}(\{v\})$. So, there exists $Q \in \mu^*O(X_\tau)$ which contains w and $Q \cap \{v\} = \phi$. As $w \in c_{\mu^*}(\{u\}$, $u \in Q$ and $u \notin c_{\mu^*}(\{v\})$, we have

$$c_{\mu^*}(\{u\}) \cap c_{\mu^*}(\{v\}) = \phi.$$

(4) $\Rightarrow$ (1) Let $u \in Q \in \mu^*O(X_\tau)$ and $v \notin Q$ then $u \neq v$. If $u \in c_{\mu^*}(\{v\})$ then $Q \cap \{v\} \neq \phi$ i.e. $v \in Q$. So, $u \notin c_{\mu^*}(\{v\})$ and $c_{\mu^*}(\{u\}) \neq c_{\mu^*}(\{v\})$. Now, $c_{\mu^*}(\{u\}) \cap c_{\mu^*}(\{v\}) = \phi, \forall\, v \notin Q$ and hence $c_{\mu^*}(\{u\}) \cap \left[\cup_{v \in X-Q} c_{\mu^*}(\{v\})\right] = \phi$. As, $u \in Q \in \mu^*O(X_\tau)$ and $v \in X - Q \in \mu^*C(X_\tau)$, we have $v \in c_{\mu^*}(\{v\}) \subseteq (X - Q)$. So, $X - Q = \cup_{v \in X-Q} c_{\mu^*}(\{v\})$ and $(X - Q) \cap c_{\mu^*}(\{u\}) = \phi$. Hence, $c_{\mu^*}(\{u\}) \subseteq Q$. □

Theorem 3.5. *A space $(X, \mu X, \tau)$ is μ^*-R_0 space iff for any $u, v \in X$, μ^*-ker$(\{u\}) \neq \mu^*$-ker$(\{v\})$ implies μ^*-ker$(\{u\}) \cap \mu^*$-ker$(\{v\}) = \phi$.*

Proof. Let $(X, \mu X, \tau)$ be a μ^*-R_0 space and $u, v \in X$ with μ^*-ker$(\{u\}) \neq \mu^*$-ker$(\{v\})$. If possible let $w \in \mu^*$-ker$(\{u\}) \cap \mu^*$-ker$(\{v\})$. Now, we claim that $u \in c_{\mu^*}(\{w\})$, because if $u \notin c_{\mu^*}(\{w\})$ then $\exists\ G \in \mu^*O(X_\tau)$ satisfying $u \in G$ and $w \notin G$, which gives $w \notin \mu^*$-ker$(\{u\})$, which is a contradiction. Hence, $u \in c_{\mu^*}(\{w\})$. Thus $u, w \in c_{\mu^*}(\{w\})$ and hence $c_{\mu^*}(\{u\}) \cap c_{\mu^*}(\{w\}) \neq \phi$. Similarly, $c_{\mu^*}(\{v\}) \cap c_{\mu^*}(\{w\}) \neq \phi$. By Theorem (3.4), $c_{\mu^*}(\{u\}) = c_{\mu^*}(\{v\})$. Now, we shall show μ^*-ker$(\{u\}) = \mu^*$-ker$(\{v\})$. If possible, let μ^*-ker$(\{u\}) \neq \mu^*$-ker$(\{v\})$ then $\exists\ w \in X$ such that $w \in \mu^*$-ker$(\{u\})$ and $w \notin \mu^*$-ker$(\{v\})$, which implies $u \in c_{\mu^*}(\{w\})$ and $v \notin c_{\mu^*}(\{w\})$. As $u \in c_{\mu^*}(\{w\})$, $c_{\mu^*}(\{u\}) \subseteq c_{\mu^*}(\{w\})$. So, $v \notin c_{\mu^*}(\{u\})$ and thus, $c_{\mu^*}(\{u\}) \neq c_{\mu^*}(\{v\})$, a contradiction to the fact that $c_{\mu^*}(\{u\}) = c_{\mu^*}(\{v\})$. So, μ^*-ker$(\{u\}) = \mu^*$-ker$(\{v\})$. Hence, μ^*-ker$(\{u\}) \cap \mu^*$-ker$(\{v\}) = \phi$.

Conversely, let the condition holds. We claim that if $c_{\mu^*}(\{u\}) \neq c_{\mu^*}(\{v\})$ then μ^*-ker$(\{u\}) \neq \mu^*$-ker$(\{v\})$. $c_{\mu^*}(\{u\}) \neq c_{\mu^*}(\{v\})$ implies $\exists$ $p \in c_{\mu^*}(\{u\})$ and $p \notin c_{\mu^*}(\{v\})$. Now, $p \notin c_{\mu^*}(\{v\})$ assures existence of $V \in \mu^*O(X_\tau)$ containing p and $u \in V$ but $v \notin V$. So, $v \notin \mu^*$-ker$(\{u\})$ thus, μ^*-ker$(\{u\}) \neq \mu^*$-ker$(\{v\})$. Using the given condition μ^*-ker$(\{u\})$ $\cap$ μ^*-ker$(\{v\}) = \phi$. Now, we prove that $c_{\mu^*}(\{v\}) \cap c_{\mu^*}(\{u\}) = \phi$. If $w \in c_{\mu^*}(\{v\}) \cap c_{\mu^*}(\{u\})$ then $u \in \mu^*$-ker$(\{w\})$ and $v \in \mu^*$-ker$(\{w\})$. So, μ^*-ker$(\{u\}) \cap \mu^*$-ker$(\{w\}) \neq \phi$ and μ^*-ker$(\{v\}) \cap \mu^*$-ker$(\{w\}) \neq \phi$. Now, μ^*-ker$(\{u\}) = \mu^*$-ker$(\{v\}) = \mu^*$-ker$(\{w\})$, a contradiction. So, $c_{\mu^*}(\{v\}) \cap c_{\mu^*}(\{u\}) = \phi$. Consequently, by Theorem (3.4), $(X, \mu X, \tau)$ is μ^*-R_0 space. □

Theorem 3.6. *In a space $(X, \mu X, \tau)$, we have the following equivalent conditions.*

(1) $(X, \mu X, \tau)$ *is a* μ^*-R_0 *space.*
(2) *For* $D(\neq \phi) \subseteq X$ *and* $R \in \mu^*O(X_\tau)$ *with* $D \cap R \neq \phi$, $\exists\ S \in \mu^*C(X_\tau)$ *such that* $D \cap S \neq \phi$ *and* $S \subseteq R$.
(3) *For any* $R \in \mu^*O(X_\tau)$, $R = \cup\{S : S \subseteq R, S \in \mu^*C(X_\tau)\}$.
(4) *For any* $S \in \mu^*C(X_\tau)$, $S = \cap\{R : S \subseteq R, R \in \mu^*O(X_\tau)\}$.
(5) *For all* $u \in X$, $c_{\mu^*}(\{u\}) \subseteq \mu^*$-$ker(\{u\})$.

Proof. (1) $\Rightarrow$ (2) Let $D(\neq \phi) \subseteq X$ and $R \in \mu^*O(X_\tau)$ with $D \cap R \neq \phi$. If $u \in D \cap R$, then $u \in R$ and $c_{\mu^*}(\{u\}) \subseteq R$. Taking $S = c_{\mu^*}(\{u\})$ we have $S \in \mu^*C(X_\tau)$, $S \subseteq R$ and $D \cap S \neq \phi$.

(2) $\Rightarrow$ (3) Let $R \in \mu^*O(X_\tau)$. Clearly,

$$\cup\{A : A \subseteq R,\ A \in \mu^*C(X_\tau)\} \subseteq R. \tag{a}$$

For any $u \in R$, assume $D = \{u\}$, then $D \cap R \neq \phi$. So, by (2) $\exists$ $S \in \mu^*C(X_\tau)$ such that $D \cap S \neq \phi$ and $S \subseteq R$. Clearly, $u \in S$ and $S \subseteq R$. Thus, $u \in S \subseteq \cup\{A : A \subseteq R, A \in \mu^*C(X_\tau)\}$.

$$\therefore R \subseteq \cup\{S : S \subseteq R,\ S \in \mu^*C(X_\tau)\}. \tag{b}$$

From equation (a) and (b) we get,

$$R = \cup\{A : A \subseteq R, A \in \mu^*C(X_\tau)\}.$$

(3) $\Rightarrow$ (4) Straightforward.
(4) $\Rightarrow$ (5) Let $u \in X$ satisfying $v \notin \mu^*\text{-ker}\{u\}$, then there exists $Q \in \mu^*O(X_\tau)$ with $u \in Q$, $Q \cap \{v\} = \phi$ which implies $u \neq v$ and $v \notin Q$. So, $C_{\mu^*}(\{v\}) \cap Q = \phi$ and by hypothesis, $\cap\{\text{R} \in \mu^*O(X_\tau)$: $C_{\mu^*}\{v\} \subseteq R\} \cap Q = \phi$. So, there exists R (not containing u) $\in \mu^*O(X_\tau)$ with $C_{\mu^*}\{v\} \subseteq R$ and $C_{\mu^*}\{u\} \cap R = \phi$, $v \notin C_{\mu^*}\{u\}$. Thus $C_{\mu^*}\{u\} \subseteq \mu^*\text{-ker}\{u\}$.
(5) $\Rightarrow$ (1) Let $R \in \mu^*O(X_\tau)$ satisfying $u \in R$, $v \in \mu^*\text{-ker}\{u\}$. This gives, $u \in c_{\mu^*}(\{v\})$ and $v \in R$. Thus, $\mu^*\text{-ker}\{u\} \subseteq R$. Also, $u \in c_{\mu^*}(\{u\}) \subseteq \mu^*\text{-ker}\{u\} \subseteq R$. Hence, the result. □

Corollary 3.7. *A space* $(X, \mu X, \tau)$ *is* μ^*-R_0 *space iff* $c_{\mu^*}(\{u\}) = \mu^*$-$ker\{u\}$ *for any all* $u \in X$.

Proof. In a μ^*-R_0 space $(X, \mu X, \tau)$, using Theorem (3.6), $c_{\mu^*}(\{u\}) \subseteq \mu^*$-ker$(\{u\})$ $\forall\ u \in X$. If $v \in \mu^*\text{-ker}(\{u\})$ then $u \in c_{\mu^*}(\{v\})$. So, $c_{\mu^*}(\{u\}) = c_{\mu^*}(\{v\})$ and $v \in c_{\mu^*}(\{u\})$. Thus, $\mu^*\text{-ker}(\{u\}) \subseteq c_{\mu^*}(\{u\})$. Consequently, $c_{\mu^*}(\{u\}) = \mu^*\text{-ker}\{u\}$. Converse part is a direct consequence of the Theorem (3.6). □

Theorem 3.8. *In a space* $(X, \mu X, \tau)$, *we have the following equivalent conditions.*

(1) $(X, \mu X, \tau)$ *is a* μ^*-R_0 *space.*
(2) $t \in c_{\mu^*}(\{s\})$ *iff* $s \in c_{\mu^*}(\{t\})$.

Proof. (1) $\Rightarrow$ (2) In a μ^*-R_0 space $(X, \mu X, \tau)$, if $t \in c_{\mu^*}(\{s\})$ then $c_{\mu^*}(\{t\}) = c_{\mu^*}(\{s\})$. So, $s \in c_{\mu^*}(\{t\})$. Similarly, another part can be shown.

(2) $\Rightarrow$ (1) Let $t \in A \in \mu^* O(X_\tau)$ and $s \notin A$ then $t \notin c_{\mu^*}(\{s\})$. So, $s \notin c_{\mu^*}(\{t\})$ and $c_{\mu^*}(\{t\}) \subseteq A$. Hence, the result. □

Theorem 3.9. *In a space $(X, \mu X, \tau)$, the following are equivalent.*

(1) *$(X, \mu X, \tau)$ is a μ^*-R_0 space.*
(2) *If $P \in \mu^* C(X_\tau)$, then $P = \mu^*$-$ker(P)$.*
(3) *If $P \in \mu^* C(X_\tau)$ and $u \in P$ then μ^*-ker$(\{u\}) \subseteq P$.*
(4) *For any $u \in X$, μ^*-$ker\{u\} \subseteq c_{\mu^*}(\{u\})$.*

Proof. (1) $\Rightarrow$ (2) Follows from Theorem (3.6).

(2) $\Rightarrow$ (3) As μ^*-ker$A \subseteq \mu^*$-ker$B$ for $A \subseteq B$, the result follows.

(3) $\Rightarrow$ (4) As $u \in c_{\mu^*}(\{u\})$ and $c_{\mu^*}(\{u\}) \in \mu^* C(X_\tau)$, the result follows.

(4) $\Rightarrow$ (1) Let $u \in c_{\mu^*}(\{v\})$ then, $v \in \mu^* - \ker(\{u\}) \subseteq c_{\mu^*}(\{u\})$. Also, let $v \in c_{\mu^*}(\{u\})$ then $u \in \mu^* - \ker(\{v\}) \subseteq c_{\mu^*}(\{v\})$. Using Theorem (3.8), the result follows. □

Now, we intend to find the image of μ^*-R_0 space under $(\mu X, \mu Y)$-continuous function. For this we consider the following example:

Example 3.10. Let us consider a space $(X, \mu X, \tau)$ and $(Y, \mu Y, \sigma)$ where $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{b\}\}$ and $\mu X = \{\phi, \{b\}, \{c, d\}, \{b, c, d\}\}$. Here,

$$\mu^* O(X_\tau) = \{\phi, \{b\}, \{c, d\}, \{b, c, d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}\}.$$

Again, consider another space $(Y, \mu Y, \sigma)$ with

$$Y = \{p, q, r\}, \qquad \sigma = \{\phi, Y, \{q\}, \{r\}, \{q, r\}\}$$

and $\mu Y = \{\phi, \{q\}, \{q, r\}\}$. Here, $\mu^* O(Y_\sigma) = \{\phi, Y, \{q\}, \{q, r\}, \{p, q\}\}$. Let us define a function f from $(X, \mu X, \tau)$ to $(Y, \mu Y, \sigma)$ as $f(a) = p, f(b) = q, f(c) = f(d) = r$. Clearly, f is $(\mu X, \mu Y)$-continuous and $(X, \mu X, \tau)$ is μ^*-R_0 but the space $(Y, \mu Y, \sigma)$ is not μ^*-R_0 because $c_{\mu^*}(\{q\}) = Y \nsubseteq \{q, r\} \in \mu^* O(Y_\sigma)$. Also, $(Y, \mu Y, \sigma)$ is not μ-R_0 as $c_\mu(\{r\}) = \{p, r\} \nsubseteq \{q, r\} \in \mu Y$.

Hence, $(\mu X, \mu Y)$-continuous image of μ^*-R_0 space is neither μ^*-R_0 nor μ-R_0.

Now, we shall find conditions under which $(\mu X, \mu Y)$-continuous image of μ^*-R_0 space becomes μ^*-R_0 and also μ-R_0.

First, let us consider the Lemmas.

Lemma 3.11. *A function $g : (X, \mu X, \tau) \to (Y, \mu Y)$ is μ^*-closed iff $c_{\mu^*}(f(A)) \subseteq f(c_{\mu^*}(A))$ for any $A \subseteq X$.*

Proof. Let f be a given map and A be any set on X. Since $c_{\mu^*}(A)$ is μ^*-closed set on X, $f(c_{\mu^*}(A))$ is μ^*-closed set on Y. As $A \subseteq c_{\mu^*}(A)$, $f(A) \subseteq f(c_{\mu^*}(A))$. So $c_{\mu^*}(f(A)) \subseteq c_{\mu^*}(f(c_{\mu^*}(A))) = f(c_{\mu^*}(A))$.
Conversely, let A be any μ^*-closed set on X, then $f(A) = f(c_{\mu^*}(A)) \supseteq c_{\mu^*}f(A)$. As $f(A) \subseteq c_{\mu^*}f(A)$, $f(A) = c_{\mu^*}f(A)$ i.e $f(A)$ is μ^*-closed set on Y, hence the result. □

Lemma 3.12. *A function $g : (X, \mu X, \tau) \to (Y, \mu Y)$ takes a μ^*-closed set A in X to μ-closed set $f(A)$ in Y iff $c_\mu(f(A)) \subseteq f(c_{\mu^*}(A))$ where $A \subseteq X$.*

Proof. Similar to Lemma (3.11) □

Theorem 3.13. *The $(\mu X, \mu Y)$-continuous image of a μ^*-R_0 space is μ^*-R_0 space if it is surjective, μ^*-closed map and $\mu^*O(Y_\sigma) = \mu Y$.*

Proof. In a space $(Y, \mu Y, \sigma)$, let $s \in V \in \mu^*O(Y_\sigma) = \mu Y$. As, $\mu^*O(Y_\sigma) = \mu Y$, $f^{-1}(V) \in \mu X$ and hence $f^{-1}(V) \in \mu^*O(X_\tau)$. $(X, \mu X, \tau)$ being μ^*-R_0 and f surjective, $\exists\ r \in f^{-1}(s) \in X$ and $c_{\mu^*}(\{r\}) \subseteq f^{-1}(V)$. By Lemma (3.11), $C_{\mu^*}(\{s\}) = C_{\mu^*}(\{f(r)\}) \subseteq f(C_{\mu^*}(\{r\})) \subseteq f(f^{-1}(V)) = V$. Hence, $(Y, \mu Y, \sigma)$ is μ^*-R_0 space. □

Theorem 3.14. *The $(\mu X, \mu Y)$-continuous image of a μ^*-R_0 space is μ-R_0 space if the function is surjective and it takes a μ^*-closed set P in X to μ-closed set $f(P)$ in Y.*

Proof. Using Lemma (3.12) and proceeding as in Theorem (3.13) the proof follows. □

4. μ^*-R_1 Spaces

In this section we initiate the notion of μ^*-R_1 space.

Definition 4.1. A space $(X, \mu X, \tau)$ is μ^*-R_1 if for any $u, v \in X$ satisfying $c_{\mu^*}\{u\} \neq c_{\mu^*}\{v\}$, $\exists\ P, Q \in \mu^*O(X_\tau)$ with $c_{\mu^*}\{u\} \subseteq P$, $c_{\mu^*}\{v\} \subseteq Q$ and $P \cap Q = \phi$.

Remark 4.2. If a space $(X, \mu X, \tau)$ is μ^*-R_1 then it is μ^*-R_0.

Proof. Let $P \in \mu^*O(X_\tau)$ and $u \in P$. If $v \notin P$ then $u \notin c_{\mu^*}\{v\}$. So, $c_{\mu^*}\{u\} \neq c_{\mu^*}\{v\}$. Thus, $\exists Q_v \in \mu^*O(X_\tau)$ with $c_{\mu^*}\{v\} \subseteq Q_v$ and $u \notin Q_v$. Hence, $v \notin c_{\mu^*}\{u\}$ and $c_{\mu^*}\{u\} \subseteq P$. □

However, the converse is not true is shown below:

Example 4.3. Let $X = \{a, b, c, d\}$,

$$\mu X = \{\phi, X, \{a, b, c\}, \{b, c, d\}, \{c, d, a\}, \{a, b, d\}\}$$

and $\tau = \{\phi, \{a\}, X\}$ then $(X, \mu X, \tau)$ is a space. Now, $\mu^* O(X_\tau) = \{\phi, X, \{a, b, c\}, \{b, c, d\}, \{c, d, a\}, \{a, b, d\}\}$ and $c_{\mu^*}\{t\} = \{t\}\ \forall t \in X$. Clearly, $(X, \mu X, \tau)$ is μ^*-R_0. Now, $c_{\mu^*}\{a\} \neq c_{\mu^*}\{b\}$ but $\nexists$ disjoint $U, V \in \mu^* O(X_\tau)$ satisfying $c_{\mu^*}\{a\} \subseteq U$ and $c_{\mu^*}\{b\} \subseteq V$. Hence, it is not μ^*-R_1 space.

Remark 4.4. Now, we show the independence of μ^*-R_1 and μ-R_1 spaces.

For this, let us consider an example below:

Example 4.5. Let us consider a space $(X, \mu X, \tau)$ where $X = \{p, q, r, s, t\}$, $\tau = \{X, \phi, \{p, q, s, t, \}, \{r\}\}$ and

$$\mu X = \{X, \phi, \{p, q, r\}, \{t\}, \{p, q, r, t\}, \{s, t\}\}.$$

Here,

$$\mu^* O(X_\tau) = \{X, \phi, \{p, q, r\}, \{p, q, r, s\}, \{s, t\}, \{t\}, \{p, q, r, t\}, \{p, t\}, \{q, t\}, \{p, q, t\}, \{p, s, t\}, \{q, s, t\}, \{p, q, s, t\}\}$$

and so,

$$\mu^* C(X_\tau) = \{X, \phi, \{s, t\}, \{p, q, r, s\}, \{s\}, \{p, q, r\}, \{t\}, \{q, r, s\}, \{p, r, s\}, \{r, s\}, \{q, r\}, \{p, r\}, \{r\}\}.$$

Clearly, this space is μ-R_1 but not μ^*-R_0 and hence not μ^*-R_1.

Remark 4.6. In Example (3.3), it can be shown that $(X, \mu X, \tau)$ is μ^*-R_1 space but not μ-R_1. Thus, μ-R_1 and μ^*-R_1 spaces do not imply each other.

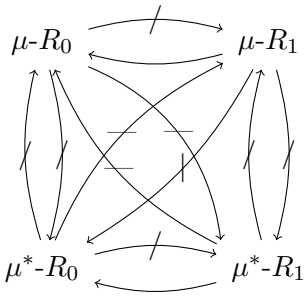
From the examples discussed above, we have the following table to represent relation among different spaces.

Example	$\mu - R_0$ space	$\mu^* - R_0$ space	$\mu - R_1$ space	$\mu^* - R_1$ space
Example 3.2	✓	×	×	×
Example 3.3	×	✓	×	✓
Example 4.3		✓		×
Example 4.5	✓	×	✓	×

The arrow diagram showing interrelations among different spaces is given as:

Now, we shall study some characterizations of μ^*-R_1 space.

Theorem 4.7. *In a space $(X, \mu X, \tau)$ we have the following equivalent conditions.*



(1) $(X, \mu X, \tau)$ *is a* μ^*-R_1 *space.*
(2) *For any* $m, n \in X$ *one of the following holds.*
   (*a*) *If* $P \in \mu^*O(X_\tau)$ *then* $m \in P$ *iff* $n \in P$.
   (*b*) *There exists disjoint* $P, Q \in \mu^*O(X_\tau)$ *such that* $m \in P$ *and* $n \in Q$.
(3) *If* $m, n \in X$ *are such that* $c_{\mu^*}\{m\} \neq c_{\mu^*}\{n\}$ *then* $\exists S_1, S_2 \in \mu^*C(X_\tau)$ *such that* $m \in S_1$, $n \notin S_1$, $n \in S_2$, $m \notin S_2$ *and* $X = S_1 \cup S_2$.

Proof. (1) $\Rightarrow$ (2) If $m, n \in X$ then either $c_{\mu^*}\{m\} = c_{\mu^*}\{n\}$ or $c_{\mu^*}\{m\} \neq c_{\mu^*}\{n\}$. If $c_{\mu^*}\{m\} = c_{\mu^*}\{n\}$, for $m \in P \in \mu^*O(X_\tau)$ we have $n \in c_{\mu^*}\{n\} = c_{\mu^*}\{m\} \subseteq P$, as the space is μ^*-R_0. Also, $n \in P$ gives $m \in c_{\mu^*}\{m\} \subseteq P$. Now, let us consider the case $c_{\mu^*}\{m\} \neq c_{\mu^*}\{n\}$ then there exists disjoint $P, Q \in \mu^*O(X_\tau)$ such that $m \in c_{\mu^*}\{m\} \subseteq P$ and $n \in c_{\mu^*}\{n\} \subseteq Q$.

(2) $\Rightarrow$ (3) Let $m, n \in X$ with $c_{\mu^*}\{m\} \neq c_{\mu^*}\{n\}$. Then $m \notin c_{\mu^*}\{n\}$ or $n \notin c_{\mu^*}\{m\}$ say $m \notin c_{\mu^*}\{n\}$ then $\exists\ G \in \mu^*O(X_\tau)$ with $m \in G$, $n \notin G$. By hypothesis, $\exists$ disjoint $P, Q \in \mu^*O(X_\tau)$ such that $m \in P$ and $n \in Q$. Let $S_1 = X - Q$ and $S_2 = X - P$ then $S_1, S_2 \in \mu^*C(X_\tau)$ such that $m \in S_1$, $n \notin S_1$ and $m \notin S_2$, $n \in S_2$, $X = S_1 \cup S_2$.

(3) $\Rightarrow$ (1) At first lets show $(X, \mu X, \tau)$ to be a μ^*-R_0 space. Let $m \in P \in \mu^*O(X_\tau)$. We claim that $c_{\mu^*}\{m\} \subseteq P$. Suppose $n \in c_{\mu^*}\{m\} \cap (X - P)$, then $c_{\mu^*}\{m\} \neq c_{\mu^*}\{n\}$ (if they are equal then $n \in P$, a contradiction) and hence by hypothesis $\exists\ S_1, S_2 \in \mu^*C(X_\tau)$ such that $m \in S_1$, $n \notin S_1$ and $n \in S_2$, $m \notin S_2$, $X = S_1 \cup S_2$. Now, $n \in X - S_1 \in \mu^*O(X_\tau)$ and $m \notin X - S_1$ which is a contradiction as $n \in c_{\mu^*}\{m\}$. Thus, (X, μ^*X, τ) is μ^*-R_0 space.

Let $p, q \in X$ with $c_{\mu^*}\{p\} \neq c_{\mu^*}\{q\}$. Now, $\exists\, U_1, U_2 \in \mu^*C(X_\tau)$ with $p \in U_1$, $q \notin U_1$, $p \notin U_2$, $q \in U_2$ and $X = U_1 \cup U_2$. Thus

$p \in X - U_2$ and $q \in X - U_1$ where $X - U_1, X - U_2 \in \mu^* O(X_\tau)$ and are disjoint. The space being μ^*-R_0, $c_{\mu^*}\{p\} \subseteq X - U_2$ and $c_{\mu^*}\{q\} \subseteq X - U_1$. Consequently, $(X, \mu X, \tau)$ is a μ^*-R_1 space. □

Theorem 4.8. *A space $(X, \mu X, \tau)$ is μ^*-R_1 iff for any $u, v \in X$ satisfying $\mu^* - \ker(\{u\}) \neq \mu^* - \ker(\{v\})$ then there exists $P, Q \in \mu^* O(X_\tau)$ with $c_{\mu^*}\{u\} \subseteq P$, $c_{\mu^*}\{v\} \subseteq Q$ and $P \cap Q = \phi$.*

Proof. Let $(X, \mu X, \tau)$ be a μ^*-R_1 space. If $u, v \in X$, $\mu^* - \ker(\{u\}) \neq \mu^*$-$\ker(\{v\})$ gives $c_{\mu^*}\{u\} \neq c_{\mu^*}\{v\}$. By definition of μ^*-R_1 space, the result follows.
The converse follows similarly. □

Theorem 4.9. *A space $(X, \mu X, \tau)$ is μ^*-R_1 iff $r \in X - c_{\mu^*}\{s\}$ implies there exists disjoint μ^*-open sets containing r and s respectively.*

Proof. Let $(X, \mu X, \tau)$ be μ^*-R_1 and $r \in X - c_{\mu^*}\{s\}$ then $c_{\mu^*}\{r\} \neq c_{\mu^*}\{s\}$ and r and s has disjoint μ^*-open sets.

Conversely, we first show that the space is μ^*-R_0. Let $r \in P \in \mu^* O(X_\tau)$ and $s \notin P$. So, $c_{\mu^*}\{s\} \cap P = \phi$ and $r \notin c_{\mu^*}\{s\}$. There exists $P_r, P_s \in \mu^* O(X_\tau)$ such that $r \in P_r, s \in P_s$ and $P_r \cap P_s = \phi$. Hence, $c_{\mu^*}\{r\} \subseteq c_{\mu^*}\{P_r\}$ and $c_{\mu^*}\{r\} \cap P_s \subseteq c_{\mu^*}\{P_r\} \cap P_s = \phi$. So, $s \notin c_{\mu^*}\{r\}$. Consequently, $c_{\mu^*}\{r\} \subseteq P$ and the space $(X, \mu X, \tau)$ is μ^*-R_0. Now, we show that $(X, \mu X, \tau)$ is μ^*-R_1. Suppose $c_{\mu^*}\{r\} \neq c_{\mu^*}\{s\}$. Then $\exists w \in c_{\mu^*}\{r\}$ and $w \notin c_{\mu^*}\{s\}$. There exists $Q_w, Q_s \in \mu^* O(X_\tau)$ such that $w \in Q_w$ and $s \in Q_s$ and $Q_w \cap Q_s = \phi$. Since, $w \in c_{\mu^*}\{r\}$, $r \in Q_w$. As $(X, \mu X, \tau)$ is μ^*-R_0, $c_{\mu^*}\{r\} \subseteq Q_w$, $c_{\mu^*}\{s\} \subseteq Q_s$ and $Q_s \cap Q_w = \phi$. Thus, $(X, \mu X, \tau)$ is μ^*-R_1. □

5. μ^*-T_0, μ^*-T_1 and μ^*-T_2 Spaces

In this section we introduce some forms of seperation axiom interms of μ^*-open sets and finally study μ^*-R_0, μ^*-R_1 spaces via these axioms.

Definition 5.1. A space $(X, \mu X, \tau)$ is called

(1) μ^*-T_0 if for any $u, v(\neq u) \in X$, $\exists P \in \mu^* O(X_\tau)$ containing one point but not other.
(2) μ^*-T_1 if for any $u, v(\neq u) \in X$, $\exists P, Q \in \mu^* O(X_\tau)$ such that $u \in P, v \notin P$ and $v \in Q, u \notin Q$.
(3) μ^*-T_2 space if for any $u, v(\neq u) \in X$, there exists disjoint $P, Q \in \mu^* O(X_\tau)$ such that $u \in P, v \in Q$.

The following theorem characterize μ^*-T_0 spaces.

Theorem 5.2. *A space $(X, \mu X, \tau)$ is μ^*-T_0 iff μ^*-closures of distinct points are distinct.*

Proof. Let u, $v(\neq u) \in X$ and X is a μ^*-T_0 space. We shall show that $c_{\mu^*}(\{u\}) \neq c_{\mu^*}(\{v\})$. X being μ^*-T_0 space, there exists $U \in \mu^*O(X_\tau)$ which contains u but not v. Also, $u \notin X-U$ and $v \in X-U \in \mu^*C(X_\tau)$. Now, $v \in c_{\mu^*}(\{v\})$ and $u \notin c_{\mu^*}(\{v\})$ as $u \notin X-U$, we have

$$c_{\mu^*}(\{u\}) \neq c_{\mu^*}(\{v\}).$$

Conversely, let u, $v(\neq u) \in X$. Let $u \in c_{\mu^*}(\{v\})$ and $v \in c_{\mu^*}(\{u\})$. $u \in c_{\mu^*}(\{v\})$ gives

$$\begin{aligned} c_{\mu^*}(\{u\}) &\subseteq c_{\mu^*}\{c_{\mu^*}(\{v\})\} \\ &= c_{\mu^*}(\{v\}) \\ &\subseteq c_{\mu^*}\{c_{\mu^*}(\{u\})\} \\ &= c_{\mu^*}(\{u\}). \end{aligned}$$

Therefore, $c_{\mu^*}(\{u\}) = c_{\mu^*}(\{v\})$. Thus if $c_{\mu^*}(\{u\}) \neq c_{\mu^*}(\{v\})$ then either $u \notin c_{\mu^*}(\{v\})$ or $v \notin c_{\mu^*}(\{u\})$. If $u \notin c_{\mu^*}(\{v\})$ then $u \in X - c_{\mu^*}(\{v\}) \in \mu^*O(X_\tau)$ and $v \notin X - c_{\mu^*}(\{v\})$. Similarly, it follows if $v \notin c_{\mu^*}(\{u\})$. Hence, X is a μ^*-T_0 space. □

Remark 5.3. The property of a space being μ^*-T_0 is preserved under bijective μ^*-open mapping.

Proof. Let $(X, \mu X, \tau_1)$ be a μ^*-T_0 space and $(Y, \mu Y, \tau_2)$ be a space. Let $f : X \to Y$ be a bijective μ^*-open mapping. Let $p, q \in Y$ with $p \neq q$. f being bijective, $\exists$ distinct points $u, v \in X$ satisfying $f(u) = p, f(v) = q$. X being μ^*-T_0 space, $\exists$ $G \in \mu^*O(X_{\tau_1})$ such that $u \in G$ and $v \notin G$. f being μ^*-open, $f(G) \in \mu^*O(Y_{\tau_2})$ which contains $f(u) = p$ but not $f(v) = q$. Hence, $(Y, \mu Y, \tau_2)$ is a μ^*-T_0 space. □

Remark 5.4. Clearly, every μ^*-T_1 space is μ^*-T_0 space but the converse does not hold is given below.

Example 5.5. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{a, c\}\}$ and $\mu X = \{X, \phi, \{c\}, \{a, c\}\}$. Clearly, the space $(X, \mu X, \tau)$ is μ^*-T_0 but not μ^*-T_1 because $c \neq b$ and there does not exists a pair of μ^*-open sets containing one point but not the other.

Theorem 5.6. *For a space $(X, \mu X, \tau)$ the following are equivalent.*

(1) *$(X, \mu X, \tau)$ is μ^*-T_1 space.*
(2) *For any $u \in X, \{u\}$ is μ^*-closed.*
(3) *$(X, \mu X, \tau)$ is μ^*-T_0 and μ^*-R_0.*

Proof. (1) $\Rightarrow$ (2) Let $u, v(\neq u) \in X, \exists\, U \in \mu^*O(X_\tau)$ which contains v but not u. Thus, $v \notin c_{\mu^*}(\{u\})$. So, $c_{\mu^*}(\{u\}) = \{u\}$ and hence $\{u\}$ is μ^*-closed.

(2) $\Rightarrow$ (1) Let $u, v(\neq u) \in X$ such that $\{u\}$ and $\{v\}$ are μ^*-closed in X. Then $X - \{u\}, X - \{v\} \in \mu^*O(X_\tau)$ such that $v \in X - \{u\}$ but $u \notin X - \{u\}$ and $u \in X - \{v\}$ but $v \notin X - \{v\}$. Hence, X is μ^*-T_1 space.
(2) $\Rightarrow$ (3) Obvious.
(3) $\Rightarrow$ (1) Let $u, v(\neq u) \in X$. As $(X, \mu X, \tau)$ is μ^*-T_0 space, $\exists U \in \mu^*O(X_\tau)$ containing u. As the space is μ^*-R_0, $u \in c_{\mu^*}(\{u\}) \subseteq U$. Clearly, $v \in X - U \in \mu^*C(X_\tau)$ and $v \in X - c_{\mu^*}(\{u\}) \in \mu^*O(X_\tau)$ and $u \notin X - c_{\mu^*}(\{u\})$. Hence, $(X, \mu X, \tau)$ is μ^*-T_1 space. □

Remark 5.7. If a space is μ^*-T_0 but not μ^*-T_1 then it cannot be μ^*-R_0.

Remark 5.8. In a μ^*-R_0 space, μ^*-T_0 and μ^*-T_1 coincides.

Theorem 5.9. *A space $(X, \mu X, \tau)$ is μ^*-T_2 iff it is μ^*-T_1 and μ^*-R_1.*

Proof. Let $(X, \mu X, \tau)$ be μ^*-T_2 space. So, it is μ^*-T_1. Let $u, v(\neq u) \in X$ with $c_{\mu^*}(\{u\}) \neq c_{\mu^*}(\{v\})$. By Theorem (5.6), $\{u\} = c_{\mu^*}(\{u\}) \neq c_{\mu^*}(\{v\}) = \{v\}$ i.e., $u \neq v$. So, there exists disjoint $U, V \in \mu^*O(X_\tau)$ satisfying $\{u\} = c_{\mu^*}(\{u\}) \subseteq U$ and $\{v\} = c_{\mu^*}(\{v\}) \subseteq V$, showing $(X, \mu X, \tau)$ is μ^*-R_1 space.

Conversely, let $(X, \mu X, \tau)$ be μ^*-R_1 and μ^*-T_1 space and $u, v(\neq u) \in X$. The space being μ^*-T_1, $\{u\} = c_{\mu^*}(\{u\}) \neq c_{\mu^*}(\{v\}) = \{v\}$ and there exists disjoint $U, V \in \mu^*O(X_\tau)$ and $u \in U$, $v \in V$. Hence, $(X, \mu X, \tau)$ is μ^*-T_2 space. □

Theorem 5.10. *A space $(X, \mu X, \tau)$ is μ^*-R_1 and μ^*-T_0 iff it is μ^*-T_2 space.*

Proof. Let $(X, \mu X, \tau)$ be μ^*-R_1 and μ^*-T_0 space and $u, v(\neq u) \in X$. The space being μ^*-T_0, $c_{\mu^*}\{u\} \neq c_{\mu^*}\{v\}$. X being μ^*-R_1 space, $\exists$ disjoint sets $U, V \in \mu^*O(X_\tau)$ with $c_{\mu^*}(\{u\}) \subseteq U$ and $c_{\mu^*}(\{v\}) \subseteq V$. So, $u \in U$ and $v \in V$. Thus, X is μ^*-T_2 space. Converse part follows from Theorem (5.9) and Remark (5.4). □

Remark 5.11. Any space which is μ^*-T_1 but not μ^*-T_2 is μ^*-R_0 and not μ^*-R_1.

Remark 5.12. In a μ^*-R_1 space, μ^*-T_0, μ^*-T_1 and μ^*-T_2 coincides.

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