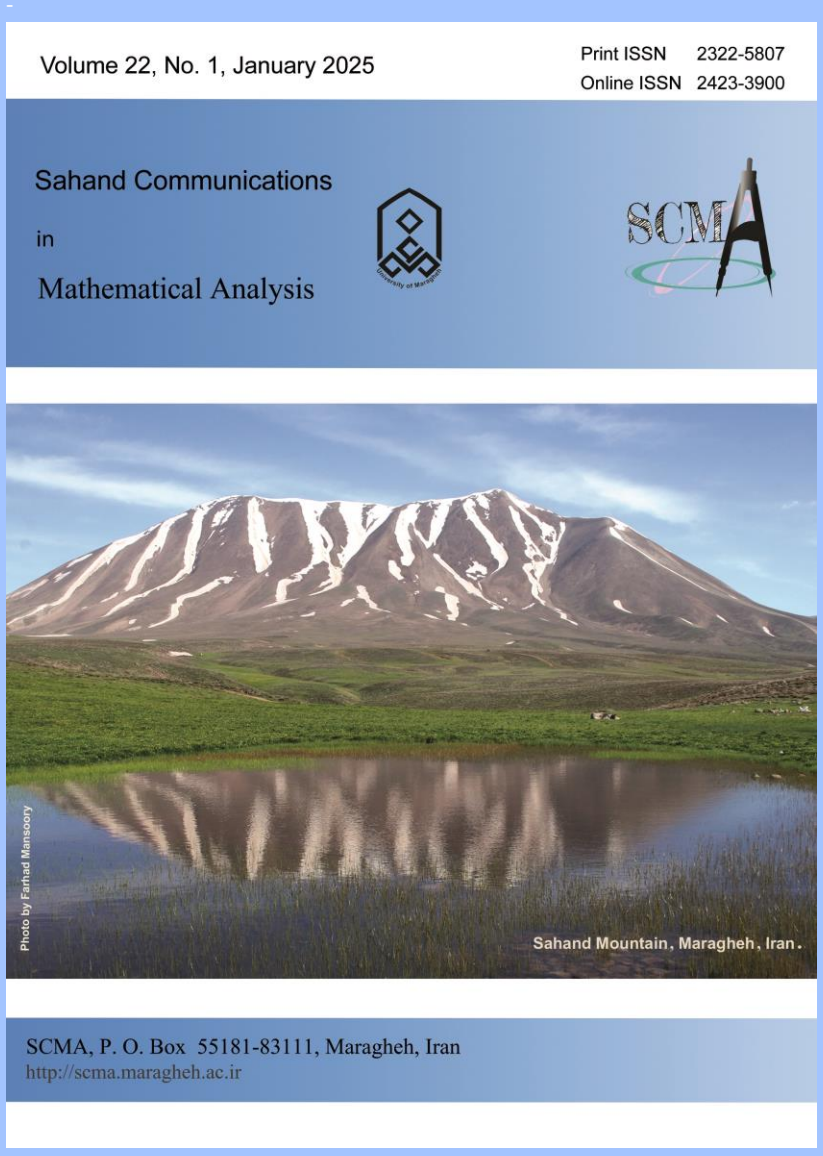
Micro αg-irresolute Maps in Micro Topological Spaces

Rangasamy Anandhi and Nataraj Balamani

Sahand Communications in Mathematical Analysis

Print ISSN: 2322-5807
Online ISSN: 2423-3900
Volume: 22
Number: 1
Pages: 75-92

Sahand Commun. Math. Anal.
DOI: 10.22130/scma.2024.2017088.1552



Sahand Communications in Mathematical Analysis (SCMA) Vol. 22 No. 1 (2025), 75-92
http://scma.maragheh.ac.ir
DOI: 10.22130/scma.2024.2017088.1552

Micro αg-irresolute Maps in Micro Topological Spaces

Rangasamy Anandhi[1*] and Nataraj Balamani[2]

Abstract. In this paper, we initiate a stronger form of Micro αg-continuous maps called Micro αg-irresolute maps and study some of their properties. Also, we introduce Micro αg-closed maps and obtain their properties. Further, we derive some composition of maps using newly defined maps. In addition to this, we develop a new class of homeomorphism called Micro αg-homeomorphism and Micro αgc-homeomorphism and discuss some of their properties.

1. Introduction

The notion of Micro topology was introduced by Sakkraiveeranan Chandrasekar [10] and he studied the concepts of Micro closed, Micro pre closed and Micro semi closed sets by using Nano topology which was introduced by Carmel Richard [3]. Chandrasekar and Swathi [4] introduced Micro α-open sets and Micro α-continuous maps and discussed some of their properties. Ibrahim [6] introduced the concept of Micro g-closed sets and Micro $T_{1/2}$-space. Further he [7, 8] defined various maps called Micro b-continuous, Micro Wb-continuous, Micro b-irresolute and Micro β-continuous maps and obtained some of their properties. Taha and Marwah [13] introduced Micro $\dot{b}$-irresolute map and analyzed their properties. Taha et al. [14] introduced and studied the properties of generalization of Micro irresolute maps. Ganesan [5] developed $m\Omega$ sets which lies between Micro closed sets and Micro generalized closed sets also he developed $m\Omega$-continuous maps and $m\Omega$-irresolute maps. Sandhiya et al. [11] studied Micro g^*-continuous maps and Micro g^*-irresolute

2020 *Mathematics Subject Classification:.* 54A05, 54B05, 54C05.
Key words and phrases. Micro topological spaces, Micro continuous maps, Micro αg-continuous maps, Micro αg-irresolute maps, Micro αg-closed map, Micro αg-open map, Micro αg-homeomorphism, Micro αgc-homeomorphism.
Received: 2023-12-04, Accepted: 2024-05-11.
* Corresponding author.

maps and investigated their properties and characterizations. Sowmiya et al. [12] introduced the concepts of Micro ψ-continuous maps and Micro ψ-irresolute maps and discussed their properties using Micro ψ-closed sets. Recently, Anandhi and Balamani [1, 2] introduced Micro αg-closed sets, Micro αg-separation axioms and discussed their properties and characterizations.

Introducing hybrid structures is important since relying just on one of these structures may not always deliver a clear idea of the concept. However, combining two or more of these structures can often improve it. In this paper, we introduce Micro αg-irresolute map in Micro topological spaces. We showed that the new concept is independent with Micro g, Micro g^* and Micro ψ-irresoluteness, so it is highly motivating to study the topological properties preserved by Micro αg-irresolute maps. We analyze the properties of the Micro αg-irresolute maps by providing more theorems and counter examples. Furthermore, we derive the necessary conditions for a Micro g-irresolute map and Micro αg-continuous map to be Micro αg-irresolute map. We also introduce Micro αg-closed maps and Micro αg-open maps and investigate their properties.

Homeomorphisms are the mappings that preserve all the topological properties of a given space. A homeomorphism is a bijective map $f\colon X \to Y$ when both f and f^{-1} are continuous. Maki et al. [9] introduced and investigated g-homeomorphism and gc-homeomorphism in topological spaces. After that, many authors have introduced different forms of homeomorphism in various other topological spaces. Following their work, we study the concept of homeomorphism in Micro topological spaces. In this paper, we introduce Micro homeomorphism, Micro αg-homeomorphism and Micro αgc-homeomorphism. Also, we find that Micro homeomorphism and Micro αgc-homeomorphism are stronger forms of Micro αg-homeomorphism.

2. Preliminaries

Throughout this paper $(\Delta, \tau_R(X^*), \mu_R(X^*))$, $(\Omega, \tau_R(Y^*), \mu_R(Y^*))$ and $(\psi, \tau_R(Z^*), \mu_R(Z^*))$(or simply Δ, Ω and ψ) denote Micro topological spaces on which no separation axioms are assumed unless explicitly stated.

Definition 2.1. [1] Let $(\Delta, \tau_R(X^*), \mu_R(X^*))$ be a Micro topological space. A subset D of Δ is said to be Micro αg-closed (briefly $Mic_{\alpha g}$-*cl.*), if Mic-$\alpha cl(D) \subseteq L$ whenever $D \subseteq L$ and L is Micro open (briefly *Mic-op.*) in Δ.

Result 2.1. (i) *Every Micro closed (briefly Mic-cl.) set is Micro α-closed (briefly Mic_α-cl.) [4].*

(ii) *Every Mic-cl. set is Micro g-closed (briefly Mic_g-cl.) [6].*
(iii) *Every Mic-cl. set is $Mic_{\alpha g}$-closed [1].*
(iv) *Every Mic_α-cl. set is $Mic_{\alpha g}$-closed [1].*
(v) *Every Mic_g-cl. set is $Mic_{\alpha g}$-closed [1].*

Definition 2.2. [6] A Micro topological space $(\Delta, \tau_R(X^*), \mu_R(X^*))$ is said to be a Micro $T_{1/2}$-space, if every Mic_g-cl. set in Δ is *Mic-closed.*

Definition 2.3. A map $v\colon (\Delta, \tau_R(X^*), \mu_R(X^*)) \to (\Omega, \tau_R(Y^*), \mu_R(Y^*))$ is called

- Micro continuous (briefly *Mic-cont.*) [10] if $v^{-1}(P)$ is *Mic-cl.* in Δ for every *Mic-cl.* set P in Ω.
- Micro α-continuous (briefly Mic_α-*cont.*) [4] if $v^{-1}(P)$ is Mic_α-*cl.* in Δ for every*Mic-cl.* set P in Ω.
- Micro g-continuous (briefly Mic_g-*cont.*) [14] if $v^{-1}(P)$ is Mic_g-*cl.* in Δ for every *Mic-cl.* set P in Ω.
- Micro g-irresolute (briefly Mic_g-*irre.*) [14] if $v^{-1}(P)$ is Mic_g-*cl.* in Δ for every Mic_g-*cl.* set P in Ω.
- Micro g^*-irresolute (briefly Mic_{g^*}-*irre.*) [11] if $v^{-1}(P)$ is Mic_{g^*}-*cl.* in Δ for every Mic_{g^*}-*cl.* set P in Ω.
- Micro ψ-irresolute (briefly Mic_ψ-*irre.*) [12] if $v^{-1}(P)$ is Mic_ψ-*cl.* in Δ for every Mic_ψ-*cl.* set P in Ω.

The following definitions and results are introduced in this article for studying the newly developed concepts.

Definition 2.4. Let $(\Delta, \tau_R(X^*), \mu_R(X^*))$ be a Micro topological space and $D \subseteq \Delta$. Then the Micro αg-closure of a set D is denoted by Mic-αg-*cl*(D) and is defined as Mic-αg-*cl*$(D) = \cap\{T : T$ is $Mic_{\alpha g}$-*cl.* in Δ and $D \subseteq T\}$.

Definition 2.5. Let $(\Delta, \tau_R(X^*), \mu_R(X^*))$ be a Micro topological space and $D \subseteq \Delta$. Then the Micro αg-interior of a set D is denoted by Mic-αg-int (D) and is defined as Mic-αg-int$(D) = \cup\{T : T$ is $Mic_{\alpha g}$-*op.* in Δ and $D \supseteq T\}$.

Definition 2.6. A Micro topological space $(\Delta, \tau_R(X^*), \mu_R(X^*))$ is said to be a

(i) Micro α-space, if every Mic_α-*cl.* set in Δ is *Mic-closed.*
(ii) Micro ${}_\alpha T_b$-space, if every $Mic_{\alpha g}$-*cl.* set in Δ is *Mic-closed.*
(iii) Micro ${}_{1/2}T_\alpha$-space, if every $Mic_{\alpha g}$-*cl.* set in Δ is Mic_α-*closed.*
(iv) Micro ${}_\alpha T_d$-space, if every $Mic_{\alpha g}$-*cl.* set in Δ is Mic_g-*closed.*

Definition 2.7. A map $v\colon (\Delta, \tau_R(X^*), \mu_R(X^*)) \to (\Omega, \tau_R(Y^*), \mu_R(Y^*))$ is called Micro αg-continuous (briefly $Mic_{\alpha g}$-*cont.*) if $v^{-1}(P)$ is $Mic_{\alpha g}$-*cl.* in Δ for every *Mic-cl.* set P in Ω.

Result 2.2. *Every Mic-cont. map* $v\colon \Delta \to \Omega$ *is a* $Mic_{\alpha g}$*-cont. map but not conversely.*

3. Micro αg-Irresolute Maps

Definition 3.1. A map $v\colon (\Delta, \tau_R(X^*), \mu_R(X^*)) \to (\Omega, \tau_R(Y^*), \mu_R(Y^*))$ is called Micro αg-irresolute (briefly $Mic_{\alpha g}$*-irre.*) if $v^{-1}(P)$ is $Mic_{\alpha g}$*-cl.* in Δ for every $Mic_{\alpha g}$*-cl.* set P in Ω.

Example 3.2. Let $\Delta = \{x, l, w, h, i\}$, $\Delta/R = \{\{x, l\}, \{w\}, \{h\}, \{i\}\}$. Let $X^* = \{x, l, w, h, i\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi\}$. Let $\mu = \{i\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{i\}, \Delta\}$. Here $Mic_{\alpha g}$*-cl.* sets in Δ are $\phi, \{x\}, \{l\}, \{w\}, \{h\}, \{x, l\}, \{x, w\}, \{x, h\}, \{x, i\}, \{l, w\}, \{l, h\}, \{l, i\}, \{w, h\}, \{w, i\}, \{h, i\}, \{x, l, w\}, \{x, l, h\}, \{x, l, i\}, \{x, w, h\}, \{x, w, i\}, \{x, h, i\}, \{l, w, h\}, \{l, w, i\}, \{l, h, i\}, \{w, h, i\}, \{x, l, w, h\}, \{x, l, w, i\}, \{x, l, h, i\}, \{x, w, h, i\}, \{l, w, h, i\}, \Delta$. Let $\Omega = \{x, l, w, h, i\}$, $\Omega/R = \{\{x\}, \{l\}, \{w\}, \{h, i\}\}$. Let $Y^* = \{w, h\} \subseteq \Omega$. Then $\tau_R(Y^*) = \{\phi, \{w\}, \{w, h, i\}, \{h, i\}, \Omega\}$. Let $\mu = \{l\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{l\}, \{w\}, \{l, w\}, \{h, i\}, \{l, h, i\}, \{w, h, i\}, \{l, w, h, i\}, \Omega\}$. Here $Mic_{\alpha g}$*-cl.* sets in Ω are $\phi, \{x\}, \{x, l\}, \{x, w\}, \{x, h\}, \{x, i\}, \{x, l, w\}, \{x, l, h\}, \{x, l, i\}, \{x, w, h\}, \{x, w, i\}, \{x, h, i\}, \{x, l, w, h\}, \{x, l, w, i\}, \{x, l, h, i\}, \{x, w, h, i\}, \Omega$. Let $v\colon (\Delta, \tau_R(X^*), \mu_R(X^*)) \to (\Omega, \tau_R(Y^*), \mu_R(Y^*))$ be a map defined by $v(x) = i$, $v(l) = x$, $v(w) = l$, $v(h) = w$, $v(i) = h$. Therefore, for every $Mic_{\alpha g}$*-cl.* set P in Ω, $v^{-1}(P)$ is $Mic_{\alpha g}$*-cl.* in Δ. Hence v is a $Mic_{\alpha g}$*-irre.* map.

Theorem 3.3. *A map* $v\colon \Delta \to \Omega$ *is* $Mic_{\alpha g}$*-irre. iff* $v^{-1}(P)$ *is* $Mic_{\alpha g}$*-op. in* Δ *for each* $Mic_{\alpha g}$*-op. set* P *in* Ω.

Proof. **(Necessity)** Let P be any $Mic_{\alpha g}$*-op.* set in Ω. Then $\Omega \backslash P$ is a $Mic_{\alpha g}$*-cl.* set in Ω. By hypothesis, $v^{-1}(\Omega \backslash P) = \Delta \backslash v^{-1}(P)$ is a $Mic_{\alpha g}$*-cl.* set in Δ. Therefore, $v^{-1}(P)$ is a $Mic_{\alpha g}$*-op.* set in Δ.

(Sufficiency) Let Q be a $Mic_{\alpha g}$*-cl.* set in Ω. Then $\Omega \backslash Q$ is a $Mic_{\alpha g}$*-op.* set in Ω. By hypothesis, $v^{-1}(\Omega \backslash Q) = \Delta \backslash v^{-1}(Q)$ is a $Mic_{\alpha g}$*-op.* set in Δ. Therefore, $v^{-1}(Q)$ is a $Mic_{\alpha g}$*-cl.* set in Δ. Therefore v is a $Mic_{\alpha g}$*-irre.* map. □

Proposition 3.4. *Every* $Mic_{\alpha g}$*-irre. map* $v\colon \Delta \to \Omega$ *is a* $Mic_{\alpha g}$*-cont. map but not conversely.*

Proof. Let P be a *Mic-cl.* set in Ω. Then P is a $Mic_{\alpha g}$*-cl.* set in Ω. $v^{-1}(P)$ is $Mic_{\alpha g}$*-cl.* in Δ as v is a $Mic_{\alpha g}$*-irre.* map. Hence v is a $Mic_{\alpha g}$*-cont.* map. □

Example 3.5. Let $\Delta = \{h, w, i, q\}$, $\Delta/R = \{\{h, w\}, \{i\}, \{q\}\}$. Let $X^* = \{i, q\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{i, q\}\}$. Let $\mu = \{w\} \notin \tau_R(X^*)$.

Then $\mu_R(X^*) = \{\phi, \{w\}, \{i,q\}\{w,i,q\}, \Delta\}$. Let $\Omega = \{h,w,i,q\}$, $\Omega/R = \{\{h\},\{w\},\{i,q\}\}$. Let $Y^* = \{w\} \subseteq \Omega$. Then $\tau_R(Y^*) = \{\Omega, \phi, \{w\}\}$. Let $\mu = \{h,w\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{w\}, \{h,w\}, \Omega\}$. Then v is a $Mic_{\alpha g}$-*cont.* map when considered as $v(h) = q$, $v(w) = i$, $v(i) = w$, $v(q) = h$, but not a $Mic_{\alpha g}$-*irre.* map, since for the $Mic_{\alpha g}$-*cl.* set $\{w,i\}$ in Ω, $v^{-1}(\{w,i\}) = \{w,i\}$ is not $Mic_{\alpha g}$-*cl.* in Δ.

Remark 3.6. Every $Mic_{\alpha g}$-*irre.* map $v\colon \Delta \to \Omega$ is a *Mic-cont.* map (resp. Mic_{α}-*cont.* map and Mic_{g}-*cont.* map), if Δ is a Micro ${}_{\alpha}T_b$-space (resp. Micro ${}_{1/2}T_{\alpha}$-space and Micro ${}_{\alpha}T_d$-space).

Proof. Let P be a *Mic-cl.* set in Ω. Then P is $Mic_{\alpha g}$-*cl.* in Ω. $v^{-1}(P)$ is $Mic_{\alpha g}$-*cl.* in Δ as v is a $Mic_{\alpha g}$-*irre.* map. Since Δ is a Micro ${}_{\alpha}T_b$-space (resp. Micro ${}_{1/2}T_{\alpha}$-space and Micro ${}_{\alpha}T_d$-space), $v^{-1}(P)$ is *Mic-cl.* (resp. Mic_{α}-*cl.* and Mic_{g}-*cl.*) in Δ. Hence v is a *Mic-cont.* map (resp. Mic_{α}-*cont.* map and Mic_{g}-*cont.* map). □

Proposition 3.7. *Every Mic_{g}-irre. map $v\colon \Delta \to \Omega$ is a $Mic_{\alpha g}$-irre. map, if Ω is a Micro ${}_{\alpha}T_d$-space.*

Proof. Let P be $Mic_{\alpha g}$-*cl.* in Ω. Since Ω is a Micro ${}_{\alpha}T_d$-space, P is a Mic_{g}-*cl.* set in Ω. Since v is a Mic_{g}-*irre.* map, $v^{-1}(P)$ is Mic_{g}-*cl.* in Δ. By Result 2.1 (v), $v^{-1}(P)$ is $Mic_{\alpha g}$-*cl.* in Δ. Hence v is a $Mic_{\alpha g}$-*irre.* map. □

Theorem 3.8. *Let $v\colon \Delta \to \Omega$ be a $Mic_{\alpha g}$-cont. map and Ω is a Micro ${}_{\alpha}T_b$-space. Then v is a $Mic_{\alpha g}$-irre. map.*

Proof. Let P be a $Mic_{\alpha g}$-*cl.* set in Ω. Since Ω is a Micro ${}_{\alpha}T_b$-space, P is a *Mic-cl.* set in Ω. Since v is $Mic_{\alpha g}$-*cont.*, $v^{-1}(P)$ is $Mic_{\alpha g}$-*cl.* in Δ. Hence v is a $Mic_{\alpha g}$-*irre.* map. □

Theorem 3.9. *If a map $v\colon \Delta \to \Omega$ is $Mic_{\alpha g}$-irre. then for every subset D of Δ such that $v(D)$ is $Mic_{\alpha g}$-cl. in Ω, v(Mic-αg-cl$(D)) \subseteq$ Mic-αg-cl$(v(D))$.*

Proof. Let $D \subseteq \Delta$ such that $v(D)$ is $Mic_{\alpha g}$-*cl.* in Ω. By hypothesis, v^{-1}(Mic-αg-cl$(v(D)))$ is $Mic_{\alpha g}$-*cl.* in Δ. Now $D \subseteq v^{-1}(v(D)) \subseteq v^{-1}$(Mic-$\alpha g$-cl$(v(D)))$. Therefore, Mic-$\alpha g$-cl$(D) \subseteq v^{-1}$(Mic-$\alpha g$-cl$(v(D)))$ which implies that v(Mic-αg-cl$(D)) \subseteq$ Mic-αg-cl$(v(D))$. □

Theorem 3.10. *Let $v\colon \Delta \to \Omega$ be $Mic_{\alpha g}$-irre. then Mic-αg-cl$(v^{-1}(E))$ $\subseteq v^{-1}(Mic$-αg-cl$(E))$ for every $Mic_{\alpha g}$-cl. set $E \subseteq \Omega$.*

Proof. Let E be a $Mic_{\alpha g}$-*cl.* set in Ω. Then *Mic-αg-cl(E)* is $Mic_{\alpha g}$-*cl.* in Ω. By hypothesis, $v^{-1}(Mic$-αg-cl$(E))$ is $Mic_{\alpha g}$-*cl.* in Δ. Since $E \subseteq Mic$-αg-cl(E), $v^{-1}(E) \subseteq v^{-1}(Mic$-$\alpha g$-cl$(E))$. By Definition 2.4, Mic-αg-cl$(v^{-1}(E)) \subseteq v^{-1}(Mic$-$\alpha g$-cl$(E))$. □

Remark 3.11. *$Mic_{\alpha g}$-irre.* maps and *Mic_g-irre.* maps are independent.

Example 3.12. Let $\Delta = \{k, f, c\}$, $\Delta/R = \{\{k\}, \{f, c\}\}$. Let $X^* = \{k, f\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{k\}\}$. Let $\mu = \{k, c\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{k\}, \{k, c\}, \Delta\}$. Let $\Omega = \{k, f, c\}$, $\Omega/R = \{\{k, f, c\}\}$. Let $Y^* = \{c\} \subseteq \Omega$. Then $\tau_R(Y^*) = \{\Omega, \phi\}$. Let $\mu = \{f\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{f\}, \Omega\}$. Then v is a *$Mic_{\alpha g}$-irre.* map when considered as $v(k) = f$, $v(f) = c$, $v(c) = k$, but not a *Mic_g-irre.* map, since for the *Mic_g-cl.* set $\{k\}$ in Ω, $v^{-1}(\{k\}) = \{c\}$ is not *Mic_g-cl.* in Δ.

Example 3.13. Let $\Delta = \{i, t, w\}$, $\Delta/R = \{\{i\}, \{t, w\}\}$. Let $X^* = \{i, t\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{i\}\}$. Let $\mu = \{i, w\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{i\}, \{i, w\}, \Delta\}$. Let $\Omega = \{i, t, w\}$, $\Omega/R = \{\{i\}, \{t, w\}\}$. Let $Y^* = \{t, w\} \subseteq \Omega$. Then $\tau_R(Y^*) = \{\Omega, \phi, \{t, w\}\}$. Let $\mu = \{t\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{t\}, \{t, w\}, \Omega\}$. Then v is a *Mic_g-irre.* map when considered as $v(i) = w$, $v(t) = i$, $v(w) = t$, but not a *$Mic_{\alpha g}$-irre.* map, since for the *$Mic_{\alpha g}$-cl.* set $\{w\}$ in Ω, $v^{-1}(\{w\}) = \{i\}$ is not *$Mic_{\alpha g}$-cl.* in Δ.

Remark 3.14. *$Mic_{\alpha g}$-irre.* maps and *Mic_{g^*}-irre.* maps are independent.

Example 3.15. Let $\Delta = \{r, o, c, l\}$, $\Delta/R = \{\{r\}, \{o\}, \{c, l\}\}$. Let $X^* = \{c, l\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{c, l\}\}$. Let $\mu = \{r\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{r\}, \{c, l\}, \{r, c, l\}, \Delta\}$. Let $\Omega = \{r, o, c, l\}$, $\Omega/R = \{\{r\}, \{o\}, \{c, l\}\}$. Let $Y^* = \{r\} \subseteq \Omega$. Then $\tau_R(Y^*) = \{\Omega, \phi, \{r\}\}$. Let $\mu = \{r, o\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{r\}, \{r, o\}, \Omega\}$. Then v is a *Mic_{g^*}-irre.* map when considered as $v(r) = r$, $v(o) = c$, $v(c) = l$, $v(l) = o$, but not a *$Mic_{\alpha g}$-irre.* map, since for the *$Mic_{\alpha g}$-cl.* set $\{o\}$ in Ω, $v^{-1}(\{o\}) = \{l\}$ is not *$Mic_{\alpha g}$-cl.* in Δ.

Example 3.16. Let $\Delta = \{r, o, l\}$, $\Delta/R = \{\{r, o\}, \{l\}\}$. Let $X^* = \{r, o\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{r, o\}\}$. Let $\mu = \{l\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{l\}, \{r, o\}, \Delta\}$. Let $\Omega = \{r, o, l\}$, $\Omega/R = \{\{r, o\}, \{l\}\}$. Let $Y^* = \{r\} \subseteq \Omega$. Then $\tau_R(Y^*) = \{\Omega, \phi\}$. Let $\mu = \{r, o\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{r, o\}, \Omega\}$. Then v is a *$Mic_{\alpha g}$-irre.* map when considered as $v(r) = r$, $v(o) = l$, $v(l) = o$, but not a *Mic_{g^*}-irre.* map, since for the *Mic_{g^*}-cl.* set $\{l\}$ in Ω, $v^{-1}(\{l\}) = \{o\}$ is not *Mic_{g^*}-cl.* in Δ.

Remark 3.17. *$Mic_{\alpha g}$-irre.* maps and *Mic_{ψ}-irre.* maps are independent.

Example 3.18. Let $\Delta = \{i, k, s, h\}$, $\Delta/R = \{\{s\}, \{h\}, \{i, k\}\}$. Let $X^* = \{i, k\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{i, k\}\}$. Let $\mu = \{h\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{h\}, \{i, k\}, \{i, k, h\}, \Delta\}$. Let $\Omega = \{i, k, s, h\}$, $\Omega/R =$

$\{\{i\},\{k\},\{s,h\}\}$. Let $Y^* = \{s,h\} \subseteq \Omega$. Then $\tau_R(Y^*) = \{\Omega,\phi,\{s,h\}\}$. Let $\mu = \{i\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi,\{i\},\{s,h\},\{i,s,h\},\Omega\}$. Then v is a $Mic_{\alpha g}$-*irre.* map when considered as $v(i) = h$, $v(k) = i$, $v(s) = k$, $v(h) = s$, but not a Mic_{ψ}-*irre.* map, since for the Mic_{ψ}-*cl.* set $\{i,k\}$ in Ω, $v^{-1}(\{i,k\}) = \{k,s\}$ is not Mic_{ψ}-*cl.* in Δ.

Example 3.19. Let $\Delta = \{h,i,r\}$, $\Delta/R = \{\{h\},\{i,r\}\}$. Let $X^* = \{h,i\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta,\phi,\{h\}\}$. Let $\mu = \{i\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi,\{h\},\{i\},\{h,i\},\Delta\}$. Let $\Omega = \{h,i,r\}$, $\Omega/R = \{\{h,i\},\{r\}\}$. Let $Y^* = \{h,i\} \subseteq \Omega$. Then $\tau_R(Y^*) = \{\Omega,\phi,\{h,i\}\}$. Let $\mu = \{r\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi,\{r\},\{h,i\},\Omega\}$. Then v is a Mic_{ψ}-*irre.* map when considered as $v(h) = r$, $v(i) = h$, $v(r) = i$, but not a $Mic_{\alpha g}$-*irre.* map, since for the $Mic_{\alpha g}$-*cl.* set $\{r\}$ in Ω, $v^{-1}(\{r\}) = \{h\}$ is not $Mic_{\alpha g}$-*cl.* in Δ.

Theorem 3.20. *Let* $(\Delta,\tau_R(X^*),\mu_R(X^*)),(\Omega,\tau_R(Y^*),\mu_R(Y^*))$ *and* $(\psi,\tau_R(Z^*),\mu_R(Z^*))$ *be the three Micro topological spaces. If* $v\colon (\Delta,\tau_R(X^*),\mu_R(X^*)) \to (\Omega,\tau_R(Y^*),\mu_R(Y^*))$ *is a* $Mic_{\alpha g}$-*irre. map and* $w\colon (\Omega,\tau_R(Y^*),\mu_R(Y^*)) \to (\psi,\tau_R(Z^*),\mu_R(Z^*))$ *is a* $Mic_{\alpha g}$-*cont. map, then* $(w \circ v)\colon (\Delta,\tau_R(X^*),\mu_R(X^*)) \to (\psi,\tau_R(Z^*),\mu_R(Z^*))$ *is a* $Mic_{\alpha g}$-*cont. map.*

Proof. Let P be *Mic-cl.* in ψ. Since w is a $Mic_{\alpha g}$-*cont.* map, $w^{-1}(P)$ is $Mic_{\alpha g}$-*cl.* in Ω. Since v is a $Mic_{\alpha g}$-*irre.* map, $(w \circ v)^{-1}(P) = v^{-1}(w^{-1}(P))$ is $Mic_{\alpha g}$-*cl.* in Δ. Therefore $w \circ v$ is a $Mic_{\alpha g}$-*cont.* map. □

Theorem 3.21. *If* $v\colon \Delta \to \Omega$ *is a* $Mic_{\alpha g}$-*cont. map and* $w\colon \Omega \to \psi$ *is a* $Mic_{\alpha g}$-*irre. map and* Ω *is a Micro* ${}_{\alpha}T_b$-*space, then* $(w \circ v)\colon \Delta \to \psi$ *is a* $Mic_{\alpha g}$-*irre. map.*

Proof. Let P be $Mic_{\alpha g}$-*cl.* in ψ. Since w is a $Mic_{\alpha g}$-*irre.* map, $w^{-1}(P)$ is $Mic_{\alpha g}$-*cl.* in Ω. Since Ω is a Micro ${}_{\alpha}T_b$-space, $w^{-1}(P)$ is *Mic-cl.* in Ω. $(w \circ v)^{-1}(P) = v^{-1}(w^{-1}(P))$ is $Mic_{\alpha g}$-*cl.* in Δ as v is a $Mic_{\alpha g}$-*cont.* map. Therefore $w \circ v$ is a $Mic_{\alpha g}$-*irre.* map. □

Theorem 3.22. *If* $v\colon \Delta \to \Omega$ *and* $w\colon \Omega \to \psi$ *are* $Mic_{\alpha g}$-*irre. maps then* $(w \circ v)\colon \Delta \to \psi$ *is a* $Mic_{\alpha g}$-*irre. map.*

Proof. Let P be $Mic_{\alpha g}$-*cl.* in ψ. Since w is a $Mic_{\alpha g}$-*irre.* map, $w^{-1}(P)$ is $Mic_{\alpha g}$-*cl.* in Ω. Since v is a $Mic_{\alpha g}$-*irre.* map, $(w \circ v)^{-1}(P) = v^{-1}(w^{-1}(P))$ is $Mic_{\alpha g}$-*cl.* in Δ. Therefore $w \circ v$ is a $Mic_{\alpha g}$-*irre.* map. □

Theorem 3.23. *If* $v\colon \Delta \to \Omega$ *is a* $Mic_{\alpha g}$-*irre. map and* $w\colon \Omega \to \psi$ *is a Mic-cont. map then* $(w \circ v)\colon \Delta \to \psi$ *is a* $Mic_{\alpha g}$-*cont. map.*

Proof. Let P be *Mic-cl.* in ψ. $w^{-1}(P)$ is *Mic-cl.* in Ω as w is a *Mic-cont.* map. By Result2.1(iii), $w^{-1}(P)$ is $Mic_{\alpha g}$-*cl.* in Ω. $(w \circ v)^{-1}(P) =$

$v^{-1}(w^{-1}(P))$ is $Mic_{\alpha g}$-*cl.* in Δ as v is a $Mic_{\alpha g}$-*irre.* map. Therefore $w \circ v$ is a $Mic_{\alpha g}$-*cont.* map. □

Theorem 3.24. *If* $v\colon \Delta \to \Omega$ *is a* $Mic_{\alpha g}$-*irre. map and* $w\colon \Omega \to \psi$ *is a* Mic_{α}-*cont.map then* $(w \circ v)\colon \Delta \to \psi$ *is a* $Mic_{\alpha g}$-*cont. map.*

Proof. Let P be *Mic-cl.* in ψ. $w^{-1}(P)$ is Mic_{α}-*cl* in Ω as w is a Mic_{α}-*cont.* map. By Result 2.1(iv), $w^{-1}(P)$ is $Mic_{\alpha g}$-*cl.* in Ω. $(w \circ v)^{-1}(P) = v^{-1}(w^{-1}(P))$ is $Mic_{\alpha g}$-*cl.* in Δ as v is a $Mic_{\alpha g}$-*irre.* map. Therefore $w \circ v$ is a $Mic_{\alpha g}$-*cont.* map. □

Theorem 3.25. *If* $v\colon \Delta \to \Omega$ *is a* $Mic_{\alpha g}$-*irre. map and* $w\colon \Omega \to \psi$ *is a* Mic_{g}-*cont. map then* $(w \circ v)\colon \Delta \to \psi$ *is a* $Mic_{\alpha g}$-*cont. map.*

Proof. Let P be *Mic-cl.* in ψ. $w^{-1}(P)$ is Mic_{g}-*cl* in Ω as w is a Mic_{g}-*cont.* map. By Result 2.1(v), $w^{-1}(P)$ is $Mic_{\alpha g}$-*cl.* in Ω. $(w \circ v)^{-1}(P) = v^{-1}(w^{-1}(P))$ is $Mic_{\alpha g}$-*cl.* in Δ as v is a $Mic_{\alpha g}$-*irre.* map. Therefore $w \circ v$ is a $Mic_{\alpha g}$-*cont.* map. □

Theorem 3.26. *Let* $v\colon \Delta \to \Omega$ *be Mic-cont. and* $w\colon \Omega \to \psi$ *be* $Mic_{\alpha g}$-*irre. maps. Then* $(w \circ v) : \Delta \to \psi$ *is a* $Mic_{\alpha g}$-*irre. map, if* Ω *is a Micro* ${}_{\alpha}T_b$-*space.*

Proof. Let P be $Mic_{\alpha g}$-*cl.* in ψ. $w^{-1}(P)$ is $Mic_{\alpha g}$-*cl.* in Ω as w is a $Mic_{\alpha g}$-*irre.* map. Since Ω is a Micro ${}_{\alpha}T_b$-space, $w^{-1}(P)$ is *Mic-cl.* in Ω. $(w \circ v)^{-1}(P) = v^{-1}(w^{-1}(P))$ is *Mic-cl.* in Δ as v is a *Mic-cont.* map. By Result 2.1(iii), $(w \circ v)^{-1}(P) = v^{-1}(w^{-1}(P))$ is a $Mic_{\alpha g}$-*cl.* set in Δ. Therefore $w \circ v$ is a $Mic_{\alpha g}$-*irre.* map. □

Theorem 3.27. *Let* $v\colon \Delta \to \Omega$ *be* Mic_{α}-*cont. and* $w\colon \Omega \to \psi$ *be* $Mic_{\alpha g}$-*irre. maps. Then* $(w \circ v) : \Delta \to \psi$ *is a* $Mic_{\alpha g}$-*irre. map, if* Ω *is a Micro* ${}_{\alpha}T_b$-*space.*

Proof. Let P be $Mic_{\alpha g}$-*cl.* in ψ. $w^{-1}(P)$ is $Mic_{\alpha g}$-*cl.* in Ω as w is a $Mic_{\alpha g}$-*irre.* map. Since Ω is a Micro ${}_{\alpha}T_b$-space, $w^{-1}(P)$ is *Mic-cl.* in Ω. $(w \circ v)^{-1}(P) = v^{-1}(w^{-1}(P))$ is Mic_{α}-*cl.* in Δ as v is a Mic_{α}-*cont.* map. By Result 2.1(iv), $(w \circ v)^{-1}(P) = v^{-1}(w^{-1}(P))$ is a $Mic_{\alpha g}$-*cl.* set in Δ. Therefore $w \circ v$ is a $Mic_{\alpha g}$-*irre.* map. □

Theorem 3.28. *Let* $v\colon \Delta \to \Omega$ *be* Mic_{g}-*cont. and* $w\colon \Omega \to \psi$ *be* $Mic_{\alpha g}$-*irre. maps. Then* $(w \circ v) : \Delta \to \psi$ *is a* $Mic_{\alpha g}$-*irre. map, if* Ω *is a Micro* ${}_{\alpha}T_b$-*space.*

Proof. Let P be $Mic_{\alpha g}$-*cl.* in ψ. $w^{-1}(P)$ is $Mic_{\alpha g}$-*cl.* in Ω as w is a $Mic_{\alpha g}$-*irre.* map. Since Ω is a Micro ${}_{\alpha}T_b$-space, $w^{-1}(P)$ is *Mic-cl.* in Ω. $(w \circ v)^{-1}(P) = v^{-1}(w^{-1}(P))$ is Mic_{g}-*cl.* in Δ as v is a Mic_{g}-*cont.* map. By Result 2.1(v), $(w \circ v)^{-1}(P) = v^{-1}(w^{-1}(P))$ is a $Mic_{\alpha g}$-*cl.* set in Δ. Therefore $w \circ v$ is a $Mic_{\alpha g}$-*irre.* map. □

4. Micro αg-Closed Maps and Micro αg-Open Maps

Definition 4.1. A map $v\colon (\Delta, \tau_R(X^*), \mu_R(X^*)) \to (\Omega, \tau_R(Y^*), \mu_R(Y^*))$ is called *Mic-cl.* if $v(P)$ is *Mic-cl.* in Ω for every *Mic-cl.* set P in Δ. i.e., if the image of every *Mic-cl.* set in Δ is *Mic-cl.* in Ω.

Definition 4.2. A map $v\colon (\Delta, \tau_R(X^*), \mu_R(X^*)) \to (\Omega, \tau_R(Y^*), \mu_R(Y^*))$ is called $Mic_{\alpha g}$*-cl.* if $v(P)$ is $Mic_{\alpha g}$*-cl.* in Ω for every *Mic-cl.* set P in Δ. i.e., if the image of every *Mic-cl.* set in Δ is $Mic_{\alpha g}$*-cl.* in Ω.

Definition 4.3. A map $v\colon (\Delta, \tau_R(X^*), \mu_R(X^*)) \to (\Omega, \tau_R(Y^*), \mu_R(Y^*))$ is called $Mic_{\alpha g}$*-op.* if $v(P)$ is $Mic_{\alpha g}$*-op.* in Ω for every *Mic-op.* set P in Δ. i.e., if the image of every *Mic-op.* set in Δ is $Mic_{\alpha g}$*-op.* in Ω.

Example 4.4. Let $\Delta = \{m, u, k, p\}$, $\Delta/R = \{\{m, u\}, \{k\}, \{p\}\}$. Let $X^* = \{m, u\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{m, u\}\}$. Let $\mu = \{k\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{k\}, \{m, u\}, \{m, u, k\}, \Delta\}$. Here *Mic-cl.* sets in Δ are $\phi, \{p\}, \{k, p\}, \{m, u, p\}, \Delta$. Let $\Omega = \{m, u, k, p\}$, $\Omega/R = \{\{m\}, \{u\}, \{k, p\}\}$. Let $Y^* = \{m\} \subseteq \Omega$ Then $\tau_R(Y^*) = \{\Omega, \phi, \{m\}\}$. Let $\mu = \{p\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{m\}, \{p\}, \{m, p\}, \Omega\}$. Here $Mic_{\alpha g}$*-cl.* sets in Ω are $\phi, \{u\}, \{k\}, \{m, u\}, \{m, k\}, \{u, k\}, \{u, p\}, \{k, p\}$, $\{m, u, k\}, \{m, u, p\}, \{m, k, p\}, \{u, k, p\}, \Omega$. Hence the image of every *Mic -cl.* set in Δ is $Mic_{\alpha g}$*-cl.* in Ω when considered v as $v(m) = k$, $v(u) = p$, $v(k) = m$, $v(p) = u$. Therefore v is a $Mic_{\alpha g}$*-cl.* map.

Example 4.5. In Example 4.4, the image of every *Mic-op.* set in Δ is $Mic_{\alpha g}$*-op.* in Ω when considered v as $v(m) = m$, $v(u) = k$, $v(k) = p$, $v(p) = u$. Hence v is a $Mic_{\alpha g}$*-op.* map.

Proposition 4.6. *Every Mic-cl. map* $v\colon \Delta \to \Omega$ *is a* $Mic_{\alpha g}$*-cl. map but not conversely.*

Proof. Let P be a *Mic-cl.* set in Δ. By hypothesis, $v(P)$ is a *Mic-cl.* set in Ω. By Result 2.1(iii), $v(P)$ is $Mic_{\alpha g}$*-cl.* set in Ω. Hence v is a $Mic_{\alpha g}$*-cl.* map. □

Example 4.7. In Example 4.4, v is a $Mic_{\alpha g}$*-cl.* map but not a *Mic-cl.* map, since for the *Mic-cl.* set $\{p\}$ in Δ, $v(\{p\}) = \{u\}$ is not *Mic-cl.* in Ω.

Theorem 4.8. *If* $v\colon \Delta \to \Omega$ *is a* $Mic_{\alpha g}$*-cl. map then Mic-*αg*-cl*$(v(D)) \subseteq v($*Mic-cl*$(D))$ *for every* $D \subseteq \Delta$.

Proof. Suppose that v is a $Mic_{\alpha g}$*-cl.* map. Since for every $D \subseteq \Delta$, Mic-*cl*(D) is a *Mic-cl.* set in Δ. By hypothesis, $v($Mic-*cl*$(D))$ is a $Mic_{\alpha g}$*-cl.* set in Ω. Since $D \subseteq$ Mic-*cl*(D), $v(D) \subseteq v($Mic-*cl*$(D))$, which implies that Mic-αg*-cl*$(v(D)) \subseteq$ *Mic-*αg*-cl*$(v($*Mic-cl*$(D)))$. Since $v($*Mic-cl*$(D))$ is a $Mic_{\alpha g}$*-cl.* set which implies *Mic-*αg*-cl*$(v(D)) \subseteq v($*Mic-cl*$(D))$. □

Theorem 4.9. *Let $v\colon \Delta \to \Omega$ be a $Mic_{\alpha g}$-op. map then $v(Mic\text{-}int(E)) \subseteq Mic\text{-}\alpha g\text{-}int(v(E))$ for every $E \subseteq \Delta$.*

Proof. Let v be a $Mic_{\alpha g}$-*op.* map and $E \subseteq \Delta$. Then $Mic\text{-}int(E)$ is Mic-*op.* in Δ. By hypothesis, $v(Mic\text{-}int(E))$ is $Mic_{\alpha g}$-*op.* in Ω. Hence $Mic\text{-}\alpha g\text{-}int(v(Mic\text{-}int(E))) = v(Mic\text{-}int(E))$. Also $Mic\text{-}int(E) \subseteq E$ implies that $v(Mic\text{-}int(E)) \subseteq v(E)$. Therefore $Mic\text{-}\alpha g\text{-}int(v(Mic\text{-}int(E))) \subseteq Mic\text{-}\alpha g\text{-}int(v(E))$. Hence $v(Mic\text{-}int(E)) \subseteq Mic\text{-}\alpha g\text{-}int(v(E))$. □

Theorem 4.10. *A map $v\colon \Delta \to \Omega$ is $Mic_{\alpha g}$-op. iff for any subset K of Ω and any Mic-cl. set T of Δ containing $v^{-1}(K)$, there exists a $Mic_{\alpha g}$-cl. set D of Ω containing K such that $v^{-1}(D) \subseteq T$.*

Proof. Suppose that v is a $Mic_{\alpha g}$-*op.* map. Let $K \subseteq \Omega$ and T be a Mic-*cl.* set of Δ such that $v^{-1}(K) \subseteq T$. Then $\Delta\backslash T$ is a Mic-*op.* set in Δ. Since v is a $Mic_{\alpha g}$-*op.* map, $v(\Delta\backslash T)$ is a $Mic_{\alpha g}$-*op.* set in Ω. Then $D = \Omega\backslash[v(\Delta\backslash T)] = v(T)$ is a $Mic_{\alpha g}$-*cl.* set in Ω. Since $v^{-1}(K) \subseteq T$ implies that $K \subseteq D$ and $v^{-1}(D) = \Delta\backslash v^{-1}(v(\Delta\backslash T)) \subseteq \Delta\backslash(\Delta\backslash T) = T$. That is, $v^{-1}(D) \subseteq T$.

Conversely, let E be a Mic-*op.* set of Δ. Then, $v^{-1}(\Omega\backslash v(E)) \subseteq \Delta\backslash E$ and $\Delta\backslash E$ is a Mic-*cl.* set in Δ. By hypothesis, there exists a $Mic_{\alpha g}$-*cl.* set D of Ω such that $\Omega\backslash v(E) \subseteq D$ and $v^{-1}(D) \subseteq \Delta\backslash E$ and $E \subseteq \Delta\backslash v^{-1}(E)$. Hence $\Omega\backslash D \subseteq v(E) \subseteq v(\Delta\backslash v^{-1}(D))$ which implies $v(E) = \Omega\backslash D$. Since $\Omega\backslash D$ is $Mic_{\alpha g}$-*op.*, $v(E)$ is $Mic_{\alpha g}$-*op.* in Ω. Therefore v is $Mic_{\alpha g}$-*op.* map. □

Theorem 4.11. *Let $v\colon (\Delta, \tau_R(X^*), \mu_R(X^*)) \to (\Omega, \tau_R(Y^*), \mu_R(Y^*))$ be a bijective map. Then the following are equivalent*

(i) *v is a $Mic_{\alpha g}$-op. map*
(ii) *v is a $Mic_{\alpha g}$-cl. map*
(iii) *v^{-1} is a $Mic_{\alpha g}$-cont. map*

Proof. (1) ⇒ (2): Let P be a Mic-*cl.* set in Δ. Then $\Delta\backslash P$ is a Mic-*op.* set in Δ. By hypothesis, $v(\Delta\backslash P)$ is $Mic_{\alpha g}$-*op.* in Ω. Since v is bijective, then $v(\Delta\backslash P) = \Omega\backslash v(P)$. Hence $v(P)$ is $Mic_{\alpha g}$-*cl.* in Ω. Therefore, v is a $Mic_{\alpha g}$-*cl.* map.

(2) ⇒ (3): Let P be a Mic-*cl.* set in Δ. By hypothesis $v(P)$ is a $Mic_{\alpha g}$-*cl.* set in Ω. Since v is bijective, $(v^{-1})^{-1}(P) = v(P)$, which is a $Mic_{\alpha g}$-*cl.* set in Ω. Hence v^{-1} is a $Mic_{\alpha g}$-*cont.* map.

(3) ⇒ (1): Let P be a Mic-*op.* set in Δ. Since v^{-1} is a $Mic_{\alpha g}$-*cont.* map, $(v^{-1})^{-1}(P) = v(P)$ is $Mic_{\alpha g}$-*op.* in Ω. Hence v is a $Mic_{\alpha g}$-*op.* map. □

Remark 4.12. The composition of two $Mic_{\alpha g}$-*cl.*(op.) maps need not be $Mic_{\alpha g}$-*cl.*(op.) as shown in the following example.

Example 4.13. Let $\Delta = \Omega = \psi = \{k, i, r, q\}$, $\Delta/R = \Omega/R = \{\{k\}, \{i\}, \{r, q\}\}$ and $\psi/R = \{\{k, i\}, \{r, q\}\}$. Let $X^* = \{i\} \subseteq \Delta, Y^* = \{i\} \subseteq \Omega, Z^* = \{k, i, r\} \subseteq \psi$. Then $\tau_R(X^*) = \{\Delta, \phi, \{i\}\}, \tau_R(Y^*) = \{\Omega, \phi, \{i\}\}$, $\tau_R(Z^*) = \{\psi, \phi, \{k, i\}\}$. Let $\mu = \{k, q\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{i\}, \{k, q\}, \{k, i, q\}, \Delta\}$. Let $\mu = \{i, r\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{i\}, \{i, r\}, \Omega\}$. Let $\mu = \{r\} \notin \tau_R(Z^*)$. Then $\mu_R(Z^*) = \{\phi, \{r\}, \{k, i\}, \{k, i, r\}, \psi\}$. Then the maps v and w are $Mic_{\alpha g}$-*cl.* when considered v as $v(k) = i$, $v(i) = k$, $v(r) = q$, $v(q) = r$ and w as $w(k) = q, w(i) = r, w(r) = i, w(q) = k$. But $w \circ v$ is not a $Mic_{\alpha g}$-*cl.* map, since for the *Mic-cl.* set $\{r\}$ in Δ, $w(v(\{r\}) = w(\{q\}) = \{k\}$ is not a $Mic_{\alpha g}$-*cl.* set in ψ.

Example 4.14. In above Example 4.13, the maps v and w are $Mic_{\alpha g}$-*op.* when considered v as $v(k) = i$, $v(i) = k$, $v(r) = q$, $v(q) = r$ and w as $w(k) = q$, $w(i) = r$, $w(r) = i$, $w(q) = k$. But $w \circ v$ is not a $Mic_{\alpha g}$-*op.* map, since for the *Mic-op.* set $\{i\}$ in Δ, $w(v(\{i\})) = w(\{k\}) = \{q\}$ is not a $Mic_{\alpha g}$-*op.* set in ψ.

Theorem 4.15. *Let $v\colon \Delta \to \Omega$ and $w\colon \Omega \to \psi$ be two Mic-cl. maps. Then $(w \circ v) : \Delta \to \psi$ is a $Mic_{\alpha g}$-cl. map.*

Proof. Let P be a *Mic-cl.* set in Δ. Since v is a *Mic-cl.* map, $v(P)$ is a *Mic-cl.* set in Ω. Since w is a *Mic-cl.* map, $(w \circ v)(P) = w(v(P))$ is a *Mic-cl.* set in ψ. By Result 2.1(iii), $(w \circ v)(P)$ is a $Mic_{\alpha g}$-*cl.* set in ψ. Hence $w \circ v$ is a $Mic_{\alpha g}$-*cl.* map. □

Theorem 4.16. *If $v\colon \Delta \to \Omega$ is a Mic-cl. map and $w\colon \Omega \to \psi$ is a $Mic_{\alpha g}$-cl. map, then $(w \circ v) : \Delta \to \psi$ is a $Mic_{\alpha g}$-cl. map.*

Proof. Let P be a *Mic-cl.* set in Δ. Since v is a *Mic-cl.* map, $v(P)$ is a *Mic-cl.* set in Ω and since w is a $Mic_{\alpha g}$-*cl.* map, $(w \circ v)(P) = w(v(P))$ is a $Mic_{\alpha g}$-*cl.* set in ψ. Therefore $w \circ v$ is a $Mic_{\alpha g}$-*cl.* map. □

Theorem 4.17. *If $v\colon \Delta \to \Omega$ is a $Mic_{\alpha g}$-cl. map and $w\colon \Omega \to \psi$ is a Mic-cl. map and Ω is a Micro ${}_{\alpha}T_b$-space, then $(w \circ v) : \Delta \to \psi$ is a Mic-cl. map.*

Proof. Let P be a *Mic-cl.* set in Δ. Since v is a $Mic_{\alpha g}$-*cl.* map, $v(P)$ is a $Mic_{\alpha g}$-*cl.* set in Ω. Since Ω is a Micro ${}_{\alpha}T_b$-space, $v(P)$ is a *Mic-cl.* set in Ω. Since w is a *Mic-cl.* map, $(w \circ v)(P) = w(v(P))$ is a *Mic-cl.* set in ψ. Therefore $w \circ v$ is a *Mic-cl.* map. □

Theorem 4.18. *Let $v\colon \Delta \to \Omega$ be any map and $w\colon \Omega \to \psi$ be an injective $Mic_{\alpha g}$-irre. map. If $(w \circ v) : \Delta \to \psi$ is a $Mic_{\alpha g}$-cl. map, then v is a $Mic_{\alpha g}$-cl. map.*

Proof. Let P be any *Mic-cl.* set in Δ. Since $w \circ v$ is a $Mic_{\alpha g}$-*cl.* map, $(w \circ v)(P)$ is a $Mic_{\alpha g}$-*cl.* set in ψ. Since w is $Mic_{\alpha g}$-*irre.* and injective,

$w^{-1}[(w \circ v)(P)] = v(P)$ is a $Mic_{\alpha g}$-*cl.* set in Ω. Hence v is a $Mic_{\alpha g}$-*cl.* map. □

Theorem 4.19. *Let $v\colon \Delta \to \Omega$ and $w\colon \Omega \to \psi$ be any two $Mic_{\alpha g}$-cl. maps and Ω is a Micro ${}_{\alpha}T_b$-space. Then $(w \circ v) : \Delta \to \psi$ is a $Mic_{\alpha g}$-cl. map.*

Proof. Suppose P is a *Mic-cl.* set in Δ. Since v is a $Mic_{\alpha g}$-*cl.* map, $v(P)$ is a $Mic_{\alpha g}$-*cl.* set in Ω. Since Ω is a Micro ${}_{\alpha}T_b$-space, $v(P)$ is a *Mic-cl.* set in Ω. Since w is a $Mic_{\alpha g}$-*cl.* map, $w(v(P))$ is a $Mic_{\alpha g}$-*cl.* set in ψ and $w(v(P)) = (w \circ v)(P)$. Therefore $w \circ v$ is a $Mic_{\alpha g}$-*cl.* map. □

Theorem 4.20. *Let $v\colon \Delta \to \Omega$ and $w\colon \Omega \to \psi$ be two maps such that $(w \circ v) : \Delta \to \psi$ is a $Mic_{\alpha g}$-cl. map.*

(i)If v is Mic-cont. and surjective, then w is a $Mic_{\alpha g}$-cl. map.

(ii) If v is Mic_{α}-cont., surjective and Δ is a Micro α-space, then w is a $Mic_{\alpha g}$-cl. map.

(iii) If v is Mic_g-cont., surjective and Δ is a Micro $T_{1/2}$-space, then w is a $Mic_{\alpha g}$-cl. map.

Proof. (i) Let P be a *Mic-cl.* set in Ω. Since v is a *Mic-cont.* map, $v^{-1}(P)$ is a *Mic-cl.* set in Δ. Since $w \circ v$ is a $Mic_{\alpha g}$-*cl.* map and v is surjective, which implies $(w \circ v)\left[(v^{-1}(P))\right] = w(P)$ is a $Mic_{\alpha g}$-*cl.* set in ψ. Therefore w is a $Mic_{\alpha g}$-*cl.* map.

(ii) Let P be a *Mic-cl.* set in Ω. Since v is a Mic_{α}-*cont.* map, $v^{-1}(P)$ is a Mic_{α}-*cl.* set in Δ and Δ is a Micro α-space, $v^{-1}(P)$ is a *Mic-cl.* set in Δ. Since $w \circ v$ is a $Mic_{\alpha g}$-*cl.* map and v is surjective, $(w \circ v)\left[(v^{-1}(P))\right] = w(P)$ is $Mic_{\alpha g}$-*cl.* in ψ. Therefore w is a $Mic_{\alpha g}$-*cl.* map.

(iii) Let P be a *Mic-cl.* set in Ω. Since v is a Mic_g-*cont.* map, $v^{-1}(P)$ is a Mic_g-*cl.* set in Δ and Δ is a Micro $T_{1/2}$-space, $v^{-1}(P)$ is a *Mic-cl.* set in Δ. Since $w \circ v$ is a $Mic_{\alpha g}$-*cl.* map and v is surjective, $(w \circ v)\left[(v^{-1}(P))\right] = w(P)$ is a $Mic_{\alpha g}$-*cl.* set in ψ. Therefore w is a $Mic_{\alpha g}$-*cl.* map. □

5. Micro αg-Homeomorphism

Definition 5.1. A bijective map $v\colon (\Delta, \tau_R(X^*), \mu_R(X^*)) \to (\Omega, \tau_R(Y^*), \mu_R(Y^*))$ is called Micro homeomorphism (briefly *Mic-homeomorphism*) if v is both *Mic-cont.* map and *Mic-op.* map.

Definition 5.2. A bijective map $v\colon (\Delta, \tau_R(X^*), \mu_R(X^*)) \to (\Omega, \tau_R(Y^*), \mu_R(Y^*))$ is called Micro αg-homeomorphism (briefly $Mic_{\alpha g}$-*homeomo rphism*) if v is both $Mic_{\alpha g}$-*cont.* map and $Mic_{\alpha g}$-*op.* map.

Example 5.3. Let $\Delta = \{h, k, l, t\}$, $\Delta/R = \{\{h, k\}, \{l\}, \{t\}\}$. Let $X^* = \{l, t\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{l, t\}\}$. Let $\mu = \{k\} \notin \tau_R(X^*)$.

Then $\mu_R(X^*) = \{\phi, \{k\}, \{l,t\}, \{k,l,t\}, \Delta\}$. Let $\Omega = \{h,k,l,t\}$, $\Omega/R = \{\{h\}, \{k\}, \{l,t\}\}$. Let $Y^* = \{k\} \subseteq \Omega$. Then $\tau_R(Y^*) = \{\Omega, \phi, \{k\}\}$. Let $\mu = \{h,k\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{k\}, \{h,k\}, \Omega\}$. Then the map v is bijective, $Mic_{\alpha g}$-*cont.* and $Mic_{\alpha g}$-*op.* when considered as $v(h) = t$, $v(k) = l$, $v(l) = k$, $v(t) = h$. Hence v is $Mic_{\alpha g}$-*homeomorphism.*

Proposition 5.4. *Every Mic-homeomorphism is* $Mic_{\alpha g}$*-homeomor phism.*

Proof. Let v be *Mic-homeomorphism.* Then v is a bijective, *Mic-cont.* and *Mic-op.* map. Since every *Mic-cont.* map is $Mic_{\alpha g}$-*cont.* and every *Mic-op.* map is $Mic_{\alpha g}$-*op.*, v is both $Mic_{\alpha g}$-*cont.* and $Mic_{\alpha g}$-*op.* map. Hence the map v is bijective, $Mic_{\alpha g}$-*cont.* and $Mic_{\alpha g}$-*op.* map. Therefore v is $Mic_{\alpha g}$-*homeomorphism.* □

Remark 5.5. The converse part of above Proposition 5.4 need not be true.

Example 5.6. From the Example 5.3, the map v is $Mic_{\alpha g}$-*homeomorphi sm* but not *Mic-homeomorphism*, since for the *Mic-op.* set $\{k\}$ in Ω, $v^{-1}(\{k\}) = \{l\}$ is not *Mic-op.* in Δ.

Theorem 5.7. *Let* $v\colon \Delta \to \Omega$ *be a bijective and* $Mic_{\alpha g}$*-cont maps. Then the following statements are equivalent.*

(i) *v is a* $Mic_{\alpha g}$*-op. map*
(ii) *v is* $Mic_{\alpha g}$*-homeomorphism*
(iii) *v is a* $Mic_{\alpha g}$*-cl. map*

Proof. (1) ⇒ (2): By hypothesis, the map v is bijective, $Mic_{\alpha g}$-*cont.* and $Mic_{\alpha g}$-*open.* Hence v is $Mic_{\alpha g}$-*homeomorphism.*

(2) ⇒ (3): Let v be $Mic_{\alpha g}$-*homeomorphism.* Then v is a $Mic_{\alpha g}$-*op.* map. Let P be a *Mic-cl.* set in Δ. Then $\Delta\backslash P$ is a *Mic-op.* set in Δ. Since v is a $Mic_{\alpha g}$-*op.* map, $v(\Delta\backslash P)$ is a $Mic_{\alpha g}$-*op.* set in Ω. That is $v(\Delta\backslash P) = \Omega\backslash v(P)$ is a $Mic_{\alpha g}$-*op.* set in Ω implies $v(P)$ is a $Mic_{\alpha g}$-*cl.* set in Ω. Hence v is a $Mic_{\alpha g}$-*cl.* map.

(3) ⇒ (1): Let P be a *Mic-op.* set in Δ. Then $\Delta\backslash P$ is a *Mic-cl.* set in Δ. By hypothesis, $v(\Delta\backslash P)$ is a $Mic_{\alpha g}$-*cl.* set in Ω. That is $v(\Delta\backslash P) = \Omega\backslash v(P)$ is a $Mic_{\alpha g}$-*cl.* set in Ω implies $v(P)$ is a $Mic_{\alpha g}$-*op.* set in Ω. Hence v is a $Mic_{\alpha g}$-*op.* map. □

Remark 5.8. The composition of two $Mic_{\alpha g}$-*homeomorphisms* need not be a $Mic_{\alpha g}$-*homeomorphism* as seen from the example.

Example 5.9. Let $\Delta = \Omega = \psi = \{l,t,r,s\}$, $\Delta/R = \Omega/R = \psi/R = \{\{l\}, \{t\}, \{r,s\}\}$. Let $X^* = \{t\} \subseteq \Delta$, $Y^* = \{t\} \subseteq \Omega$, $Z^* = \{t\} \subseteq \psi$. Then $\tau_R(X^*) = \{\Delta, \phi, \{t\}\}$, $\tau_R(Y^*) = \{\Omega, \phi, \{t\}\}$, $\tau_R(Z^*) = \{\psi, \phi, \{t\}\}$. Let $\mu = \{l,s\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{t\}, \{l,s\}, \{l,t,s\}, \Delta\}$.

Let $\mu = \{t, r\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{t\}, \{t, r\}, \Omega\}$. Let $\mu = \{t, s\} \notin \tau_R(Z^*)$. Then $\mu_R(Z^*) = \{\phi, \{t\}, \{t, s\}, \psi\}$. Then the maps v and w are $Mic_{\alpha g}$-*homeomorphism*, when considered v as $v(l) = t$, $v(t) = l$, $v(r) = s$, $v(s) = r$ and w as identity map. But $w \circ v$ is not $Mic_{\alpha g}$-*homeomorphism*, since for the *Mic-cl.* set $\{l, r\}$ in ψ, $(w \circ v)^{-1}(\{l, r\}) = \{t, s\}$ is not a $Mic_{\alpha g}$-*cl* set in Δ.

Theorem 5.10. *Let $v \colon \Delta \to \Omega$ be $Mic_{\alpha g}$-homeomorphism. Then v is a Mic-homeomorphism whenever Δ and Ω are Micro ${}_{\alpha}T_b$-space.*

Proof. Let P be a *Mic-cl.* set in Ω. Since v is $Mic_{\alpha g}$-*cont.*, $v^{-1}(P)$ is a $Mic_{\alpha g}$-*cl.* set in Δ. Since Δ is a Micro ${}_{\alpha}T_b$-space, $v^{-1}(P)$ is a *Mic-cl.* set in Δ. Hence v is a *Mic-cont.* map.
Let Q be a *Mic-cl.* set in Δ. Since v is a $Mic_{\alpha g}$-*cl.* map, $v(Q)$ is a $Mic_{\alpha g}$-*cl.* set in Ω. Since Ω is a Micro ${}_{\alpha}T_b$-space, $v(Q)$ is a *Mic-cl.* set in Ω. Hence v is a *Mic-cl.* map. Therefore v is *Mic-homeomorphism.* □

Theorem 5.11. *Let $v \colon \Delta \to \Omega$ and $w \colon \Omega \to \psi$ be two $Mic_{\alpha g}$-homeomor phisms. Then $(w \circ v) : \Delta \to \psi$ is a $Mic_{\alpha g}$-homeomorphism, if Ω is a Micro ${}_{\alpha}T_b$-space.*

Proof. Let P be a *Mic-cl.* set in Δ. Since v is a $Mic_{\alpha g}$-*cl.* map, $v(P)$ is a $Mic_{\alpha g}$-*cl.* set in Ω. Since Ω is a Micro ${}_{\alpha}T_b$-space, $v(P)$ is a *Mic-cl.* set in Ω. Since w is a $Mic_{\alpha g}$-*cl.* map, $w(v(P)) = (w \circ v)(P)$ is a $Mic_{\alpha g}$-*cl.* set in ψ. Hence $w \circ v$ is a Micro $Mic_{\alpha g}$-*cl.* map.
Let Q be a *Mic-cl.* set in ψ. Since w is $Mic_{\alpha g}$-*cont.*, $w^{-1}(Q)$ is a $Mic_{\alpha g}$-*cl.* set in Ω. Since Ω is a Micro ${}_{\alpha}T_b$-space, $w^{-1}(Q)$ is a *Mic-cl.* set in Ω. Since v is $Mic_{\alpha g}$-*cont.*, $(w \circ v)^{-1}(Q) = v^{-1}(w^{-1}(Q))$ is a $Mic_{\alpha g}$-*cl.* set in Δ. Hence $w \circ v$ is a $Mic_{\alpha g}$-*cont.* map. Therefore $w \circ v$ is $Mic_{\alpha g}$-*homeomorphism.* □

Definition 5.12. A bijection map $v \colon \Delta \to \Omega$ is called $Mic_{\alpha g}$-*homeomor phism* if both v and v^{-1} are $Mic_{\alpha g}$-*cont.* map.

Definition 5.13. A bijective map $v \colon \Delta \to \Omega$ is called Micro αgc-homeom
orphism (briefly $Mic_{\alpha gc}$-*homeomorphism*) if v is $Mic_{\alpha g}$-*irre.* and its inverse v^{-1} is also a $Mic_{\alpha g}$-*irre.* map. We say that the spaces Δ and Ω are $Mic_{\alpha gc}$-*homeomorphic* if there exists $Mic_{\alpha gc}$-*homeomorphism* from Δ onto Ω. The family of all $Mic_{\alpha gc}$-*homeomorphism* of a Micro topological space Δ onto itself is denoted by $M\alpha gc$-$h(\Delta)$.

Proposition 5.14. *Every $Mic_{\alpha gc}$-homeomorphism is $Mic_{\alpha g}$-homeomor phism.*

Proof. Let v be $Mic_{\alpha gc}$-*homeomorphism.* Then v and v^{-1} are the $Mic_{\alpha g}$-*irre.* maps. Since every $Mic_{\alpha g}$-*irre.* is $Mic_{\alpha g}$-*cont.*, v and v^{-1}

are $Mic_{\alpha g}$-*cont.* maps. Then by Definition 5.12, v is $Mic_{\alpha g}$-*homeomorphism.* □

Remark 5.15. The reverse implication of the Proposition 5.14 need not be true.

Example 5.16. Let $\Delta = \{h, k, l, t\}$, $\Delta/R = \{\{h\}, \{k\}, \{l, t\}\}$. Let $X^* = \{k\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{k\}\}$. Let $\mu = \{k, l\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{k\}, \{k, l\}, \Delta\}$. Let $\Omega = \{h, k, l, t\}$, $\Omega/R = \{\{h\}, \{k\}, \{l, t\}\}$. Let $Y^* = \{k\} \subseteq \Omega$. Then $\tau_R(Y^*) = \{\Omega, \phi, \{k\}\}$. Let $\mu = \{k, t\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{k\}, \{k, t\}, \Omega\}$. Then the map v is $Mic_{\alpha g}$-*homeomorphism* when considered as identity map. Hence v is $Mic_{\alpha g}$-*homeomorphism*, but not $Mic_{\alpha gc}$-*homeomorphism*, because for the $Mic_{\alpha g}$-*cl.* set $\{k, l\}$ in Ω, $v^{-1}(\{k, l\}) = \{k, l\}$ is not $Mic_{\alpha g}$-*cl.* in Δ.

Theorem 5.17. *Let* $v \colon \Delta \to \Omega$ *and* $w \colon \Omega \to \psi$ *be two* $Mic_{\alpha gc}$-*homeomor phisms. Then* $(w \circ v) : \Delta \to \psi$ *is also a* $Mic_{\alpha gc}$-*homeomorphism.*

Proof. Suppose that v and w are two $Mic_{\alpha gc}$-*homeomorphisms*. Then by Definition 5.12, v, v^{-1}, w and w^{-1} are $Mic_{\alpha g}$-*irre.* maps. Let P be a $Mic_{\alpha g}$-*cl.* set in ψ. Since w is $Mic_{\alpha g}$-*irre.*, $w^{-1}(P)$ is a $Mic_{\alpha g}$-*cl.* set in Ω. Since v is $Mic_{\alpha g}$-*irre.*, $(w \circ v)^{-1}(P)$ is a $Mic_{\alpha g}$-*cl.* set in Δ. Therefore $w \circ v$ is a $Mic_{\alpha g}$-*irre.* map.
Let Q be a $Mic_{\alpha g}$-*cl.* set in Δ. Since v^{-1} is $Mic_{\alpha g}$-*irre.*, $(v^{-1})^{-1}(Q) = v(Q)$ is a $Mic_{\alpha g}$-*cl.* set in Ω. Since w^{-1} is $Mic_{\alpha g}$-*irre.*, $(w^{-1})^{-1}(v(Q)) = w(v(Q)) = (w \circ v)(Q)$ is a $Mic_{\alpha g}$-*cl.* set in ψ. Therefore $(w \circ v)^{-1}$ is a $Mic_{\alpha g}$-*irre.* map. Since v and w are bijective, $w \circ v$ is also bijective. Hence $w \circ v$ is $Mic_{\alpha gc}$-*homeomorphism.* □

Theorem 5.18. *The set* $M\alpha gc$-$h(\Delta)$ *is a group under the composition of maps.*

Proof. Let * be a binary operation. Define $*$: $M\alpha gc$-$h(\Delta) \times M\alpha gc$-$h(\Delta) \to M\alpha gc$-$h(\Delta)$ by $v * w = w \circ v$, for every $v, w \in M\alpha gc$-$h(\Delta)$, where $\circ$ is the usual operation of composition of maps. Then by Theorem 5.17, $w \circ v \in M\alpha gc$-$h(\Delta)$. The associative property holds in composition of maps. The identity mapping $I \colon \Delta \to \Delta$ is $Mic_{\alpha gc}$-*homeomorphism*, $I \in M\alpha gc$-$h(\Delta)$. Also $I \circ v = v \circ I = v$ for every $v \in M\alpha gc$-$h(\Delta)$. For every $v \in M\alpha gc$-$h(\Delta)$, the inverse map $v^{-1} \in M\alpha gc$-$h(\Delta)$ such that $v \circ v^{-1} = v^{-1} \circ v = I$. Therefore $M\alpha gc$-$h(\Delta)$ is a group under the composition of maps. □

Theorem 5.19. *Let* $v \colon \Delta \to \Omega$ *be* $Mic_{\alpha gc}$-*homeomorphism, then* v *induces an isomorphism from the group* $M\alpha gc$-$h(\Delta)$ *onto the group* $M\alpha gc$-$h(\Omega)$.

Proof. Let $v \in M\alpha gc\text{-}h(\Delta)$. Define $\psi_v \colon M\alpha gc\text{-}h(\Delta) \to M\alpha gc\text{-}h(\Omega)$ by $\psi_v(s) = v^{-1} \circ h \circ v$ for every $s \in M\alpha gc\text{-}h(\Delta)$. For $s_1 \neq s_2$, if $\psi_v(s_1) = \psi_v(s_2)$ then $v^{-1} \circ s_1 \circ v = v^{-1} \circ s_2 \circ v$. Hence $s_1 = s_2$, which is a contradiction. Therefore ψ_v is one to one. For $t \in M\alpha gc\text{-}h(\Omega)$, choose $s \in M\alpha gc\text{-}h(\Delta)$ such that $\psi_v(s) = v^{-1} \circ s \circ v = t$. That is $s = v \circ t \circ v^{-1} \in M\alpha gc\text{-}h(\Delta)$. Therefore ψ_v is onto. Hence ψ_v is bijective. Also for all $s_1, s_2 \in M\alpha gc\text{-}h(\Delta)$, $\psi_v(s_1 \circ s_2) = v^{-1} \circ (s_1 \circ s_2) \circ v = (v^{-1} \circ s_1 \circ v) \circ (v^{-1} \circ s_2 \circ v) = \psi_v(s_1) \circ \psi_v(s_2)$. Hence ψ_v is a homeomorphism and so it is an isomorphism induced by v. □

Theorem 5.20. *If $v \colon \Delta \to \Omega$ is $Mic_{\alpha gc}$-homeomorphism, then Mic-αg -cl$(v^{-1}(D)) = v^{-1}(Mic\text{-}\alpha g\text{-}cl(D))$ for every $Mic_{\alpha g}$-cl. set $D \subseteq \Omega$.*

Proof. As v is $Mic_{\alpha gc}$-homeomorphism, v and v^{-1} are the $Mic_{\alpha g}$-irre. maps. Since D is $Mic_{\alpha g}$-cl. in Ω, $D = Mic\text{-}\alpha g\text{-}cl(D)$. Since v is $Mic_{\alpha g}$-irre., $v^{-1}(D)$ is $Mic_{\alpha g}$-cl. in Δ. Since $D \subseteq Mic\text{-}\alpha g\text{-}cl(D)$, $v^{-1}(Mic\text{-}\alpha g\text{-}cl(D))$ is $Mic_{\alpha g}$-cl. in Δ. Obviously, $v^{-1}(D) \subseteq v^{-1}(Mic\text{-}\alpha g\text{-}cl(D))$. Therefore $Mic\text{-}\alpha g\text{-}cl(v^{-1}(D)) \subseteq v^{-1}(Mic\text{-}\alpha g\text{-}cl(D))$. Since v^{-1} is $Mic_{\alpha g}$-irre. and $Mic\text{-}\alpha g\text{-}cl(v^{-1}(D))$ is $Mic_{\alpha g}$-cl. in Δ, $(v^{-1})^{-1}(Mic\text{-}\alpha g\text{-}cl(v^{-1}(D))) = v(Mic\text{-}\alpha g\text{-}cl(v^{-1}(D)))$ is $Mic_{\alpha g}$-cl. in Ω. Since $D \subseteq (v^{-1})^{-1}(v^{-1}(D)) \subseteq (v^{-1})^{-1}(Mic\text{-}\alpha g\text{-}cl(v^{-1}(D))) = v(Mic\text{-}\alpha g\text{-}cl(v^{-1}(D)))$. Therefore $Mic\text{-}\alpha g\text{-}cl(D) \subseteq v(Mic\text{-}\alpha g\text{-}cl(v^{-1}(D)))$. That is, $v^{-1}(Mic\text{-}\alpha g\text{-}cl(D)) \subseteq v^{-1}(v(Mic\text{-}\alpha g\text{-}cl(v^{-1}(D)))) \subseteq Mic\text{-}\alpha g\text{-}cl(v^{-1}(D))$. Hence $Mic\text{-}\alpha g\text{-}cl(v^{-1}(D)) = v^{-1}(Mic\text{-}\alpha g\text{-}cl(D))$. □

Corollary 5.21. *If $v \colon \Delta \to \Omega$ is a $Mic_{\alpha gc}$-homeomorphism, then $Mic\text{-}\alpha g\text{-}cl(v(D)) = v(Mic\text{-}\alpha g\text{-}cl(D))$ for every $Mic_{\alpha g}$-cl. set $D \subseteq \Delta$.*

Proof. Since $v \colon \Delta \to \Omega$ is a $Mic_{\alpha gc}$-homeomorphism, $v^{-1} \colon \Omega \to \Delta$ is also a $Mic_{\alpha gc}$-homeomorphism. Therefore, by Theorem 5.20, $Mic\text{-}\alpha g\text{-}cl((v^{-1})^{-1}(D)) = (v^{-1})^{-1}(Mic\text{-}\alpha g\text{-}cl(D))$ for every $D \subseteq \Delta$. Hence $Mic\text{-}\alpha g\text{-}cl(v(D)) = v(Mic\text{-}\alpha g\text{-}cl(D))$. □

Theorem 5.22. *Every $Mic_{\alpha g}$-homeomorphism $v \colon \Delta \to \Omega$ is $Mic_{\alpha gc}$-homeomorphism whenever both Δ and Ω are Micro ${}_{\alpha}T_b$-space.*

Proof. Let P be a $Mic_{\alpha g}$-cl. set in Ω. Then P is a Mic-cl. set as Ω is a Micro ${}_{\alpha}T_b$-space. Since v is $Mic_{\alpha g}$-cont., $v^{-1}(P)$ is a $Mic_{\alpha g}$-cl. set in Δ. Hence v is a $Mic_{\alpha g}$-irre. map. Let Q be a $Mic_{\alpha g}$-op. set in Δ. Then Q Mic-op. set as Δ is a Micro ${}_{\alpha}T_b$-space. Since v is a $Mic_{\alpha g}$-op. map, $v(Q)$ is a $Mic_{\alpha g}$-op. set in Ω. That is $(v^{-1})^{-1}(Q)$ is a $Mic_{\alpha g}$-op. set in Ω and hence v^{-1} is a $Mic_{\alpha g}$-irre. map. Thus v is $Mic_{\alpha gc}$-homeomorphism. □

Theorem 5.23. *$Mic_{\alpha gc}$-homeomorphism is an equivalence relation in the collection of all Micro topological spaces.*

Proof. Let Π be the collection of Micro topological spaces. For $\Delta, \Omega \in \Pi$ define a relation R on Π such that Δ R Ω if and only if Δ and Ω are $Mic_{\alpha gc}$-*homeomorphic* to each other.

Reflexive: Δ R Δ is immediate.

Symmetry: Δ R $\Omega \Rightarrow \Omega$ R Δ.

Transitivity: Follows from Theorem 5.17.

Hence $Mic_{\alpha gc}$-*homeomorphism* is an equivalence relation in the collection of all Micro topological spaces. □

6. Conclusion

In this work, we have introduced Micro αg-irresolute maps and investigated their properties. Also, we defined Micro αg-homeomorphism and obtained some of their properties. In future, this concept can be extended to study real life problems in Micro topological spaces.

Acknowledgment. The author wish to thank the editor and reviewers for their valuable comments and suggestions that helped us to improve the paper.

Abbreviations of Names of Serials

bull. fukuoka univ. ed. - Bulletin of Fukuoka University of Education.

References

1. R. Anandhi and N. Balamani, *Micro α-Generalized Closed Sets in Micro Topological Spaces*, AIP Conference Proceedings, 2385 (2022), pp. 1300071-1300078.
2. R. Anandhi and N. Balamani, *Micro αg-Separation Axioms*, Adv. Appl. Math. Sci., 21(4) (2022), pp. 1925-1936.
3. Carmel Richard, *Studies on Nano Topological Spaces*, Ph.D. Thesis, Madurai Kamaraj University, Madurai, India, 2013.
4. S. Chandrasekar and G. Swathi, *Micro α-open sets in Micro Topological Spaces*, International Journal of Research in Advent Technology, 6 (2018), pp. 2633-2637.
5. S. Ganesan, *$m\Omega$-closed sets in Micro Topological Spaces*, International Journal of Scientific Research and Engineering Development, 4 (3) (2021), pp. 285-303.
6. H.Z. Ibrahim, *On Micro $T_{1/2}$-Space*, Int. J. Appl. Math., 33 (2020), pp. 369-384.
7. H.Z. Ibrahim, *Micro b-open sets*, Asia Mathematika, 6 (2) (2022), pp. 20-32.
8. H.Z. Ibrahim, *Micro β-open sets in Micro topology*, General Letters in Mathematics, 8 (1) (2020), pp. 8-15.

9. H. Maki, P. Sundaram and K. Balachandran, *On Generalized Homeomorphisms in topological spaces*, Bull. Fukuoka Univ. Ed. Part III, 40 (1991), pp. 13-21.
10. Sakkraiveeranan Chandrasekar, *On Micro Topological Spaces*, Journal of New Theory, 26 (2019), pp. 23-31.
11. S. Sandhiya, R. Anandhi and N. Balamani, *Micro g^*-continuous maps and Micro g^*-irresolute maps in Micro Topological Spaces*, Advances in Mathematical and Statistical Science, 1 (2022), pp. 115-125.
12. T. Sowmiya, R. Anandhi and N. Balamani, *Micro ψ-continuous maps and Micro ψ-irresolute maps in Micro Topological Spaces*, Advances in Mathematical and Statistical Science, 1 (2022), pp. 126-136.
13. T.H. Jasim and M.D. Sultan, *On Micro-$\dot{b}$-open sets and Micro-$\dot{b}$-continuity*, Tikrit Journal of Pure Science, 25(5) (2020), pp. 124-129.
14. T.H. Jasim, S.S. Mohsen and K.S. Eke, *On Micro-generalized closed sets and Micro-generalized continuity in Micro Topological Spaces*, Eur. J. Pure Appl. Math., 14(4) (2021), pp. 1507-1516.

---

[1] Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, India.
Email address: anandhi08081985@gmail.com

[2] Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, India.
Email address: nbalamani77@gmail.com