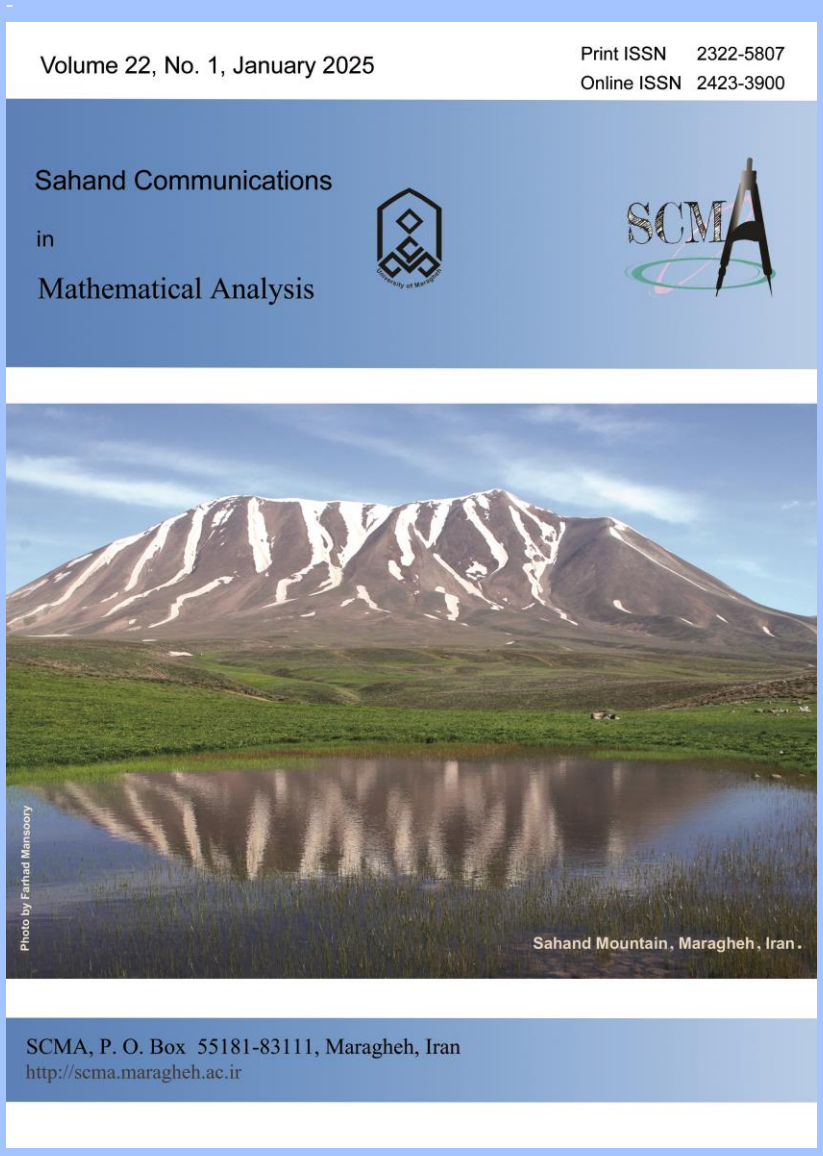
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Conditionally Compatible Mappings, Non-Compatible Mappings, Faintly Compatible Mappings and Some Results on Common Fixed Point in C^*-Algebra Valued Metric Space

Rishi Dhariwal[1] and Deepak Kumar[2*]

Abstract. This manuscript introduces and investigates various types of mappings in the context of a C^*-algebra valued metric space. These include conditionally compatible, non-compatible, and faintly compatible mappings. Additionally, the manuscript establishes fixed-point results for certain combinations of these mappings. Specifically, it considers pairs of self-mappings that satisfy either a $(\phi-\psi)$-Kannan-type or Chatterjea-type contractive condition and proves common fixed point theorems for faintly compatible continuous or reciprocal continuous along with non-compatible pairs of such mappings. Also, non-compatibility is relaxed by using the $E.A.$ property and faintly compatible continuous property or reciprocal continuous property for pairs of self-mappings. To demonstrate the practicality of these results, the manuscript includes examples that illustrate their usefulness.

1. Introduction and Preliminaries

In 2014, Ma et al. [15] extended the Banach contraction principle to encompass C^*-algebra valued metric spaces by substituting the set of real numbers with positive members of an unital C^*-algebra. Their work resulted in several fixed-point theorems for such spaces. Subsequently, Xin et al. [11] introduced the concepts of compatible and weakly compatible mappings within the C^*-algebra valued metric space framework. They employed these concepts to establish further fixed-point theorems to solve integral equations on a Lebesgue measurable set. Since then,

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a number of researchers have contributed to this field, as evidenced by references [1–3, 5–8, 14, 17] to prior work in the literature.

In the present study, we introduced three new types of mappings, namely conditionally compatible, non-compatible, and faintly compatible mappings, within the C^*-algebra valued metric space framework. We proved several common fixed point theorems for faintly compatible continuous or reciprocal continuous along with non-compatible pairs of self-mappings that satisfy the $(\phi-\psi)$-contractive condition. To reassure the practical applicability of our results, we provided examples.

To begin with, let us start with some basic definitions, which will be used later.

Throughout the paper, by $\mathbb{B}$, we denote an unital C^*-algebra with the unity element $I_{\mathbb{B}}$. We call an element $\varkappa \in \mathbb{B}$, a positive element denote it by $\varkappa \succeq \theta_{\mathbb{B}}$, where $\theta_{\mathbb{B}}$ means zero element in $\mathbb{B}$. From now on, by $\mathbb{B}^+$, we denote the set $\{\varkappa \in \mathbb{B} : \varkappa \succeq \theta_{\mathbb{B}}\}$.

Definition 1.1 ([15])**.** Suppose $\mho$ is a non-empty set. The mapping $d_{\mathbb{B}} : \mho \times \mho \to \mathbb{B}$ satisfies:

- (i) $d_{\mathbb{B}}(\varpi, \varsigma) \succeq \theta_{\mathbb{B}}$ and $d_{\mathbb{B}}(\varpi, \varsigma) = \theta_{\mathbb{B}}$ if and only if $\varpi = \varsigma$;
- (ii) $d_{\mathbb{B}}(\varpi, \varsigma) = d_{\mathbb{B}}(\varsigma, \varpi)$;
- (iii) $d_{\mathbb{B}}(\varpi, \varkappa) \preceq d_{\mathbb{B}}(\varpi, \varsigma) + d_{\mathbb{B}}(\varsigma, \varkappa)$ for all $\varpi, \varsigma, \varkappa \in \mho$.

Then, $d_{\mathbb{B}}$ is called a C^*-algebra metric on $\mho$ and $(\mho, \mathbb{B}, d_{\mathbb{B}})$ is called a C^*-algebra valued metric space.

Definition 1.2 ([15])**.** Let $(\mho, \mathbb{B}, d_{\mathbb{B}})$ be a C^*-algebra valued metric space and let $\{\vartheta_n\}_{n\in\mathbb{N}}$ be sequence in $(\mho, \mathbb{B}, d_{\mathbb{B}})$, then

- (i) $\{\vartheta_n\}_{n\in\mathbb{N}}$ is said to be convergent with respect to $\mathbb{B}$, if for given $\epsilon > 0$, there exists a positive integer k such that $\|d_{\mathbb{B}}(\vartheta_n, \vartheta)\| < \epsilon$, for all $n > k$;
- (ii) $\{\vartheta_n\}_{n\in\mathbb{N}}$ is said to be Cauchy sequence with respect to $\mathbb{B}$ if for any $\epsilon > 0$, there exists $k \in \mathbb{N}$ such that $\|d_{\mathbb{B}}(\vartheta_n, \vartheta_m)\| < \epsilon$, for all $n, m > k$;
- (iii) $(\mho, \mathbb{B}, d_{\mathbb{B}})$ is said to be a complete C^*-algebra valued metric space if every Cauchy sequence with respect to $\mathbb{B}$ is convergent.

Definition 1.3 ([11])**.** For two mappings Γ_1 and Γ_2 on $(\mho, \mathbb{B}, d_{\mathbb{B}})$,

- (i) Γ_1 and Γ_2 are said to be compatible if for any arbitrary sequence $\{\vartheta_n\}_{n\in\mathbb{N}} \subseteq \mho$ such that $\lim_{n\to\infty} \Gamma_1\vartheta_n = \lim_{n\to\infty} \Gamma_2\vartheta_n = \tau$ for some $\tau \in \mho$, then
$$\lim_{n\to\infty} \|d_{\mathbb{B}}(\Gamma_1\Gamma_2\vartheta_n, \Gamma_2\Gamma_1\vartheta_n)\| = 0;$$
- (ii) if $\varkappa = \Gamma_1\vartheta = \Gamma_2\vartheta$ for some ϑ, $\varkappa \in \mho$, then ϑ is called a coincidence point of Γ_1 and Γ_2 and additionally, $\varkappa$ is called point of coincidence of Γ_1 and Γ_2;

(*iii*) Γ_1 and Γ_2 are said to be weakly compatible if $\Gamma_1\Gamma_2\vartheta = \Gamma_2\Gamma_1\vartheta$ for all $\vartheta \in \{\vartheta \in \mho : \Gamma_1\vartheta = \Gamma_2\vartheta\}$.

Definition 1.4 ([9])**.** Let Γ_1 and Γ_2 are two self-mappings of a metric space $(\mho, d_{\mathbb{B}})$. Then, the pair (Γ_1, Γ_2) is said to satisfy $E.A.$ property if there exists a sequence $\{\vartheta_n\}_{n\in\mathbb{N}}$ in $\mho$ such that

$$\lim_{n\to\infty} \Gamma_1\vartheta_n = \lim_{n\to\infty} \Gamma_2\vartheta_n = \tau, \quad \text{for some } \tau \in \mho.$$

Definition 1.5 ([13])**.** A pair of self-mappings (Γ_1, Γ_2) of C^*-algebra valued metric space is called reciprocally continuous if and only if $\lim_{n\to\infty} \Gamma_1\Gamma_2 \vartheta_n = \Gamma_1\tau$ and $\lim_{n\to\infty} \Gamma_2\Gamma_1\vartheta_n = \Gamma_2\tau$ whenever $\{\vartheta_n\}_{n\in\mathbb{N}}$ is a sequence in $\mho$ such that $\lim_{n\to\infty} \Gamma_1\vartheta_n = \lim_{n\to\infty} \Gamma_2\vartheta_n = \tau$ for some $\tau \in \mho$.

Remark 1.6. If Γ_1 and Γ_2 are both continuous, they are reciprocally continuous, but the converse need not be true.

Example 1.7. Let $\mho = [0, \infty[$, $\mathbb{B} = \mathbb{C}$ and $d_{\mathbb{B}}$ be the usual metric on $\mho$. Defining the mappings $\Gamma_1, \Gamma_2 : \mho \to \mho$ by

$$\Gamma_1\vartheta = \begin{cases} \frac{1}{3} + \vartheta, & \text{if } 0 \le \vartheta < \frac{1}{3} \\ 1, & \text{if } \vartheta = \frac{1}{3} \\ 0, & \text{otherwise} \end{cases}, \qquad \Gamma_2\vartheta = \begin{cases} \frac{1}{3} - \vartheta, & \text{if } 0 \le \vartheta < \frac{1}{3} \\ \frac{1}{3}, & \text{if } \vartheta = \frac{1}{3} \\ 1, & \text{otherwise.} \end{cases}$$

Hence, Γ_1 and Γ_2 are discontinuous at $\vartheta = \frac{1}{3}$. However, Γ_1 and Γ_2 are reciprocal continuous (taking a sequence ϑ_n converging to zero).

Definition 1.8 ([14])**.** Let $\Phi_{\mathbb{B}}$ be the set of positive functions, $\phi_{\mathbb{B}} : \mathbb{B}^+ \to \mathbb{B}^+$ satisfying the following conditions:

(ϕ_1) $\phi_{\mathbb{B}}$ is continuous and non-decreasing;
(ϕ_2) $\phi_{\mathbb{B}}(a) = \theta_{\mathbb{B}}$ if and only if $a = \theta_{\mathbb{B}}$.

Definition 1.9 ([6])**.** Let $\{X_i\}$ be nonempty closed subsets of a metric space (X, d), and suppose $T : \bigcup_{i=1}^{p} X_i \to \bigcup_{i=1}^{p} X_i$ is a cyclical operator. Then, T is said to be a

(*i*) cyclic $(\phi - \psi)$-Kannan contraction if there exists constants α, β with $0 \le \beta < 1$ and $0 < \alpha + \beta \le 1$, such that for any $x \in X_i$, $y \in A_{i+1}, i = 1, 2, \ldots, p$, we have

$$\phi\left(d(Tx, Ty)\right) \le \phi\left(\alpha d(x, Tx) + \beta d(y, Ty)\right) - \psi\left(d(x, Tx), d(y, Ty)\right);$$

(*ii*) cyclic $(\phi - \psi)$-Chatterjea contraction if there exists constants α, β with $0 \le \alpha < \frac{1}{2}$ and $0 < \alpha + \beta \le 1$, such that for any

$x \in X_i$, $y \in A_{i+1}, i = 1, 2, \ldots, p$, we have

$$\phi\left(d(Tx, Ty)\right) \leq \phi\left(\alpha d(x, Ty) + \beta d(y, Tx)\right) - \psi\left(d(x, Ty), d(y, Tx)\right),$$

where $\phi : [0, \infty) \to [0, \infty)$ is an altering distance function, and $\psi : [0, \infty)^2 \to [0, \infty)$ is a continuous function with $\psi(t, s) = 0$ if and only if $t = s = 0$.

2. Main Results

In this section, we prove some definitions and common fixed point theorems in a complete C^*-algebra valued metric space (in short, C^*_{av}-metric space).

Definition 2.1. The two mappings Γ_1 and Γ_2 on a C^*_{av}-metric space $(\mho, \mathbb{B}, d_{\mathbb{B}})$ are said to be

- (i) non-compatible if there exists a sequence $\{\vartheta_n\}_{n\in\mathbb{N}} \subseteq \mho$ such that $\lim_{n\to\infty} \Gamma_1\vartheta_n = \lim_{n\to\infty} \Gamma_2\vartheta_n = \tau$ for some $\tau \in \mho$, but $\lim_{n\to\infty} \|d_{\mathbb{B}}(\Gamma_1\Gamma_2\vartheta_n, \Gamma_2\Gamma_1\vartheta_n)\|$ is either non-zero or non-existent;
- (ii) conditionally commuting if the pair commutes on a nonempty subset of the set of coincidence points whenever the set of coincidence point is nonempty;
- (iii) conditionally compatible if and only if the set of sequences $\{\vartheta_n\}_{n\in\mathbb{N}}$ satisfying $\lim_{n\to\infty} \Gamma_1\vartheta_n = \lim_{n\to\infty} \Gamma_2\vartheta_n$ is nonempty, then there exists a sequence $\{\varsigma_n\}_{n\in\mathbb{N}}$ such that $\lim_{n\to\infty} \Gamma_1\varsigma_n = \lim_{n\to\infty} \Gamma_2\varsigma_n = \tau$ for some $\tau \in \mho$ and $\lim_{n\to\infty} \|d_{\mathbb{B}}(\Gamma_1\Gamma_2\varsigma_n, \Gamma_2\Gamma_1\varsigma_n)\| = 0$.

Remark 2.2. Compatibility implies conditionally compatibility, but the converse need not be true.

Example 2.3. Let $\mho = [1, 5]$, $\mathbb{B} = \mathbb{C}$ and $d_{\mathbb{B}}$ be the usual metric on $\mho$. Defining the mappings $\Gamma_1, \Gamma_2 : \mho \to \mho$ by

$$\Gamma_1\vartheta = \begin{cases} 3, & \text{if } \vartheta \leq 3 \\ 10, & \text{otherwise} \end{cases}, \qquad \Gamma_2\vartheta = \begin{cases} 12 - 3\vartheta, & \text{if } \vartheta \leq 3 \\ 4, & \text{otherwise.} \end{cases}$$

Consider the constant sequence $\vartheta_n = 3$ for all $n \in \mathbb{N}$, then

$$\lim_{n\to\infty} \Gamma_1(\vartheta_n) = \lim_{n\to\infty} \Gamma_2(\vartheta_n) = \lim_{n\to\infty} \Gamma_1\Gamma_2(\vartheta_n) = \lim_{n\to\infty} \Gamma_2\Gamma_1(\vartheta_n) = 3,$$

and $\lim_{n\to\infty} d_{\mathbb{B}}(\Gamma_1\Gamma_2\vartheta_n, \Gamma_2\Gamma_1\vartheta_n) = \theta_{\mathbb{B}}$. Again if we consider $\varsigma_n = 3 - \dfrac{2}{n^2}$, where $n \in \mathbb{N}$, then

$$\lim_{n\to\infty} \Gamma_1(\varsigma_n) = \lim_{n\to\infty} \Gamma_2(\varsigma_n) = \lim_{n\to\infty} \Gamma_2\Gamma_1(\varsigma_n) = 3,$$

whereas $\lim_{n\to\infty} \Gamma_1\Gamma_2(\varsigma_n) = 10$ and $\lim_{n\to\infty} d_{\mathbb{B}}(\Gamma_1\Gamma_2\varsigma_n, \Gamma_2\Gamma_1\varsigma_n) \neq \theta_{\mathbb{B}}$. Thus, Γ_1 and Γ_2 are conditionally compatible but not compatible.

Remark 2.4. Conditional compatibility need not be commutativity at the coincidence points.

Example 2.5. Let $\mho = [0, \infty)$, $\mathbb{B} = \mathbb{C}$ and $d_{\mathbb{B}}$ be the usual metric on γ. Define the mappings $\Gamma_1, \Gamma_2 : \mho \to \mho$ by

$$\Gamma_1\vartheta = \vartheta^3 \text{ and } \Gamma_2\vartheta = \begin{cases} \vartheta + 6, & \text{if } \vartheta \in [0, 8] \cup (20, \infty) \\ 520 - \vartheta, & \text{otherwise.} \end{cases}$$

Consider the sequence $\varsigma_n = 2 + \dfrac{1}{n}$, where $n \in \mathbb{N}$, then $\lim\limits_{n\to\infty} \Gamma_1\varsigma_n = \lim\limits_{n\to\infty} \Gamma_2\varsigma_n = 8$ and $\lim\limits_{n\to\infty} \|d_{\mathbb{B}}(\Gamma_1\Gamma_2\varsigma_n, \Gamma_2\Gamma_1\varsigma_n)\| = 0$. Thus, Γ_1 and Γ_2 are conditionally compatible. However, we have $\Gamma_1(\vartheta) = \Gamma_2(\vartheta)$ if and only if $\vartheta = 2$ and $\Gamma_1\Gamma_2(2) = 512$ and $\Gamma_2\Gamma_1(2) = 14$. Then, $\Gamma_1(2) = \Gamma_2(2)$ but $\Gamma_1\Gamma_2(2) \neq \Gamma_2\Gamma_1(2)$.

In this paper, we define the notion of conditionally compatible mappings in a slightly different manner as follows:

Definition 2.6. Two self-mappings Γ_1 and Γ_2 of a C^*_{av}-metric space $(\mho, \mathbb{B}, d_{\mathbb{B}})$ will be called faintly compatible if and only if Γ_1 and Γ_2 are conditionally compatible and Γ_1 and Γ_2 commute on a nonempty subset of coincidence points whenever the set of coincidence points are nonempty.

If Γ_1 and Γ_2 are compatible, they are faintly compatible, but the converse need not be true.

Example 2.7. Let $\mho = [1, 6]$, $\mathbb{B} = \mathbb{C}$ and $d_{\mathbb{B}}$ be the usual metric on $\mho$. Defining the mappings $\Gamma_1, \Gamma_2 : \mho \to \mho$ by

$$\Gamma_1\vartheta = \begin{cases} 2, & \text{if } \vartheta = 2 \text{ or } \vartheta > 4 \\ \vartheta^2 + 1, & \text{otherwise} \end{cases}, \qquad \Gamma_2\vartheta = \begin{cases} 2, & \text{if } \vartheta = 2 \\ \dfrac{2\vartheta + 3}{2}, & \text{if } 2 < \vartheta \leq 4 \\ \dfrac{\vartheta + 2}{3}, & \text{otherwise.} \end{cases}$$

Consider the constant sequence $\{\varsigma_n = 2\}_{n\in\mathbb{N}}$, then Γ_1 and Γ_2 are faintly compatible. However, if the sequence $\vartheta_n = 4 + \dfrac{1}{n^2 + 1}$, where $n \in \mathbb{N}$, then $\lim\limits_{n\to\infty} \Gamma_1\vartheta_n = 2 = \lim\limits_{n\to\infty} \Gamma_2\vartheta_n$ but $\lim\limits_{n\to\infty} \|d_{\mathbb{B}}(\Gamma_1\Gamma_2\vartheta_n, \Gamma_2\Gamma_1\vartheta_n)\| \neq 0$. Thus, Γ_1 and Γ_2 are not compatible.

Remark 2.8. Faintly compatibility and non-compatibility are independent concepts.

Example 2.9. Let $\mho = [1, 10]$, $\mathbb{B} = \mathbb{C}$ and $d_{\mathbb{B}}$ be the usual metric on $\mho$. Defining the mappings $\Gamma_1, \Gamma_2 : \mho \to \mho$ by

$$\Gamma_1\vartheta = \begin{cases} 15, & \text{if } 1 \leq \vartheta \leq 3 \\ 3, & \text{otherwise} \end{cases}, \qquad \Gamma_2\vartheta = \begin{cases} 3, & \text{if } 1 \leq \vartheta \leq 3 \\ 6 - \vartheta, & \text{otherwise.} \end{cases}$$

Consider a sequence $\vartheta_n = 3 + \frac{1}{n}$, where $n \in \mathbb{N}$, then $\lim_{n\to\infty} \Gamma_1\vartheta_n = 3 = \lim_{n\to\infty} \Gamma_2\vartheta_n$. But $\lim_{n\to\infty} \|d_{\mathbb{B}}(\Gamma_1\Gamma_2\vartheta_n, \Gamma_2\Gamma_1\vartheta_n)\| = 12$. Thus, Γ_1 and Γ_2 are non-compatible but not faintly compatible.

Example 2.10. Let $\mho = [1, \infty)$, $\mathbb{B} = \mathbb{C}$ and $d_{\mathbb{B}}$ be the usual metric on $\mho$. Defining the mappings $\Gamma_1, \Gamma_2 : \mho \to \mho$ by $\Gamma_1\vartheta = \vartheta^3$ and $\Gamma_2\vartheta = 4\vartheta - 3$ for all $\vartheta \in \mho$.
In this example, Γ_1 and Γ_2 are faintly compatible but not non-compatible.

Definition 2.11. Let $\Psi_{\mathbb{B}}$ be the set of positive functions, $\psi_{\mathbb{B}} : \mathbb{B}^+ \times \mathbb{B}^+ \to \mathbb{B}^+$ satisfying the following conditions:

(ψ_1) $\psi_{\mathbb{B}}$ is continuous;
(ψ_2) $\psi_{\mathbb{B}}(a, b) = \theta_{\mathbb{B}}$ if and only if $a = b = \theta_{\mathbb{B}}$.

Theorem 2.12. *Let $(\mho, \mathbb{B}, d_{\mathbb{B}})$ be a C^*_{av}-metric space and $\Gamma_1, \Gamma_2, \Gamma_3$ and Γ_4 are four continuous self mappings on $\mho$ satisfying:*

(i) $\Gamma_1(\mho) \subseteq \Gamma_4(\mho)$ *and* $\Gamma_2(\mho) \subseteq \Gamma_3(\mho)$*;*
(ii) pairs (Γ_1, Γ_3) *and* (Γ_2, Γ_4) *are faintly compatible;*
(iii) for every $\vartheta, \varsigma \in \mho$, $\alpha, \beta \in \mathbb{B}^+$ *with* $0 < \|\alpha + \beta\| < 1$,

$$\phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\vartheta, \Gamma_2\varsigma)\right) \preceq \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\Gamma_1\vartheta, \Gamma_4\varsigma) + \beta d_{\mathbb{B}}(\Gamma_2\varsigma, \Gamma_3\vartheta)\right) - \psi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\vartheta, \Gamma_4\varsigma), d_{\mathbb{B}}(\Gamma_2\varsigma, \Gamma_3\vartheta)\right);$$

(iv) one of the pair (Γ_1, Γ_3) *or* (Γ_2, Γ_4) *are non-compatible.*

Then, $\Gamma_1, \Gamma_2, \Gamma_4$ *and* Γ_3 *have a unique common fixed point.*

Proof. The pair (Γ_2, Γ_4) is non-compatible, then there exists a sequence $\{\varsigma_n\}_{n\in\mathbb{N}} \in \mho$ such that $\lim_{n\to\infty} \Gamma_2\varsigma_n = \lim_{n\to\infty} \Gamma_4\varsigma_n = \tau \in \mho$ but $\lim_{n\to\infty} \|d_{\mathbb{B}}(\Gamma_2\Gamma_4\varsigma_n, \Gamma_4\Gamma_2\varsigma_n)\|$ is either non-zero or non-existent. Since, (Γ_2, Γ_4) are faintly compatible and $\lim_{n\to\infty} \Gamma_2\varsigma_n = \lim_{n\to\infty} \Gamma_4\varsigma_n = \tau \in \mho$, then there exists a sequence $\{\varphi_n\}_{n\in\mathbb{N}} \in \mho$ satisfying $\lim_{n\to\infty} \Gamma_2\varphi_n = \Gamma_4\varphi_n = \mu'$ such that

$$\lim_{n\to\infty} \|d_{\mathbb{B}}(\Gamma_2\Gamma_4\varphi_n, \Gamma_4\Gamma_2\varphi_n)\| = 0.$$

Further, Γ_2 is continuous, then $\lim_{n\to\infty} \Gamma_2\Gamma_4\varphi_n = \Gamma_2\mu'$ and $\lim_{n\to\infty} \Gamma_2\Gamma_2\varphi_n = \Gamma_2\mu'$. The last three limit show that $\lim_{n\to\infty} \Gamma_4\Gamma_2\varphi_n = \Gamma_2\mu'$. Since, $\Gamma_2(\mho) \subseteq \Gamma_3(\mho)$, therefore there exists $\nu' \in \mho$ such that $\Gamma_2\mu' = \Gamma_3\nu'$. Further, $\Gamma_2(\mho) \subseteq \Gamma_3(\mho)$, there exists a sequence $\{\vartheta_n\}_{n\in\mathbb{N}}$ in $\mho$ such that $\Gamma_2(\varsigma_n) = \Gamma_3(\vartheta_n)$. Hence, $\lim_{n\to\infty} \Gamma_3(\vartheta_n) = \tau$. We claim that $\lim_{n\to\infty} \Gamma_1(\vartheta_n) = \tau$. Let if possible $\lim_{n\to\infty} \Gamma_1(\vartheta_n) = \tau_1 \neq \tau$. Then, putting $\vartheta = \vartheta_n$ and $\varsigma = \varsigma_n$ in condition *(iii)*, we have

$$\phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\vartheta_n, \Gamma_2\varsigma_n)\right) \preceq \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\Gamma_1\vartheta_n, \Gamma_4\varsigma_n) + \beta d_{\mathbb{B}}(\Gamma_2\varsigma_n, \Gamma_3\vartheta_n)\right) \tag{2.1}$$

$$- \psi_{\mathbb{B}} \left(d_{\mathbb{B}}(\Gamma_1 \vartheta_n, \Gamma_4 \varsigma_n), d_{\mathbb{B}}(\Gamma_2 \varsigma_n, \Gamma_3 \vartheta_n) \right).$$

Taking limit as $n \to \infty$ in (2.1), we get

$$\begin{aligned} \phi_{\mathbb{B}} \left(d_{\mathbb{B}}(\tau_1, \tau) \right) &\preceq \phi_{\mathbb{B}} \left(\alpha d_{\mathbb{B}}(\tau_1, \tau) + \beta d_{\mathbb{B}}(\tau, \tau) \right) - \psi_{\mathbb{B}} \left(d_{\mathbb{B}}(\tau_1, \tau), d_{\mathbb{B}}(\tau, \tau) \right) \\ &\prec \phi_{\mathbb{B}} \left(\alpha d_{\mathbb{B}}(\tau_1, \tau) \right). \end{aligned}$$

Since, $\phi_{\mathbb{B}}$ is non-decreasing. Therefore, $\|d_{\mathbb{B}}(\tau_1, \tau)\| < \|\alpha\| \, \|d_{\mathbb{B}}(\tau_1, \tau)\|$, which is a contradiction. Hence, $\tau_1 = \tau$. Subsequently, we get

$$\lim_{n \to \infty} \Gamma_1(\vartheta_n) = \lim_{n \to \infty} \Gamma_2(\varsigma_n) = \lim_{n \to \infty} \Gamma_4(\varsigma_n) = \lim_{n \to \infty} \Gamma_3(\vartheta_n) = \tau.$$

Since, (Γ_1, Γ_3) are faintly compatible and $\lim_{n \to \infty} \Gamma_1 \vartheta_n = \lim_{n \to \infty} \Gamma_3 \vartheta_n = \tau \in \mho$, then there exists a sequence $\{\varrho_n\} \in \mho$ satisfying $\lim_{n \to \infty} \Gamma_1 \varrho_n = \Gamma_3 \varrho_n = \mu$ such that $\lim_{n \to \infty} \|d_{\mathbb{B}}(\Gamma_1 \Gamma_3 \varrho_n, \Gamma_3 \Gamma_1 \varrho_n)\| = 0$.

Further, Γ_1 is continuous, then $\lim_{n \to \infty} \Gamma_1 \Gamma_3 \varrho_n = \Gamma_1 \mu$ and $\lim_{n \to \infty} \Gamma_1 \Gamma_1 \varrho_n = \Gamma_1 \mu$. The last three limit show that $\lim_{n \to \infty} \Gamma_3 \Gamma_1 \varrho_n = \Gamma_1 \mu$. Since, $\Gamma_1 \mho \subseteq \Gamma_4 \mho$, therefore $\Gamma_1 \mu = \Gamma_4 \nu$.

Now, if $\mu \neq \mu'$ then substituting $\vartheta = \varrho_n$ and $\varsigma = \varphi_n$ in condition (iii), we get

$$\begin{aligned} \phi_{\mathbb{B}} \left(d_{\mathbb{B}}(\Gamma_1 \varrho_n, \Gamma_2 \varphi_n) \right) &\preceq \phi_{\mathbb{B}} \left(\alpha d_{\mathbb{B}}(\Gamma_1 \varrho_n, \Gamma_4 \varphi_n) + \beta d_{\mathbb{B}}(\Gamma_2 \varphi_n, \Gamma_3 \varrho_n) \right) \\ &\quad - \psi_{\mathbb{B}} \left(d_{\mathbb{B}}(\Gamma_1 \varrho_n, \Gamma_4 \varphi_n), d_{\mathbb{B}}(\Gamma_2 \varphi_n, \Gamma_3 \varrho_n) \right). \end{aligned} \tag{2.2}$$

Taking limit as $n \to \infty$ and norm in (2.2), we get

$$\begin{aligned} \left\| \phi_{\mathbb{B}} \left(d_{\mathbb{B}}(\mu, \mu') \right) \right\| &\leq \left\| \phi_{\mathbb{B}} \left(\alpha d_{\mathbb{B}}(\mu, \mu') + \beta d_{\mathbb{B}}(\mu, \mu') \right) \right. \\ &\quad \left. - \psi_{\mathbb{B}} \left(d_{\mathbb{B}}(\mu, \mu'), d_{\mathbb{B}}(\mu, \mu') \right) \right\| \\ &\leq \left\| \phi_{\mathbb{B}} \left((\alpha + \beta) d_{\mathbb{B}}(\mu, \mu') \right) \right\|. \end{aligned}$$

Since, ϕ is non-decreasing function, therefore we get $\|\alpha + \beta\| > 1$, which is a contradiction. Hence, $\mu = \mu'$.
Also,

$$\lim_{n \to \infty} \Gamma_1 \varrho_n = \lim_{n \to \infty} \Gamma_3 \varrho_n = \lim_{n \to \infty} \Gamma_4 \varphi_n = \lim_{n \to \infty} \Gamma_2 \varphi_n = \mu,$$

with $\Gamma_1 \mu = \Gamma_4 \nu$ and $\Gamma_2 \mu = \Gamma_3 \nu'$. Using the continuity of Γ_4 and Γ_3 together with condition (i), we get

$$\begin{aligned} \lim_{n \to \infty} \Gamma_3 \Gamma_3 \varrho_n = \lim_{n \to \infty} \Gamma_3 \Gamma_1 \varrho_n = \lim_{n \to \infty} \Gamma_1 \Gamma_3 \varrho_n = \Gamma_3 \mu, \\ \lim_{n \to \infty} \Gamma_4 \Gamma_4 \varphi_n = \lim_{n \to \infty} \Gamma_4 \Gamma_2 \varphi_n = \lim_{n \to \infty} \Gamma_2 \Gamma_4 \varphi_n = \Gamma_4 \mu. \end{aligned}$$

Now, substituting $\vartheta = \varsigma = \mu$ in condition (iii), we get

$$\begin{aligned} \phi_{\mathbb{B}} \left(d_{\mathbb{B}}(\Gamma_1 \mu, \Gamma_2 \mu) \right) &\preceq \phi_{\mathbb{B}} \left(\alpha d_{\mathbb{B}}(\Gamma_1 \mu, \Gamma_4 \mu) + \beta d_{\mathbb{B}}(\Gamma_2 \mu, \Gamma_3 \mu) \right) \\ &\quad - \psi_{\mathbb{B}} \left(d_{\mathbb{B}}(\Gamma_1 \mu, \Gamma_4 \mu), d_{\mathbb{B}}(\Gamma_2 \mu, \Gamma_3 \mu) \right) \\ &\prec \phi_{\mathbb{B}} \left((\alpha + \beta) d_{\mathbb{B}}(\Gamma_1 \mu, \Gamma_2 \mu) \right). \end{aligned}$$

Since, $\phi_{\mathbb{B}}$ is a non-decreasing function. Therefore, $\|d_{\mathbb{B}}(\Gamma_1\mu, \Gamma_2\mu)\| \leq \|(\alpha+\beta)\| \, \|d_{\mathbb{B}}(\Gamma_1\mu, \Gamma_2\mu)\|$, which is a contradiction. Hence, $\Gamma_1\mu = \Gamma_2\mu = \Gamma_4\mu = \Gamma_3\mu$, i.e. μ, is a common coincidence point of the mappings. Let, if possible, $\Gamma_2\mu \neq \mu$. Substituting $\vartheta = \varrho_n$ and $\varsigma = \mu$ in condition (iii), we get

$$\begin{aligned}\phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\varrho_n, \Gamma_2\mu)\right) \preceq \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\Gamma_1\varrho_n, \Gamma_4\mu) + \beta d_{\mathbb{B}}(\Gamma_2\mu, \Gamma_3\varrho_n)\right) \\ - \psi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\varrho_n, \Gamma_4\mu), d_{\mathbb{B}}(\Gamma_2\mu, \Gamma_3\varrho_n)\right).\end{aligned} \tag{2.3}$$

Taking limit as $n \to \infty$ on both side in (2.3), we get

$$\begin{aligned}\phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\mu, \Gamma_2\mu)\right) &\preceq \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\mu, \Gamma_2\mu) + \beta d_{\mathbb{B}}(\Gamma_2\mu, \mu)\right) \\ &\quad - \psi_{\mathbb{B}}\left(d_{\mathbb{B}}(\mu, \Gamma_2\mu), d_{\mathbb{B}}(\Gamma_2\mu, \mu)\right) \\ &\prec \phi_{\mathbb{B}}\left((\alpha+\beta) d_{\mathbb{B}}(\mu, \Gamma_2\mu)\right).\end{aligned}$$

Since, $\phi_{\mathbb{B}}$ is non-decreasing therefore $\|d_{\mathbb{B}}(\mu, \Gamma_2\mu)\| < \|\alpha+\beta\| \, \|d_{\mathbb{B}}(\mu, \Gamma_2\mu)\|$, which is a contradiction. Hence, $\mu = \Gamma_2\mu = \Gamma_1\mu = \Gamma_4\mu = \Gamma_3\mu$.
Uniqueness: Let q be another common fixed point of $\Gamma_1, \Gamma_2, \Gamma_3$ and Γ_4. Consider,

$$\begin{aligned}\phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\mu, q)\right) &= \phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\mu, \Gamma_2 q)\right) \\ &\preceq \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\Gamma_1\mu, \Gamma_4 q) + \beta d_{\mathbb{B}}(\Gamma_2 q, \Gamma_3\mu)\right) \\ &\quad - \psi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\mu, \Gamma_4 q), d_{\mathbb{B}}(\Gamma_2 q, \Gamma_3\mu)\right) \\ &\prec \phi_{\mathbb{B}}\left((\alpha+\beta) d_{\mathbb{B}}(\mu, q)\right).\end{aligned}$$

Since, $\phi_{\mathbb{B}}$ is non-decreasing function therefore $\|\alpha+\beta\| > 1$ which is contradiction. Hence, $\mu = q$ which proves the uniqueness. □

Corollary 2.13. *Let $(\mho, \mathbb{B}, d_{\mathbb{B}})$ be C^*_{av}-metric space and $\Gamma_1, \Gamma_2, \Gamma_3$ and Γ_4 are four continuous self mappings on $\mho$ satisfying:*

(i) $\Gamma_1(\mho) \subseteq \Gamma_4(\mho)$ and $\Gamma_2(\mho) \subseteq \Gamma_3(\mho)$;
(ii) pairs (Γ_1, Γ_3) and (Γ_2, Γ_4) are faintly compatible;
(iii) for every $\vartheta, \varsigma \in \mho$, $\alpha, \beta \in \mathbb{B}^+$ with $0 < \|\alpha+\beta\| < 1$,

$$\begin{aligned}\phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\vartheta, \Gamma_2\varsigma)\right) \preceq \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\Gamma_1\vartheta, \Gamma_3\vartheta) + \beta d_{\mathbb{B}}(\Gamma_2\varsigma, \Gamma_4\varsigma)\right) \\ - \psi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\vartheta, \Gamma_4\varsigma), d_{\mathbb{B}}(\Gamma_2\varsigma, \Gamma_3\vartheta)\right);\end{aligned}$$

(iv) one of the pair (Γ_1, Γ_3) or (Γ_2, Γ_4) are non-compatible.

Then, $\Gamma_1, \Gamma_2, \Gamma_4$ and Γ_3 have a unique common fixed point.

Remark 2.14. On taking $\phi(t) = t$ and $\psi(a, b) = \theta_{\mathbb{B}}$ in above corollary, we get the following result.

Corollary 2.15. *Let $(\mho, \mathbb{B}, d_{\mathbb{B}})$ be C^*_{av}-metric space and $\Gamma_1, \Gamma_2, \Gamma_3$ and Γ_4 are four continuous self mappings on $\mho$ satisfying:*

(i) $\Gamma_1(\mho) \subseteq \Gamma_4(\mho)$ and $\Gamma_2(\mho) \subseteq \Gamma_3(\mho)$;

(*ii*) *pairs* (Γ_1, Γ_3) *and* (Γ_2, Γ_4) *are faintly compatible;*
(*iii*) *for every* $\vartheta, \varsigma \in \mho$, $\alpha, \beta \in \mathbb{B}^+$ *with* $0 < \|\alpha + \beta\| < 1$,

$$d_{\mathbb{B}}(\Gamma_1\vartheta, \Gamma_2\varsigma) \preceq \alpha d_{\mathbb{B}}(\Gamma_1\vartheta, \Gamma_3\vartheta) + \beta d_{\mathbb{B}}(\Gamma_2\varsigma, \Gamma_4\varsigma);$$

(*iv*) *one of the pair* (Γ_1, Γ_3) *or* (Γ_2, Γ_4) *are non-compatible.*

Then, $\Gamma_1, \Gamma_2, \Gamma_4$ *and* Γ_3 *have a unique common fixed point.*

Corollary 2.16. *Let* $(\mho, \mathbb{B}, d_{\mathbb{B}})$ *be* C^*_{av}*-metric space and* $\Gamma_1, \Gamma_2, \Gamma_3$ *and* Γ_4 *are four continuous self mappings on* $\mho$ *satisfying:*

(*i*) $\Gamma_1(\mho) \subseteq \Gamma_4(\mho)$ *and* $\Gamma_2(\mho) \subseteq \Gamma_3(\mho)$*;*
(*ii*) *pairs* (Γ_1, Γ_3) *and* (Γ_2, Γ_4) *are faintly compatible;*
(*iii*) *for every* $\vartheta, \varsigma \in \mho$, $\alpha, \beta \in \mathbb{B}^+$ *with* $\theta_{\mathbb{B}} < \|\alpha + \beta\| < 1$,

$$d(\Gamma_1\vartheta, \Gamma_2\varsigma) \preceq \alpha^* \frac{d(\Gamma_3\vartheta, \Gamma_1\vartheta)d(\Gamma_3\vartheta, \Gamma_2\varsigma) + d(\Gamma_4\varsigma, \Gamma_2\varsigma) + d(\Gamma_4\varsigma, \Gamma_1\varsigma)}{1 + d(\Gamma_3\vartheta, \Gamma_2\varsigma) + d(\Gamma_4\varsigma, \Gamma_1\vartheta)}\alpha;$$

(*iv*) *one of the pair* (Γ_1, Γ_3) *or* (Γ_2, Γ_4) *are non-compatible.*

Then, $\Gamma_1, \Gamma_2, \Gamma_4$ *and* Γ_3 *have a unique common fixed point.*

Theorem 2.17. *Let* $(\mho, \mathbb{B}, d_{\mathbb{B}})$ *be* C^*_{av}*-metric space and* $\Gamma_1, \Gamma_2, \Gamma_3$ *and* Γ_4 *are four self mappings on* $\mho$ *satisfying:*

(*i*) $\Gamma_1(\mho) \subseteq \Gamma_4(\mho)$ *and* $\Gamma_2(\mho) \subseteq \Gamma_3(\mho)$*;*
(*ii*) *pairs* (Γ_1, Γ_3) *and* (Γ_2, Γ_4) *are reciprocal continuous and faintly compatible;*
(*iii*) *for every* $\vartheta, \varsigma \in \mho$, $\alpha, \beta \in \mathbb{B}^+$ *with* $0 < \|\alpha + \beta\| < 1$,

$$\begin{aligned}\phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\vartheta, \Gamma_2\varsigma)\right) \preceq \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\Gamma_1\vartheta, \Gamma_4\varsigma) + \beta d_{\mathbb{B}}(\Gamma_2\varsigma, \Gamma_3\vartheta)\right)\\ - \psi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\vartheta, \Gamma_4\varsigma), d_{\mathbb{B}}(\Gamma_2\varsigma, \Gamma_3\vartheta)\right);\end{aligned}$$

(*iv*) *one of the pair* (Γ_1, Γ_3) *or* (Γ_2, Γ_4) *are non-compatible.*

Then, $\Gamma_1, \Gamma_2, \Gamma_4$ *and* Γ_3 *have a unique common fixed point.*

Proof. The pair (Γ_2, Γ_4) is non-compatible, then there exists a sequence $\{\varsigma_n\}_{n\in\mathbb{N}} \in \mho$ such that $\lim_{n\to\infty} \Gamma_2\varsigma_n = \lim_{n\to\infty} \Gamma_4\varsigma_n = \tau \in \mho$ but $\lim_{n\to\infty} \|d_{\mathbb{B}}(\Gamma_2\Gamma_4\varsigma_n, \Gamma_4\Gamma_2\varsigma_n)\|$ is either non-zero or non-existent. Since, (Γ_2, Γ_4) are faintly compatible and $\lim_{n\to\infty} \Gamma_2\varsigma_n = \lim_{n\to\infty} \Gamma_4\varsigma_n = \tau \in \mho$, then there exists a sequence $\{\varphi_n\}_{n\in\mathbb{N}} \in \mho$ satisfying $\lim_{n\to\infty} \Gamma_2\varphi_n = \Gamma_4\varphi_n = \mu'$ such that

$$\lim_{n\to\infty} \|d_{\mathbb{B}}(\Gamma_2\Gamma_4\varphi_n, \Gamma_4\Gamma_2\varphi_n)\| = 0.$$

Further, (Γ_2, Γ_4) are reciprocal continuous, then $\lim_{n\to\infty} \Gamma_2\Gamma_4\varphi_n = \Gamma_2\mu'$ and $\lim_{n\to\infty} \Gamma_4\Gamma_2\varphi_n = \Gamma_4\mu'$. The last three limit show that $\Gamma_2\mu' = \Gamma_4\mu'$. Since, $\Gamma_2(\mho) \subseteq \Gamma_3(\mho)$, therefore there exists $\nu' \in \mho$ such that $\Gamma_2\mu' = \Gamma_3\nu' = \Gamma_4\mu'$. Again, $\Gamma_2(\mho) \subseteq \Gamma_3(\mho)$, there exists a sequence $\{\vartheta_n\}_{n\in\mathbb{N}}$ in

$\mho$ such that $\Gamma_2(\varsigma_n) = \Gamma_3(\vartheta_n)$. Hence, $\lim_{n\to\infty} \Gamma_3(\vartheta_n) = \tau$. We claim that $\lim_{n\to\infty} \Gamma_1(\vartheta_n) = \tau$. Let if possible $\lim_{n\to\infty} \Gamma_1(\vartheta_n) = \tau_1 \neq \tau$. Then, putting $\vartheta = \vartheta_n$ and $\varsigma = \varsigma_n$ in condition (iii), we have

$$\begin{aligned}(2.4)\quad \phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\vartheta_n, \Gamma_2\varsigma_n)\right) &\preceq \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\Gamma_1\vartheta_n, \Gamma_4\varsigma_n) + \beta d_{\mathbb{B}}(\Gamma_2\varsigma_n, \Gamma_3\vartheta_n)\right)\\ &\quad - \psi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\vartheta_n, \Gamma_4\varsigma_n), d_{\mathbb{B}}(\Gamma_2\varsigma_n, \Gamma_3\vartheta_n)\right).\end{aligned}$$

Taking limit as $n \to \infty$ in (2.4), we get

$$\begin{aligned}\phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\tau_1, \tau)\right) &\preceq \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\tau_1, \tau) + \beta d_{\mathbb{B}}(\tau, \tau)\right) - \psi_{\mathbb{B}}\left(d_{\mathbb{B}}(\tau_1, \tau), d_{\mathbb{B}}(\tau, \tau)\right)\\ &\prec \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\tau_1, \tau)\right).\end{aligned}$$

Since, $\phi_{\mathbb{B}}$ is non-decreasing, therefore $\|d_{\mathbb{B}}(\tau_1, \tau)\| < \|\alpha\| \, \|d_{\mathbb{B}}(\tau_1, \tau)\|$, which is a contradiction. Hence, $\tau_1 = \tau$. Subsequently, we get

$$\lim_{n\to\infty} \Gamma_1(\vartheta_n) = \lim_{n\to\infty} \Gamma_2(\varsigma_n) = \lim_{n\to\infty} \Gamma_4(\varsigma_n) = \lim_{n\to\infty} \Gamma_3(\vartheta_n) = \tau.$$

Since, (Γ_1, Γ_3) are faintly compatible and $\lim_{n\to\infty} \Gamma_1\vartheta_n = \lim_{n\to\infty} \Gamma_3\vartheta_n = \tau \in \mho$, then there exists a sequence $\{\varrho_n\} \in \mho$ satisfying $\lim_{n\to\infty} \Gamma_1\varrho_n = \Gamma_3\varrho_n = \mu$ such that $\lim_{n\to\infty} \|d_{\mathbb{B}}(\Gamma_1\Gamma_3\varrho_n, \Gamma_3\Gamma_1\varrho_n)\| = 0$.

Further, (Γ_1, Γ_3) are reciprocal continuous, then $\lim_{n\to\infty} \Gamma_1\Gamma_3\varrho_n = \Gamma_1\mu$ and $\lim_{n\to\infty} \Gamma_3\Gamma_1\varrho_n = \Gamma_3\mu$. The last three limit show that $\Gamma_1\mu = \Gamma_3\mu$. Since, $\Gamma_1\mho \subseteq \Gamma_4\mho$, therefore, $\Gamma_1\mu = \Gamma_3\mu = \Gamma_4\nu$.

Now, if $\mu \neq \mu'$ then substituting $\vartheta = \varrho_n$ and $\varsigma = \varphi_n$ in condition (iii), we get

$$\begin{aligned}(2.5)\quad \phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\varrho_n, \Gamma_2\varphi_n)\right) &\preceq \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\Gamma_1\varrho_n, \Gamma_4\varphi_n) + \beta d_{\mathbb{B}}(\Gamma_2\varphi_n, \Gamma_3\varrho_n)\right)\\ &\quad - \psi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\varrho_n, \Gamma_4\varphi_n), d_{\mathbb{B}}(\Gamma_2\varphi_n, \Gamma_3\varrho_n)\right).\end{aligned}$$

Taking limit as $n \to \infty$ and norm in (2.5), we get

$$\begin{aligned}\left\|\phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\mu, \mu')\right)\right\| &\leq \left\|\phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\mu, \mu') + \beta d_{\mathbb{B}}(\mu, \mu')\right)\right.\\ &\quad \left. - \psi_{\mathbb{B}}\left(d_{\mathbb{B}}(\mu, \mu'), d_{\mathbb{B}}(\mu, \mu')\right)\right\|\\ &\leq \left\|\phi_{\mathbb{B}}((\alpha + \beta) d_{\mathbb{B}}(\mu, \mu'))\right\|.\end{aligned}$$

Since, ϕ is non-decreasing therefore $\|\alpha + \beta\| > 1$, which is a contradiction. Hence, $\mu = \mu'$.

Since, $\lim_{n\to\infty} \Gamma_1\varrho_n = \lim_{n\to\infty} \Gamma_3\varrho_n = \lim_{n\to\infty} \Gamma_4\varphi_n = \lim_{n\to\infty} \Gamma_2\varphi_n = \mu$ with $\Gamma_1\mu = \Gamma_3\mu = \Gamma_4\nu$ and $\Gamma_4\mu = \Gamma_2\mu = \Gamma_3\nu'$. Consider,

$$\begin{aligned}\phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_4\nu, \Gamma_3\nu')\right) &= \phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\mu, \Gamma_2\mu)\right)\\ &\preceq \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\Gamma_1\mu, \Gamma_4\mu) + \beta d_{\mathbb{B}}(\Gamma_2\mu, \Gamma_3\mu)\right)\\ &\quad - \psi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\mu, \Gamma_4\mu), d_{\mathbb{B}}(\Gamma_2\mu, \Gamma_3\mu)\right)\\ &\preceq \phi_{\mathbb{B}}\left((\alpha + \beta) d_{\mathbb{B}}(\Gamma_1\mu, \Gamma_2\mu)\right).\end{aligned}$$

Since, $\phi_{\mathbb{B}}$ is non-decreasing function therefore

$$\left\|d_{\mathbb{B}}(\Gamma_4\nu, \Gamma_3\nu')\right\| = \|d_{\mathbb{B}}(\Gamma_1\mu, \Gamma_2\mu)\| \leq \|(\alpha+\beta)\| \, \|d_{\mathbb{B}}(\Gamma_1\mu, \Gamma_2\mu)\|,$$

which is a contradiction. Hence, $\Gamma_4\nu = \Gamma_3\nu'$ or $\Gamma_1\mu = \Gamma_2\mu = \Gamma_3\mu = \Gamma_4\mu$. Now, if $\Gamma_2\mu \neq \mu$, then substituting $\vartheta = \varrho_n$ and $\varsigma = \mu$, we get

$$\begin{aligned}(2.6) \qquad \phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\varrho_n, \Gamma_2\mu)\right) &\preceq \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\Gamma_1\varrho_n, \Gamma_4\mu) + \beta d_{\mathbb{B}}(\Gamma_2\mu, \Gamma_3\varrho_n)\right) \\ &\quad - \psi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\varrho_n, \Gamma_4\mu), d_{\mathbb{B}}(\Gamma_2\mu, \Gamma_3\varrho_n)\right).\end{aligned}$$

Taking limit as $n \to \infty$ in (2.6), we get

$$\begin{aligned}\psi_{\mathbb{B}}\left(d_{\mathbb{B}}(\mu, \Gamma_2\mu)\right) &\preceq \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\mu, \Gamma_2\mu) + \beta d_{\mathbb{B}}(\Gamma_2\mu, \mu)\right) \\ &\quad - \psi_{\mathbb{B}}\left(d_{\mathbb{B}}(\mu, \Gamma_2\mu), d_{\mathbb{B}}(\Gamma_2\mu, \mu)\right) \\ &\prec \phi_{\mathbb{B}}\left((\alpha+\beta) d_{\mathbb{B}}(\mu, \Gamma_2\mu)\right).\end{aligned}$$

Since, $\phi_{\mathbb{B}}$ is non-decreasing function, therefore, we have $\|\alpha+\beta\| > 1$, which is a contradiction. Hence, $\mu = \Gamma_2\mu = \Gamma_1\mu = \Gamma_3\mu = \Gamma_4\mu$. Hence, μ is a common fixed point of the mappings $\Gamma_1, \Gamma_2, \Gamma_3$ and Γ_4.

Uniqueness: Let q be another common fixed point of $\Gamma_1, \Gamma_2, \Gamma_3$ and Γ_4. Consider,

$$\begin{aligned}\phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\mu, q)\right) &= \phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\mu, \Gamma_2 q)\right) \\ &\preceq \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\Gamma_1\mu, \Gamma_3 q) + \beta d_{\mathbb{B}}(\Gamma_2 q, \Gamma_3\mu)\right) \\ &\quad - \psi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\mu, \Gamma_3 q), d_{\mathbb{B}}(\Gamma_2 q, \Gamma_3\mu)\right) \\ &\prec \phi_{\mathbb{B}}\left((\alpha+\beta) d_{\mathbb{B}}(\mu, q)\right).\end{aligned}$$

Since, $\phi_{\mathbb{B}}$ is non-decreasing function, therefore,

$$\|d_{\mathbb{B}}(\mu, q)\| \leq \|\alpha+\beta\| \, \|d_{\mathbb{B}}(\mu, q)\|,$$

which is a contradiction. Hence, $\mu = q$. □

Corollary 2.18. *Let $(\mho, \mathbb{B}, d_{\mathbb{B}})$ be C^*_{av}-metric space and $\Gamma_1, \Gamma_2, \Gamma_3$ and Γ_4 are four self mappings on $\mho$ satisfying:*

(i) $\Gamma_1(\mho) \subseteq \Gamma_4(\mho)$ and $\Gamma_2(\mho) \subseteq \Gamma_3(\mho)$;

(ii) pairs (Γ_1, Γ_3) and (Γ_2, Γ_4) are reciprocal continuous and faintly compatible;

(iii) for every $\vartheta, \varsigma \in \mho$, $\alpha, \beta \in \mathbb{B}^+$ with $0 < \|\alpha+\beta\| < 1$,

$$\begin{aligned}\phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\vartheta, \Gamma_2\varsigma)\right) &\preceq \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\Gamma_1\vartheta, \Gamma_3\vartheta) + \beta d_{\mathbb{B}}(\Gamma_2\varsigma, \Gamma_4\varsigma)\right) \\ &\quad - \psi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\vartheta, \Gamma_4\varsigma), d_{\mathbb{B}}(\Gamma_2\varsigma, \Gamma_3\vartheta)\right);\end{aligned}$$

(iv) one of the pair (Γ_1, Γ_3) or (Γ_2, Γ_4) are non-compatible.

Then, $\Gamma_1, \Gamma_2, \Gamma_4$ and Γ_3 have a unique common fixed point.

Corollary 2.19. *Let $(\mho, \mathbb{B}, d_{\mathbb{B}})$ be C^*_{av}-metric space and $\Gamma_1, \Gamma_2, \Gamma_3$ and Γ_4 are four self mappings on $\mho$ satisfying:*

(i) $\Gamma_1(\mho) \subseteq \Gamma_4(\mho)$ and $\Gamma_2(\mho) \subseteq \Gamma_3(\mho)$;

(*ii*) *pairs* (Γ_1, Γ_3) *and* (Γ_2, Γ_4) *are reciprocal continuous and faintly compatible;*

(*iii*) *for every* $\vartheta, \varsigma \in \mho$, $\alpha, \beta \in \mathbb{B}^+$ *with* $0 < \|\alpha + \beta\| < 1$,

$$d(\Gamma_1\vartheta, \Gamma_2\varsigma) \preceq \alpha^* \frac{d(\Gamma_3\vartheta, \Gamma_1\vartheta)d(\Gamma_3\vartheta, \Gamma_2\varsigma) + d(\Gamma_4\varsigma, \Gamma_2\varsigma) + d(\Gamma_4\varsigma, \Gamma_1\varsigma)}{1 + d(\Gamma_3\vartheta, \Gamma_2\varsigma) + d(\Gamma_4\varsigma, \Gamma_1\vartheta)}\alpha;$$

(*iv*) *one of the pair* (Γ_1, Γ_3) *or* (Γ_2, Γ_4) *are non-compatible.*

Then, $\Gamma_1, \Gamma_2, \Gamma_4$ *and* Γ_3 *have a unique common fixed point.*

Theorem 2.20. *Let* $(\mho, \mathbb{B}, d_{\mathbb{B}})$ *be* C^*_{av}*-metric space and* $\Gamma_1, \Gamma_2, \Gamma_3$ *and* Γ_4 *are four self mappings on* $\mho$ *satisfying:*

(*i*) $\Gamma_1(\mho) \subseteq \Gamma_4(\mho)$ *and* $\Gamma_2(\mho) \subseteq \Gamma_3(\mho)$*;*

(*ii*) *the pairs* (Γ_1, Γ_3) *and* (Γ_2, Γ_4) *are reciprocal continuous and faintly compatible;*

(*iii*) *for every* $\vartheta, \varsigma \in \mho$, $\alpha, \beta \in \mathbb{B}^+$ *with* $0 < \|\alpha + \beta\| < 1$,

$$\begin{aligned}\phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\vartheta, \Gamma_2\varsigma)\right) \preceq{}& \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\Gamma_1\vartheta, \Gamma_4\varsigma) + \beta d_{\mathbb{B}}(\Gamma_2\varsigma, \Gamma_3\vartheta)\right)\\ &- \psi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\vartheta, \Gamma_4\varsigma), d_{\mathbb{B}}(\Gamma_2\varsigma, \Gamma_3\vartheta)\right);\end{aligned}$$

(*iv*) *one of the pair* (Γ_1, Γ_3) *or* (Γ_2, Γ_4) *satisfy E.A. property.*

Then, $\Gamma_1, \Gamma_2, \Gamma_4$ *and* Γ_3 *have a unique common fixed point.*

Proof. The pair (Γ_2, Γ_4) satisfy *E.A.* property then, there exists a sequence $\{\varsigma_n\}_{n\in\mathbb{N}} \in \mho$ such that $\lim_{n\to\infty} \Gamma_2\varsigma_n = \lim_{n\to\infty} \Gamma_4\varsigma_n = \tau \in \mho$. Since, (Γ_2, Γ_4) are faintly compatible and $\lim_{n\to\infty} \Gamma_2\varsigma_n = \lim_{n\to\infty} \Gamma_4\varsigma_n = \tau \in \mho$, then there exists a sequence $\{\varphi_n\}_{n\in\mathbb{N}} \in \mho$ satisfying $\lim_{n\to\infty} \Gamma_2\varphi_n = \Gamma_4\varphi_n = u$ such that $\lim_{n\to\infty} \|d_{\mathbb{B}}(\Gamma_2\Gamma_4\varphi_n, \Gamma_4\Gamma_2\varphi_n)\| = 0$.

Further, (Γ_2, Γ_4) are reciprocal continuous, then $\lim_{n\to\infty} \Gamma_2\Gamma_4\varphi_n = \Gamma_2\mu$ and $\lim_{n\to\infty} \Gamma_4\Gamma_2\varphi_n = \Gamma_4\mu$. The last three limit show that $\Gamma_2\mu = \Gamma_4\mu$. Again, $\Gamma_2(\mho) \subseteq \Gamma_3(\mho)$, there exists a sequence $\{\vartheta_n\}_{n\in\mathbb{N}}$ in $\mho$ such that $\Gamma_2(\varsigma_n) = \Gamma_3(\vartheta_n)$. Hence, $\lim_{n\to\infty} \Gamma_3(\vartheta_n) = \tau$. We claim that $\lim_{n\to\infty} \Gamma_1(\vartheta_n) = \tau$. Let if possible $\lim_{n\to\infty} \Gamma_1(\vartheta_n) = \tau_1 \neq \tau$. Then, putting $\vartheta = \vartheta_n$ and $\varsigma = \varsigma_n$ in condition (*iii*), we have

$$\begin{aligned}\phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\vartheta_n, \Gamma_2\varsigma_n)\right) \preceq{}& \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\Gamma_1\vartheta_n, \Gamma_4\varsigma_n) + \beta d_{\mathbb{B}}(\Gamma_2\varsigma_n, \Gamma_3\vartheta_n)\right)\\ &- \psi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\vartheta_n, \Gamma_4\varsigma_n), d_{\mathbb{B}}(\Gamma_2\varsigma_n, \Gamma_3\vartheta_n)\right).\end{aligned} \tag{2.7}$$

Taking limit as $n \to \infty$ in (2.7), we get

$$\begin{aligned}\phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\tau_1, \tau)\right) &\preceq \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\tau_1, \tau) + \beta d_{\mathbb{B}}(\tau, \tau)\right) - \psi_{\mathbb{B}}\left(d_{\mathbb{B}}(\tau_1, \tau), d_{\mathbb{B}}(\tau, \tau)\right)\\ &\prec \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\tau_1, \tau)\right).\end{aligned}$$

Since, $\phi_{\mathbb{B}}$ is non-decreasing therefore $\|d_{\mathbb{B}}(\tau_1,\tau)\| < \|\alpha\|\,\|d_{\mathbb{B}}(\tau_1,\tau)\|$, which is a contradiction. Hence, $\tau_1 = \tau$. Subsequently, we get

$$\lim_{n\to\infty}\Gamma_1(\vartheta_n) = \lim_{n\to\infty}\Gamma_2(\varsigma_n) = \lim_{n\to\infty}\Gamma_4(\varsigma_n) = \lim_{n\to\infty}\Gamma_3(\vartheta_n) = \tau.$$

Since, (Γ_1,Γ_3) are faintly compatible and $\lim_{n\to\infty}\Gamma_1\vartheta_n = \lim_{n\to\infty}\Gamma_3\vartheta_n = \tau \in \mho$, then there exists a sequence $\{\varrho_n\} \in \mho$ satisfying $\lim_{n\to\infty}\Gamma_1\varrho_n = \Gamma_3\varrho_n = h$, such that

$$\lim_{n\to\infty}\|d_{\mathbb{B}}(\Gamma_1\Gamma_3\varrho_n, \Gamma_3\Gamma_1\varrho_n)\| = 0.$$

Further, (Γ_1,Γ_3) are reciprocal continuous, then $\lim_{n\to\infty}\Gamma_1\Gamma_3\varrho_n = \Gamma_1 h$ and $\lim_{n\to\infty}\Gamma_3\Gamma_1\varrho_n = \Gamma_3 h$. The last three limit show that $\Gamma_3 h = \Gamma_1 h$. Now, on the lines of Theorem (2.17), one can easily prove the uniqueness of common fixed point of the mappings $\Gamma_1,\Gamma_2,\Gamma_4$ and Γ_3. □

Corollary 2.21. *Let $(\mho,\mathbb{B},d_{\mathbb{B}})$ be C^*_{av}-metric space and $\Gamma_1,\Gamma_2,\Gamma_3$ and Γ_4 are four self mappings on $\mho$ satisfying:*

(i) $\Gamma_1(\mho) \subseteq \Gamma_4(\mho)$ *and* $\Gamma_2(\mho) \subseteq \Gamma_3(\mho)$*;*

(ii) the pairs (Γ_1,Γ_3) *and* (Γ_2,Γ_4) *are reciprocal continuous and faintly compatible;*

(iii) for every $\vartheta,\varsigma \in \mho$, $\alpha,\beta \in \mathbb{B}^+$ *with* $0 < \|\alpha+\beta\| < 1$,

$$\phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\vartheta,\Gamma_2\varsigma)\right) \preceq \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\Gamma_1\vartheta,\Gamma_3\vartheta) + \beta d_{\mathbb{B}}(\Gamma_2\varsigma,\Gamma_4\varsigma)\right) - \psi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\vartheta,\Gamma_4\varsigma), d_{\mathbb{B}}(\Gamma_2\varsigma,\Gamma_3\vartheta)\right);$$

(iv) one of the pair (Γ_1,Γ_3) *or* (Γ_2,Γ_4) *satisfy E.A. property.*

Then, $\Gamma_1,\Gamma_2,\Gamma_4$ and Γ_3 have a unique common fixed point.

Corollary 2.22. *Let $(\mho,\mathbb{B},d_{\mathbb{B}})$ be C^*_{av}-metric space and $\Gamma_1,\Gamma_2,\Gamma_3$ and Γ_4 are four self mappings on $\mho$ satisfying:*

(i) $\Gamma_1(\mho) \subseteq \Gamma_4(\mho)$ *and* $\Gamma_2(\mho) \subseteq \Gamma_3(\mho)$*;*

(ii) the pairs (Γ_1,Γ_3) *and* (Γ_2,Γ_4) *are reciprocal continuous and faintly compatible;*

(iii) for every $\vartheta,\varsigma \in \mho$, $\alpha,\beta \in \mathbb{B}^+$ *with* $0 < \|\alpha+\beta\| < 1$,

$$d(\Gamma_1\vartheta,\Gamma_2\varsigma) \preceq \alpha^* \frac{d(\Gamma_3\vartheta,\Gamma_1\vartheta)d(\Gamma_3\vartheta,\Gamma_2\varsigma) + d(\Gamma_4\varsigma,\Gamma_2\varsigma) + d(\Gamma_4\varsigma,\Gamma_1\varsigma)}{1 + d(\Gamma_3\vartheta,\Gamma_2\varsigma) + d(\Gamma_4\varsigma,\Gamma_1\vartheta)}\alpha;$$

(iv) one of the pair (Γ_1,Γ_3) *or* (Γ_2,Γ_4) *satisfy E.A. property.*

Then, $\Gamma_1,\Gamma_2,\Gamma_4$ and Γ_3 have a unique common fixed point.

Example 2.23. Let $\mho = [0,\infty)$ and $\mathbb{B} = \mathbb{C}$. Define $d : \mho \times \mho \to \mathbb{B}$ by $d_{\mathbb{B}}(\vartheta,\varsigma) = |\vartheta - \varsigma|$ Then, $(\mho,\mathbb{B},d_{\mathbb{B}})$ is C^*_{av}-metric space. Defining four self

mappings $\Gamma_1, \Gamma_2, \Gamma_3$ and Γ_4 on $\mho$ by

$$\Gamma_1(\vartheta) = \begin{cases} \vartheta & \text{if } \vartheta = 1 \\ 2 & \text{otherwise} \end{cases}, \qquad \Gamma_2(\vartheta) = \begin{cases} 1 & \text{if } \vartheta \in [0,1] \\ 0 & \text{otherwise,} \end{cases}$$

$$\Gamma_3(\vartheta) = \begin{cases} \vartheta + 1 & \text{if } \vartheta \in [0,1) \\ 1 & \text{if } \vartheta = 1 \\ 4 & \text{otherwise.} \end{cases}, \qquad \Gamma_4(\vartheta) = \begin{cases} \vartheta & \text{if } \vartheta = 1 \\ \vartheta + 1 & \text{otherwise.} \end{cases}$$

For the sequence $\vartheta_n = 1 - \dfrac{1}{n}$, where $n \in \mathbb{N}$, the pair (Γ_1, Γ_3) satisfy *E.A.* property and non-compatibility condition. Then, there exists a sequence $\varsigma_n = 1$ for all $n \in \mathbb{N}$, the pairs (Γ_1, Γ_3) and (Γ_2, Γ_4) are faintly compatible. Also,

(*i*) $\Gamma_1\mho \subseteq \Gamma_4\mho$ and $\Gamma_2\mho \subseteq \Gamma_3\mho$,

(*ii*) for every $\vartheta, \varsigma \in \mho$, $\alpha, \beta \in \mathbb{B}^+$ with $0 < \|\alpha + \beta\| \leq 1$ and for choosing $\phi_{\mathbb{B}}(\tau) = \tau$ and $\psi_{\mathbb{B}}(\tau) = \theta_{\mathbb{B}}$, we have

$$\begin{aligned} \phi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\vartheta, \Gamma_2\varsigma)\right) \preceq \phi_{\mathbb{B}}\left(\alpha d_{\mathbb{B}}(\Gamma_1\vartheta, \Gamma_4\varsigma) + \beta d_{\mathbb{B}}(\Gamma_2\varsigma, \Gamma_3\vartheta)\right) \\ - \psi_{\mathbb{B}}\left(d_{\mathbb{B}}(\Gamma_1\vartheta, \Gamma_4\varsigma), d_{\mathbb{B}}(\Gamma_2\varsigma, \Gamma_3\vartheta)\right), \end{aligned}$$

(*iii*) the pairs (Γ_1, Γ_3) and (Γ_2, Γ_4) are reciprocal continuous.

Hence, the above pair of mappings (Γ_1, Γ_3) and (Γ_2, Γ_4) satisfy all the conditions of Theorem (2.17) and Theorem (2.20), therefore the mappings Γ_1, Γ_2, Γ_4 and Γ_3 have a unique common fixed point. Moreover, $\vartheta = 1$ is the common fixed point of Γ_1, Γ_2, Γ_4 and Γ_3.

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