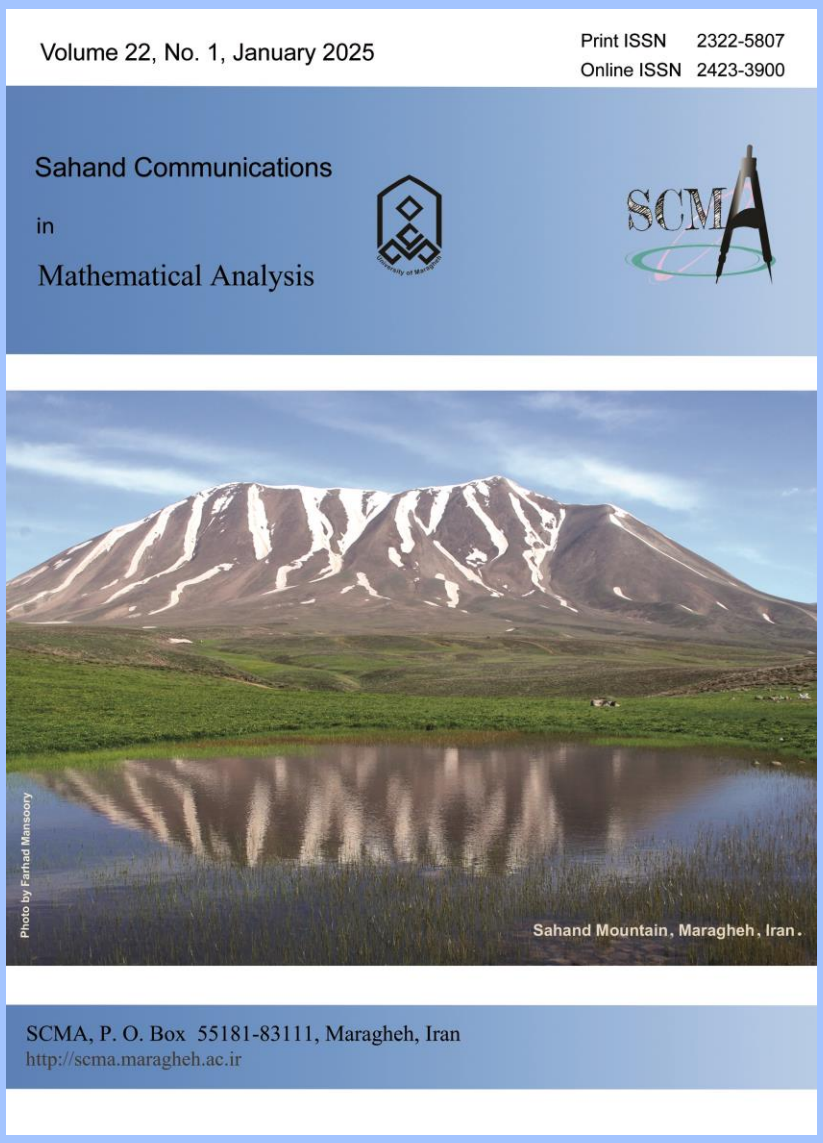
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λ-Statistical Pointwise and Uniform Convergence of Sequences of Functions on Neutrosophic Normed Spaces

Hari Shankar[1], Ayaz Ahmad[2], Chandan Chaurasiya[3] and Ayhan Esi[4*]

Abstract. This insightful article examines the λ-statistical convergence of function sequences within neutrosophic norm spaces. It introduces novel concepts such as λ-statistical pointwise convergence and λ-statistical uniform convergence specifically tailored for neutrosophic norm spaces, representing a notable advancement in the field. The article elucidates the foundational properties of these innovative concepts, offering valuable insights into their theoretical foundations and practical applications.

1. Introduction

The innovative concept of fuzzy set (FS) theory, initially introduced by Lofti A. Zadeh [14], serves as a generalization of classical set theory, by operating on a continuum of membership degrees within the range of $[0, 1]$. Since its inception, FS theory has undergone significant evolution. Kaleva and Seikkala [19] pioneered the concept of fuzzy metric spaces (FMS), while Abu Osman [18] independently approached the concept from a different perspective. Additionally, Cheng and Moderson [24] contributed to the field by introducing the notion of fuzzy norms in linear spaces. These advancements have substantially contributed to development and elucidation of theoretical frameworks that remain widely used today, such as similarity relations [15], fuzzy physics [11], as well as applications in image processing, robotics and management.

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Remarkably, these achievements were realized within a relatively brief period, marking a significant milestone in the field's progress.

The theory of FSs encounters challenges when dealing with incomplete knowledge concerning membership degrees. To overcome this limitation, Atanassov [12] explored the theory of intuitionistic fuzzy sets (IFS). This expansion of fuzzy sets not only accounts for membership but also integrates non-membership degrees. By incorporating both aspects, IFS offers a more comprehensive framework for addressing uncertainty and imprecision across various applications. This concept finds numerous applications in diverse fields, including decision-making [13]. Intuitionistic fuzzy metric spaces (IFMSs) [10] have been extensively studied, encompassing topics such as fixed point theory [4, 26]. Saadai and Park [22] introduced the concept of intuitionistic fuzzy normed space utilizing the notions of T-norm and T-conorm, originally defined by Schweizer and Sklar [3].

Samarandache [6] extensively investigated the Neutrosophic Set (NS), which serves as a comprehensive generalization encompassing various other set theories like FS, intuitive fuzzy sets (IFS), interval-valued intuitive fuzzy sets (IVIFS), among others. This exploration aimed to establish a broader framework for addressing uncertainty, imprecision and indeterminacy across diverse fields. An NS comprises of three components: truth-membership (T), indeterminacy-membership (I) and falsity-membership (F) functions, each ranging from 0 to 1. These functions quantify the degree of membership, indeterminacy, or belonging to the complementary set. NSs find applications in decision-making, pattern recognition, image processing, medical diagnosis and more, providing a flexible and robust framework for modelling complex phenomena with intrinsic uncertainty, enabling nuanced representation and analysis compared to traditional crisp sets.

In analysis, whether classical or fuzzy, convergent sequences imply that nearly all elements of the sequence cluster within a small neighbourhood of a point known as the limit point. Statistical convergence relaxes this condition, requiring the convergence criterion to hold for a majority of elements. This concept aligns with classical analysis, where statistics focus on large quantities and "majority" implies "almost all." Statistical convergence was initially introduced by Fast [7] and Steinhaus [8] and later revisited by Schoenberg [9], who examined its basic properties and its application as a summability method. Mursaleen's [17] contribution involves defining a specific type of convergence called λ-statistical convergence, while Karakus' [25] work investigates how statistical convergence behaves within the context of IFNSs. λ-statistical convergence in IFNSs was defined and investigated by Mohiuddine and

Lohani [23]. The concept of statistical convergence of sequences of functions in IFNSs was introduced and explored by Karakaya et al [31]. Many studies have been done with different types of convergence in this space [5, 27, 29, 32]. In their latest study published in 2020, Kirisci and Simsek [16] introduced and explored the notion of statistical convergence in neutrosophic normed spaces. Further, Khan et al. [30] explore the statistical convergence of a sequence of functions in the neutrosophic norm. A lot of work has been done in neutrosophic normed space see in [1, 2, 20, 21, 33, 34].

The primary objective of this research is to introduce the concept of λ-statistical convergence within the context of neutrosophic normed spaces (NNS) and to explore various properties associated with these concepts. This paper is organized as follows:

(i) Introduction
(ii) Preliminaries
(iii) Main work
(iv) Conclusion.

2. Preliminaries

In this section, we review some basic definitions and results that are needed in the present study.

Definition 2.1 ([28])**.** Define a binary operation as $* : [0,1]^2 \to [0,1]$. It is said to be a continuous T-norm if it satisfies the following properties:

(1) $x * 1 = x \ \forall \ x \in [0,1]$,
(2) $x*y \leq z*w$ whenever $x \leq z$ and $y \leq w$ for each $x, y, z, w \in [0,1]$,
(3) $*$ is associative and commutative,
(4) $*$ is continuous.

Example 2.2. $x * y = \min\{x, y\}$ and $x * y = x.y$; $\forall x, y \in [0,1]$.

Definition 2.3 ([28])**.** Define a binary operation as $\boxdot : [0,1]^2 \to [0,1]$. It is said to be a continuous T-conorm if it satisfies the following properties:

(1) $x \boxdot 0 = x \ \forall \ x \in [0,1]$,
(2) $x \boxdot y \leq z \boxdot w$ whenever $x \leq z$ and $y \leq w$ for each $x, y, z, w \in [0,1]$,
(3) $\boxdot$ is associative and commutative,
(4) $\boxdot$ is continuous.

Example 2.4. $x \boxdot y = \max\{x, y\}$ and $x \boxdot y = x + y - xy$; $\forall x, y \in [0,1]$.

Remark 2.5 ([16])**.** if $0 < \nu_j < 1$, $j = 1$ to 7, $*$ and $\boxdot$ are continuous-T-norm and continuous-T-conorm, respectively. Then,

(1) If $0 \leq \nu_1, \nu_2 < 1$ for $\nu_1 > \nu_2$, then there exists $0 \leq \nu_3, \nu_4 < 1$ such that $\nu_1 * \nu_3 \geq \nu_2$ and $\nu_1 \geq \nu_4 \boxdot \nu_2$.
(2) if $0 \leq \nu_5 < 1$ then there exist $0 \leq \nu_6, \nu_7 < 1$ such that $\nu_6 * \nu_6 \geq \nu_5$, $\nu_7 \boxdot \nu_7 \leq \nu_5$.

Definition 2.6 ([30])**.** Let Y be a non-empty set, the $\mathcal{H}$ intuitionistic fuzzy set, $\mathcal{H} \subset Y$ is defined by

$$\mathcal{H} = \{< \vartheta, \Lambda(\vartheta), \Pi(\vartheta) >: \vartheta \in Y\},$$

where $\Lambda(\vartheta), \Pi(\vartheta) : Y \to [0,1]$, $\Lambda(\vartheta)$ =Truth and $\Pi(\vartheta)$ =Falsehood, respectively.

$$0 \leq \Lambda(\vartheta) + \Pi(\vartheta) \leq 1.$$

Definition 2.7 ([30])**.** Let Y be a non-empty set and $\mathcal{M}_{NS} \subset Y$. Then, neutrosophic set is defined by

$$\mathcal{M}_{NS} = \{< \vartheta, \Lambda(\vartheta), \Gamma(\vartheta), \Pi(\vartheta) >: \vartheta \in Y\}.$$

where $\Lambda(\vartheta), \Gamma(\vartheta), \Pi(\vartheta) : Y \to [0,1]$, $\Lambda(\vartheta)$ =Truth, $\Gamma(\vartheta)$ =Indeterminacy and $\Pi(\vartheta)$ = Falsehood, respectively.

$$0 \leq \Lambda(\vartheta) + \Gamma(\vartheta) + \Pi(\vartheta) \leq 3.$$

The component of the neutrosophic set $\mathcal{M}_{NS}$ are $\Lambda(\vartheta)$, $\Gamma(\vartheta)$ and $\Pi(\vartheta)$ independent of each other.

Definition 2.8 ([30])**.** Let Y be the vector space.
$\mathcal{M} = \{< \vartheta, \Lambda(\vartheta), \Gamma(\vartheta), \Pi(\vartheta) >: \vartheta \in Y\}$ be a neutrosophic set such that $\mathbb{M} : Y \times \mathbb{R}^+ \to [0,1]$. Let the continuous T-norm and continuous T-conorm be represented by $*$ and $\boxdot$, respectively. The four-tuple $V = (Y, \mathcal{M}, *, \boxdot)$ is referred to as NNS for every $\vartheta, y \in Y$, $l, s > 0$ and for each $\alpha \neq 0$ if the following conditions are hold:

(1) $0 \leq \Lambda(\vartheta, l) \leq 1$, $0 \leq \Gamma(\vartheta, l) \leq 1$, $0 \leq \Pi(\vartheta, l) \leq 1$ $\forall$ $l \in \mathbb{R}^+$,
(2) $\Lambda(\vartheta, l) + \Gamma(\vartheta, l) + \Pi(\vartheta, l) \leq 3$ for $l \in \mathbb{R}^+$,
(3) $\Lambda(\vartheta, l) = 1$, for $l > 0$ iff $\vartheta = 0$,
(4) $\Lambda(\alpha\vartheta, l) = \Lambda(\vartheta, \frac{l}{|\alpha|})$,
(5) $\Lambda(\vartheta, l) * \Lambda(y, s) \leq \Lambda(\vartheta + y, l + s)$,
(6) $\Lambda(\vartheta, .)$ is a continuous, non-decreasing function,
(7) $\lim_{l \to \infty} \Lambda(\vartheta, l) = 1$,
(8) $\Gamma(\vartheta, l) = 0$, for $l > 0$ iff $\vartheta = 0$,
(9) $\Gamma(\alpha\vartheta, l) = \Gamma(\vartheta, \frac{l}{|\alpha|})$,
(10) $\Gamma(\vartheta, l) \boxdot \Gamma(y, s) \geq \Gamma(\vartheta + y, l + s)$,
(11) $\Gamma(\vartheta, .)$ is a continuous, non-increasing function,
(12) $\lim_{l \to \infty} \Gamma(\vartheta, l) = 0$,
(13) $\Pi(\vartheta, l) = 0$, for $l > 0$ iff $\vartheta = 0$,
(14) $\Pi(\alpha\vartheta, l) = \Pi\left(\vartheta, \frac{l}{|\alpha|}\right)$,

(15) $\Pi(\vartheta, l) \boxdot \Pi(y, s) \geq \Pi(\vartheta + y, l + s)$,
(16) $\Pi(\vartheta, .)$ is a continuous, non-increasing function,
(17) $\lim_{l\to\infty} \Pi(\vartheta, l) = 0$,
(18) if $l \leq 0$, then $\Lambda(\vartheta, l) = 0$, $\Gamma(\vartheta, l) = 1$, $\Pi(\vartheta, l) = 1$,

So, $\mathcal{M} = (\Lambda, \Gamma, \Pi)$ is called neutrosophic normed (NN).

Example 2.9 ([30])**.** Let us suppose $(Y, ||.||)$ is a normed space . Assign the operation $*$ and $\boxdot$ as T-norm $\vartheta * y = \vartheta y$, T-conorm $\vartheta \boxdot y = \vartheta + y - \vartheta y$ for $l > \|\vartheta\|$.

$$\Lambda(\vartheta, l) = \frac{l}{l + \|\vartheta\|}, \qquad \Gamma(\vartheta, l) = \frac{\|\vartheta\|}{l + \|\vartheta\|}, \qquad \Pi(\vartheta, l) = \frac{\|\vartheta\|}{l},$$

$\forall \vartheta, y \in Y$ and $l > 0$. If we consider $l \leq \|\vartheta\|$, then $\Lambda(\vartheta, l) = 0$, $\Gamma(\vartheta, l) = 1$ and $\Pi(\vartheta, l) = 1$. Hence, $(Y, \mathcal{M}, *, \boxdot)$ is neutrosophic norm space such that $\mathcal{M} : Y \times \mathbb{R}^+ \to [0, 1]$.

Definition 2.10 ([31])**.** Let $K \subset \mathbb{N}$ and $K_n = \{k \in K : k \leq n\}$. Then, the natural density is defined by $\delta(K) = \lim_{n\to\infty} \frac{|K_n|}{n}$, where $|K_n|$ denotes the cardinality of the set K_n.

Definition 2.11 ([31])**.** A sequence $a = (a_k)$ is said to be statistically convergent to the number $\Im$ if for every $\sigma > 0$, the set $N(\sigma)$ has asymptotic density zero, where $N(\sigma) = \{k \in \mathbb{N} : |a_k - \Im| \geq \sigma\}$ and it is denoted by: $st - \lim a = \Im$.

Definition 2.12. Consider two NNS $(Y, \mathcal{M}, *, \boxdot)$ and $(Z, \mathcal{M}', *, \boxdot)$. Let $p_k : (Y, \mathcal{M}, *, \boxdot) \to (Z, \mathcal{M}', *, \boxdot)$ be a sequence of functions. If for every $\vartheta \in Y$ and every $\sigma > 0, l > 0$,

$$\delta\Big(\Big\{k \in \mathbb{N} : \Lambda'(p_k(\vartheta) - p(\vartheta), l) \leq 1 - \sigma \ \text{ or } \ \Gamma'(p_k(\vartheta) - p(\vartheta), l) \geq \sigma,$$
$$\Pi'(p_k(\vartheta) - p(\vartheta), l) \geq \sigma\Big\}\Big) = 0.$$

Then, we denote $st_{\mathcal{M}} - p_k \to p$, indicating that the sequence of functions (p_k) is pointwise statistically convergent to p with respect to NN-$(\mathcal{M})$.

Definition 2.13 ([17])**.** Let $\lambda = (\lambda_n)$ be a non-decreasing sequence of positive numbers tending to ∞ such that $\lambda_{(n+1)} \leq \lambda_n + 1$, $\lambda_1 = 1$.

Let $K \subset \mathbb{N}$

$$\delta_\lambda(K) = \lim_{n\to\infty} \frac{1}{\lambda_n} |\{k \in I_n : k \in K\}|$$

is said to be the λ-density of K, where $I_n = [n - \lambda_n + 1, n]$.

If $\lambda_n = n$, the λ-density coincides with the asymptotic density. A sequence $a = (a_k)$ is said to be λ-statistical convergent to $\Im$ if for every $\sigma > 0$, the set $N(\sigma)$ has λ-density zero, where $N(\sigma) = \{k \in \mathbb{N} : |a_k - \Im| \geq \sigma\}$, and it is denoted by: $st_\lambda - \lim a = \Im$.

Definition 2.14. Let a NNS be $(Y, \mathcal{M}, *, \boxdot)$. Then, with regard to NN -$(\mathcal{M})$, a sequence $a = (a_k)$ is said to be λ-statistically convergent to $\Im \in Y$, given that for each $\sigma > 0$ and $l > 0$,

$$\delta_\lambda\Big(\Big\{k \in \mathbb{N} : \Lambda(a_k - \Im, l) \leq 1 - \sigma \text{ or } \Gamma(a_k - \Im, l) \geq \sigma, \\ \Pi(a_k - \Im, l) \geq \sigma\Big\}\Big) = 0,$$

or equivalently

$$\delta_\lambda\Big(\Big\{k \in \mathbb{N} : \Lambda(a_k - \Im, l) > 1 - \sigma, \Gamma(a_k - \Im, l) < \sigma \\ \text{and } \Pi(a_k - \Im, l) < \sigma\Big\}\Big) = 1,$$

and it is denoted by $st^\lambda_{\mathcal{M}} - \lim a = \Im$.

3. Main Work

In this section, we introduce the concepts of pointwise and uniform λ-statistical convergence for function sequences in NNS. Furthermore, we delve into the connection between these concepts and continuity. Additionally, we present the λ-statistical adaptation of the Cauchy convergence criterion. We also define neutrosophic equicontinuity and its correlation with continuity. Our discussion commences with the definition of pointwise λ-statistical convergence of function sequences in NNS.

Definition 3.1. Let $(Y, \mathcal{M}, *, \boxdot)$ and $(Z, \mathcal{M}', *, \boxdot)$ be two NNS and let $p_k : (Y, \mathcal{M}, *, \boxdot) \to (Z, \mathcal{M}', *, \boxdot)$ be a sequence of functions. If for each $\vartheta \in Y$ and for all $\sigma > 0$ and $l > 0$,

$$\delta_\lambda\Big(\Big\{k \in \mathbb{N} : \Lambda'(p_k(\vartheta) - p(\vartheta), l) \leq 1 - \sigma \text{ or } \Gamma'(p_k(\vartheta) - p(\vartheta), l) \geq \sigma, \\ \Pi'(p_k(\vartheta) - p(\vartheta), l) \geq \sigma\Big\}\Big) = 0,$$

or equivalently

$$\delta_\lambda\Big(\Big\{k \in \mathbb{N} : \Lambda'(p_k(\vartheta) - p(\vartheta), l) > 1 - \sigma, \Gamma'(p_k(\vartheta) - p(\vartheta), l) < \sigma \\ \text{and } \Pi'(p_k(\vartheta) - p(\vartheta), l) < \sigma\Big\}\Big) = 1.$$

Then, with regard to NN-$(\mathcal{M})$, we state that the sequence of functions (p_k) is pointwise λ-statistically convergent to $p(\vartheta)$, which is denoted as $st^\lambda_{\mathcal{M}} - \ p_k \to p$.

This indicates that for each $\delta > 0$, there is an integer N such that, for each $\sigma > 0$ and for each $n > N(\sigma, \delta, l, \vartheta)$,

$$\frac{1}{\lambda_n}\left|\left\{k \in I_n : \Lambda'(p_k(\vartheta) - p(\vartheta), l) \leq 1 - \sigma \text{ or } \right.\right.$$
$$\left.\left.\Gamma'(p_k(\vartheta) - p(\vartheta), l) \geq \sigma, \Pi'(p_k(\vartheta) - p(\vartheta), l) \geq \sigma \text{ for each } \vartheta \in Y\right\}\right| < \delta.$$

Remark 3.2. Consider a sequence of functions $p_k : (Y, \mathcal{M}, *, \boxdot) \to (Z, \mathcal{M}', *, \boxdot)$. When $\lambda_n = n$, the λ-density simplifies to the asymptotic density. In this case, the sequence of functions (p_k) converges pointwise statistically to $p(\vartheta)$ on Y with respect to NN-$(\mathcal{M})$, denoted as $st_{\mathcal{M}} - p_k \to p$.

Example 3.3. Let $(\mathbb{R}, \|\cdot\|)$ denote the space of real numbers with the usual norm and let $a * b = ab$ and $a \boxdot b = min\{a+b, 1\}$ for all $a, b \in [0, 1]$. For all $\vartheta \in \mathbb{R}$ and for every $l > 0$, suppose

$$\Lambda(\vartheta, l) = \frac{l}{l + \|\vartheta\|}, \qquad \Gamma(\vartheta, l) = \frac{\|\vartheta\|}{l + \|\vartheta\|}, \qquad \Pi(\vartheta, l) = \frac{\|\vartheta\|}{l}.$$

In this case, $(\mathbb{R}, \mathcal{M}, *, \boxdot)$ is NNS. Also, $([0, 1], \mathcal{M}, *, \boxdot)$ is NNS. Now suppose that $p_k : [0, 1] \to \mathbb{R}$ be the sequence of functions whose terms are given as

$$p_k(\vartheta) = \begin{cases} \vartheta^k + 1 & \text{if } k \in \left[n - \sqrt{\lambda_n} + 1, n\right], \vartheta \in \left[0, \frac{1}{2}\right) \\ 0 & \text{if } k \notin \left[n - \sqrt{\lambda_n} + 1, n\right], \vartheta \in \left[0, \frac{1}{2}\right) \\ \vartheta^k + \frac{1}{3} & \text{if } k \in \left[n - \sqrt{\lambda_n} + 1, n\right], \vartheta \in \left[\frac{1}{2}, 1\right) \\ 1 & \text{if } k \notin \left[n - \sqrt{\lambda_n} + 1, n\right], \vartheta \in \left[\frac{1}{2}, 1\right) \\ 3 & \text{if } \vartheta = 1 \end{cases}$$

(p_k) is pointwise λ-statistically convergent on $[0, 1]$ with respect to NN-$(\mathcal{M})$. Evidently, for $\vartheta \in [0, \frac{1}{2})$, since

$$\mathcal{K}(\sigma, l) = \Big\{k \in I_n : \Lambda(p_k(\vartheta) - p(\vartheta), l) \leq 1 - \sigma \text{ or } \Gamma(p_k(\vartheta) - p(\vartheta), l) \geq \sigma,$$
$$\Pi(p_k(\vartheta) - p(\vartheta), l) \geq \sigma\Big\}.$$

Hence, we have

$$\mathcal{K}(\sigma, l) = \left\{k \in I_n : \frac{l}{l + \|p_k(\vartheta) - 0\|} \leq 1 - \sigma \text{ or } \frac{||p_k(\vartheta) - 0||}{l + \|p_k(\vartheta) - 0\|} \geq \sigma, \right.$$
$$\left. \frac{\|p_k(\vartheta) - 0\|}{l} \geq \sigma\right\}$$
$$= \left\{k \in I_n : \|p_k(\vartheta)\| \geq \frac{\sigma l}{1 - \sigma} \text{ or } \|p_k(\vartheta)\| \geq \sigma l\right\}$$

$$= \left\{k \in I_n : \|p_k(\vartheta)\| = \vartheta^k + 1\right\}$$

$$|\mathcal{K}(\sigma, l)| \leq \sqrt{\lambda_n}.$$

Thus, for each $\vartheta \in [0, \frac{1}{2})$, we have

$$\delta_\lambda(\mathcal{K}(\sigma, l)) = \lim_{n\to\infty} \frac{|\mathcal{K}(\sigma, l)|}{\lambda_n} = \lim_{n\to\infty} \frac{\sqrt{\lambda_n}}{\lambda_n} = 0,$$

so, (p_k) is λ-statistically converges to 0 with respect to NN-$(\mathcal{M})$. Now, if we take $\vartheta \in \left[\frac{1}{2}, 1\right)$, we have

$$\mathcal{K}'(\sigma, l) = \left\{k \in I_n : \frac{l}{l + \|p_k(\vartheta) - 1\|} \leq 1 - \sigma \text{ or } \frac{\|p_k(\vartheta) - 1\|}{l + \|p_k(\vartheta) - 1\|} \geq \sigma, \right.$$
$$\left. \frac{\|p_k(\vartheta) - 1\|}{l} \geq \sigma \right\}$$
$$= \left\{k \in I_n : \|p_k(\vartheta) - 1\| \geq \frac{\sigma l}{1 - \sigma} \text{ or } \|p_k(\vartheta) - 1\| \geq \sigma l \right\}$$
$$= \left\{k \in I_n : \|p_k(\vartheta)\| = \vartheta^k + \frac{1}{3} \right\}$$

$$|\mathcal{K}'(\sigma, l)| \leq \sqrt{\lambda_n}.$$

Thus, for each $\vartheta \in \left[\frac{1}{2}, 1\right)$, we have

$$\delta_\lambda(\mathcal{K}'(\sigma, l)) = \lim_{n\to\infty} \frac{|\mathcal{K}'(\sigma, l)|}{\lambda_n} = \lim_{n\to\infty} \frac{\sqrt{\lambda_n}}{\lambda_n} = 0.$$

So, (p_k) is λ-statistically converges to 1 with respect to NN-$(\mathcal{M})$.

Now, if we take the case when $\vartheta = 1$, it can be seen easily that,

$$\mathcal{K}''(\sigma, l) = \left\{k \in I_n : \frac{l}{l + \|p_k(\vartheta) - 3\|} \leq 1 - \sigma \text{ or } \frac{\|p_k(\vartheta) - 3\|}{l + \|p_k(\vartheta) - 3\|} \geq \sigma, \right.$$
$$\left. \frac{\|p_k(\vartheta) - 3\|}{l} \geq \sigma \right\}$$
$$= \left\{k \in I_n : \|p_k(\vartheta) - 3\| \geq \frac{\sigma l}{1 - \sigma} \text{ or } \|p_k(\vartheta) - 3\| \geq \sigma l \right\}$$
$$= \left\{k \in I_n : 0 \geq \frac{\sigma l}{1 - \sigma} \text{ or } 0 \geq \sigma l \right\}$$
$$= \emptyset,$$

$$|\mathcal{K}''(\sigma, l)| = 0.$$

So,

$$\delta_\lambda(\mathcal{K}''(\sigma,l)) = \lim_{n\to\infty} \frac{|\mathcal{K}''(\sigma,l)|}{\lambda_n} = \lim_{n\to\infty} \frac{0}{\lambda_n} = 0.$$

Hence, at $\vartheta = 1$, (p_k) is λ-statistically convergent to 3 with respect to NN-$(\mathcal{M})$. That is,

$$st^{\lambda}_{\mathcal{M}} - p_k(\vartheta) = \begin{cases} 0 & \text{if } \vartheta \in \left[0, \frac{1}{2}\right) \\ 1 & \text{if } \vartheta \in \left[\frac{1}{2}, 1\right) \\ 3 & \text{if } \vartheta = 1. \end{cases}$$

It is evident that (p_k) is pointwise λ-statistically neutrosophic convergent on $[0,1]$, since (p_k) is λ-statistically convergent to distinct points with regard to NN-$(\mathcal{M})$ for each $\vartheta \in Y$.

Lemma 3.4. *Let $p_k : (Y, \mathcal{M}, *, \boxdot) \to (Z, \mathcal{M}', *, \boxdot)$ be a sequence of functions. Then, for every $\sigma > 0$ and $l > 0$, the following statements are equivalent:*

(1) $st^{\lambda}_{\mathcal{M}} - \lim a = \Im$,

(2)

$$\delta_\lambda\left(\left\{k \in \mathbb{N} : \Lambda'(p_k(\vartheta) - p(\vartheta), l) \le 1 - \sigma \right.\right.$$
$$\left.\left. \text{or } \Gamma'(p_k(\vartheta) - p(\vartheta), l) \ge \sigma, \Pi'(p_k(\vartheta) - p(\vartheta), l) \ge \sigma\right\}\right) = 0$$

for each $\vartheta \in Y$,

(3)

$$\delta_\lambda\left(\left\{k \in \mathbb{N} : \Lambda'(p_k(\vartheta) - p(\vartheta), l) > 1 - \sigma, \Gamma'(p_k(\vartheta) - p(\vartheta), l) < \sigma \right.\right.$$
$$\left.\left. \text{and } \Pi(p_k(\vartheta) - p(\vartheta), l) < \sigma\right\}\right) < \sigma$$

for each $\vartheta \in Y$,

(4) *For each $\vartheta \in Y$, we have*

$$st_\lambda - \lim \Lambda'(p_k(\vartheta) - p(\vartheta), l) = 1,$$
$$st_\lambda - \lim \Gamma'(p_k(\vartheta) - p(\vartheta), l) = 0,$$
$$st_\lambda - \lim \Pi'(p_k(\vartheta) - p(\vartheta), l) = 0.$$

Theorem 3.5. *Suppose $(Y, \mathcal{M}, *, \boxdot)$ and $(Z, \mathcal{M}', *, \boxdot)$ are two NNS and let $p_k : (Y, \mathcal{M}, *, \boxdot) \to (Z, \mathcal{M}', *, \boxdot)$ be a sequence of functions. If the sequence of functions (p_k) converges pointwise in a neutrosophic sense on Y to a function $p(\vartheta)$ with respect to NN-$(\mathcal{M})$, then the sequence of functions (p_k) converges λ-statistically pointwise within NN-$(\mathcal{M})$.*

Proof. Let (p_k) denote a sequence of functions that is pointwise neutrosophic convergent on Y for all $k \in \mathbb{N}$. This implies that the sequence of functions (p_k) converges with respect to NN-$(\mathcal{M}')$ for each $\vartheta \in Y$. Thus, for every $\sigma > 0$ and $l > 0$, there exists an $n_0 \in \mathbb{N}$ such that

$$\begin{bmatrix} \Lambda'(p_k(\vartheta) - p(\vartheta), l) > 1 - \sigma, \\ \Gamma'(p_k(\vartheta) - p(\vartheta), l) < \sigma, \\ \text{and } \Pi'(p_k(\vartheta) - p(\vartheta), l) < \sigma \end{bmatrix}$$

for every $k \geq n_0$ and each $\vartheta \in Y$. Thus, for each $\vartheta \in Y$, the set

$$\mathcal{K}_\vartheta(\sigma, l) = \left\{ k \in \mathbb{N} : \begin{bmatrix} \Lambda'(p_k(\vartheta) - p(\vartheta), l) \leq 1 - \sigma \text{ or } \Gamma'(p_k(\vartheta) - p(\vartheta), l) \geq \sigma, \\ \Pi'(p_k(\vartheta) - p(\vartheta), l) \geq \sigma \end{bmatrix} \right\}$$

contains only finitely many terms. Since the λ-density of any finite subset of $\mathbb{N}$ is 0, i.e., $\delta_\lambda(\mathcal{K}_\vartheta(\sigma, l)) = 0$, we conclude that $st^\lambda_{\mathcal{M}} - \ p_k \to p$. □

Theorem 3.6. *Let (p_k) and (q_k) be two sequence of function from $(Y, \mathcal{M}, *, \boxdot)$ to $(Z, \mathcal{M}', *, \boxdot)$. if $st^\lambda_{\mathcal{M}} - \ p_k \to p$ and $st^\lambda_{\mathcal{M}} - \ q_k \to q$ then $st^\lambda_{\mathcal{M}} - (\alpha p_k + \beta q_k) \to (\alpha p + \beta q)$ where $\alpha, \beta \in \mathbb{R}$ or $\mathbb{C}$.*

Proof. We will prove this theorem by taking two cases.
Case (i): When $\alpha = 0$ and $\beta = 0$. The proof is obvious.
Case (ii): When $\alpha \neq 0$ and $\beta \neq 0$. Since we have $st^\lambda_{\mathcal{M}} - \ p_k \to p$ and $st^\lambda_{\mathcal{M}} - \ q_k \to q$, for each $\vartheta \in Y$ we can define:

$$\mathfrak{A}_1 = \left\{ k \in \mathbb{N} : \Lambda' \left(p_k(\vartheta) - p(\vartheta), \frac{l}{2|\alpha|} \right) \leq 1 - \sigma \right.$$
$$\left. \text{or } \Gamma' \left(p_k(\vartheta) - p(\vartheta), \frac{l}{2|\alpha|} \right) \geq \sigma, \ \Pi' \left(p_k(\vartheta) - p(\vartheta), \frac{l}{2|\alpha|} \right) \geq \sigma \right\}$$

$$\mathfrak{A}_2 = \left\{ k \in \mathbb{N} : \Lambda' \left(q_k(\vartheta) - q(\vartheta), \frac{l}{2|\beta|} \right) \leq 1 - \sigma \right.$$
$$\left. \text{or } \Gamma' \left(q_k(\vartheta) - q(\vartheta), \frac{l}{2|\beta|} \right) \geq \sigma, \ \Pi' \left(q_k(\vartheta) - q(\vartheta), \frac{l}{2|\beta|} \right) \geq \sigma \right\}.$$

Since $\mathfrak{A}_1$ and $\mathfrak{A}_2$ have a finite number of terms, then $\delta_\lambda(\mathfrak{A}_1) = 0$ and $\delta_\lambda(\mathfrak{A}_2) = 0$. If we define $\mathfrak{A} = \mathfrak{A}_1 \cup \mathfrak{A}_2$,

$$\delta_\lambda(\mathfrak{A}) = 0.$$

Hence $\mathfrak{A} \neq \mathbb{N}$, so there exist $k_0 \in \mathbb{N}$ such that

$$\Lambda' \left(p_{k_0}(\vartheta) - p(\vartheta), \frac{l}{2|\alpha|} \right) > 1 - \sigma,$$

$$\Gamma'\left(p_{k_0}(\vartheta)-p(\vartheta),\frac{l}{2|\alpha|}\right)<\sigma,$$
$$\Pi'\left(p_{k_0}(\vartheta)-p(\vartheta),\frac{l}{2|\alpha|}\right)<\sigma,$$
$$\Lambda'\left(q_{k_0}(\vartheta)-q(\vartheta),\frac{l}{2|\beta|}\right)>1-\sigma,$$
$$\Gamma'\left(q_{k_0}(\vartheta)-q(\vartheta),\frac{l}{2|\beta|}\right)<\sigma,$$
$$\Pi'\left(q_{k_0}(\vartheta)-q(\vartheta),\frac{l}{2|\beta|}\right)<\sigma.$$

Now, let

$$\begin{aligned}\mathfrak{B}=\Big\{&k\in\mathbb{N}:\Lambda'((\alpha p_k+\beta q_k)(\vartheta)-(\alpha p+\beta q)(\vartheta),l)>1-\sigma,\\&\Gamma'((\alpha p_k+\beta q_k)(\vartheta)-(\alpha p+\beta q)(\vartheta),l)<\sigma\\&\text{and }\Pi'((\alpha p_k+\beta q_k)(\vartheta)-(\alpha p+\beta q)(\vartheta),l)<\sigma\Big\}.\end{aligned}$$

We will show that for each $\vartheta\in Y$, $\mathfrak{A}^{\complement}\subset\mathfrak{B}$. Now, let $k_0\in\mathfrak{A}^{\complement}$. Then,

$$\Lambda'\left(p_{k_0}(\vartheta)-p(\vartheta),\frac{l}{2|\alpha|}\right)>1-\sigma,$$
$$\Gamma'\left(p_{k_0}(\vartheta)-p(\vartheta),\frac{l}{2|\alpha|}\right)<\sigma,$$
$$\Pi'\left(p_{k_0}(\vartheta)-p(\vartheta),\frac{l}{2|\alpha|}\right)<\sigma,$$
$$\Lambda'\left(q_{k_0}(\vartheta)-q(\vartheta),\frac{l}{2|\beta|}\right)>1-\sigma,$$
$$\Gamma'\left(q_{k_0}(\vartheta)-q(\vartheta),\frac{l}{2|\beta|}\right)<\sigma,$$
$$\Pi'\left(q_{k_0}(\vartheta)-q(\vartheta),\frac{l}{2|\beta|}\right)<\sigma.$$

Using the above definition of NNS, we have,

$$\begin{aligned}&\Lambda'\left((\alpha p_k+\beta q_k)(\vartheta)-(\alpha p+\beta q)(\vartheta),l\right)\\&\quad\geq\Lambda'\left((\alpha p_k(\vartheta)-\alpha p(\vartheta)),\frac{l}{2}\right)*\Lambda'\left((\beta q_k(\vartheta)-\beta q(\vartheta)),\frac{l}{2}\right)\\&\quad=\Lambda'\left((p_k(\vartheta)-p(\vartheta)),\frac{l}{2|\alpha|}\right)*\Lambda'\left((q_k(\vartheta)-q(\vartheta)),\frac{l}{2|\beta|}\right)\\&\quad>(1-\sigma)*(1-\sigma)\end{aligned}$$

$$= (1-\sigma),$$

$$\begin{aligned}
&\Gamma'\left((\alpha p_k+\beta q_k)(\vartheta)-(\alpha p+\beta q)(\vartheta),l\right)\\
&\quad\leq \Gamma'\left((\alpha p_k(\vartheta)-\alpha p(\vartheta)),\frac{l}{2}\right)\boxdot\Gamma'\left((\beta q_k(\vartheta)-\beta q(\vartheta)),\frac{l}{2}\right)\\
&\quad= \Gamma'\left((p_k(\vartheta)-p(\vartheta)),\frac{l}{2|\alpha|}\right)\boxdot\Gamma'\left((q_k(\vartheta)-q(\vartheta)),\frac{l}{2|\beta|}\right)\\
&\quad< \sigma\boxdot\sigma\\
&\quad= \sigma,
\end{aligned}$$

and

$$\begin{aligned}
&\Pi'\left((\alpha p_k+\beta q_k)(\vartheta)-(\alpha p+\beta q)(\vartheta),l\right)\\
&\quad\leq \Pi'\left((\alpha p_k(\vartheta)-\alpha p(\vartheta)),\frac{l}{2}\right)\boxdot\Pi'\left((\beta q_k(\vartheta)-\beta q(\vartheta)),\frac{l}{2}\right)\\
&\quad= \Pi'\left((p_k(\vartheta)-p(\vartheta)),\frac{l}{2|\alpha|}\right)\boxdot\Pi'\left((q_k(\vartheta)-q(\vartheta)),\frac{l}{2|\beta|}\right)\\
&\quad< \sigma\boxdot\sigma\\
&\quad= \sigma.
\end{aligned}$$

This implies that, $\mathfrak{A}^{\complement}\subset\mathfrak{B}$. Since, $\mathfrak{B}^{\complement}\subset\mathfrak{A}$ and $\delta_\lambda(\mathfrak{A})=0$, hence

$$\delta_\lambda\left(\mathfrak{B}^{\complement}\right)=0.$$

That is,

$$\begin{aligned}
\delta_\lambda\Big(\Big\{&k\in\mathbb{N}:\Lambda'((\alpha p_k+\beta q_k)(\vartheta)-(\alpha p+\beta q)(\vartheta),l)\leq 1-\sigma \text{ or}\\
&\Gamma'((\alpha p_k+\beta q_k)(\vartheta)-(\alpha p+\beta q)(\vartheta),l)\geq\sigma,\\
&\Pi'((\alpha p_k+\beta q_k)(\vartheta)-(\alpha p+\beta q)(\vartheta),l)\geq\sigma\Big\}\Big)=0.
\end{aligned}$$

So,

$$st^{\lambda}_{\mathcal{M}}-(\alpha p_k+\beta q_k)\to(\alpha p+\beta q).$$ □

Definition 3.7. Let $p_k:(Y,\mathcal{M},*,\boxdot)\to(Z,\mathcal{M}',*,\boxdot)$ be a sequence of functions. The sequence of functions (p_k) is a pointwise λ-statistical Cauchy sequence with respect to NN-$(\mathcal{M})$ provided that for every $\sigma>0$ and $l>0$, there exists $\mathcal{N}=\mathcal{N}(\sigma,l,\vartheta)$ such that,

$$\begin{aligned}
\delta_\lambda\Big(\Big\{&k\in\mathbb{N}:\Lambda'(p_k(\vartheta)-p_{\mathcal{N}}(\vartheta),l)\leq 1-\sigma \text{ or } \Gamma'(p_k(\vartheta)-p_{\mathcal{N}}(\vartheta),l)\geq\sigma,\\
&\Pi'(p_k(\vartheta)-p_{\mathcal{N}}(\vartheta),l)\geq\sigma \text{ for each } \vartheta\in Y\Big\}\Big)=0.
\end{aligned}$$

That is, there exists a number $\mathcal{N} = \mathcal{N}(\sigma, l, \vartheta)$ for every $\vartheta \in Y$ such that, $\Lambda'(p_k(\vartheta) - p(\vartheta), l) > 1 - \sigma$, $\Gamma'(p_k(\vartheta) - p(\vartheta), l) < \sigma$ and $\Pi'(p_k(\vartheta) - p(\vartheta), l) < \sigma$ for almost all k.

Theorem 3.8. *Let $p_k : (Y, \mathcal{M}, *, \boxdot) \to (Z, \mathcal{M}', *, \boxdot)$ be a sequence of functions. If (p_k) is a pointwise λ-statistical convergent sequence of function with respect to NN-$(\mathcal{M})$, then (p_k) is a pointwise λ-statistical Cauchy sequence of function with respect to NN-$(\mathcal{M})$.*

Proof. Suppose that $st^{\lambda}_{\mathcal{M}} - \ p_k \to p$ and let $\sigma > 0$, $l > 0$. For a given $\sigma > 0$, choose $r > 0$ such that $(1 - \sigma) * (1 - \sigma) > 1 - r$ and $\sigma \boxdot \sigma < r$. If we state $\mathcal{A}_\vartheta(\sigma, l)$ and $\mathcal{A}^{\complement}{}_\vartheta(\sigma, l)$ by,

$$\left\{k \in \mathbb{N} : \Lambda'\left(p_k(\vartheta) - p(\vartheta), \frac{l}{2}\right) \leq 1 - \sigma \ \text{ or } \ \Gamma'\left(p_k(\vartheta) - p(\vartheta), \frac{l}{2}\right) \geq \sigma, \right.$$
$$\left. \Pi'\left(p_k(\vartheta) - p(\vartheta), \frac{l}{2}\right) \geq \sigma\right\}$$

$$\left\{k \in \mathbb{N} : \Lambda'\left(p_k(\vartheta) - p(\vartheta), \frac{l}{2}\right) > 1 - \sigma, \Gamma'\left(p_k(\vartheta) - p(\vartheta), \frac{l}{2}\right) < \sigma \ \text{ and } \right.$$
$$\left. \Pi'\left(p_k(\vartheta) - p(\vartheta), \frac{l}{2}\right) < \sigma\right\}$$

respectively, for each $\vartheta \in Y$. Then we have, $\delta_\lambda(\mathcal{A}_\vartheta(\sigma, l)) = 0$ which implies that $\delta_\lambda(\mathcal{A}^{\complement}{}_\vartheta(\sigma, l)) = 1$. Now, let $\mathcal{N} \in \mathcal{A}^{\complement}{}_\vartheta(\sigma, l)$. Then,

$$\Lambda'\left(p_{\mathcal{N}}(\vartheta) - p(\vartheta), \frac{l}{2}\right) > 1 - \sigma,$$
$$\Gamma'\left(p_{\mathcal{N}}(\vartheta) - p(\vartheta), \frac{l}{2}\right) < \sigma$$
$$\Pi'\left(p_{\mathcal{N}}(\vartheta) - p(\vartheta), \frac{l}{2}\right) < \sigma.$$

We want to show that $\exists$ a number $\mathcal{N} = \mathcal{N}(\sigma, l, \vartheta)$ such that,

$$\delta_\lambda\Big(\Big\{k \in \mathbb{N} : \Lambda'(p_k(\vartheta) - p_{\mathcal{N}}(\vartheta), l) \leq 1 - r \text{ or } \Gamma'(p_k(\vartheta) - p_{\mathcal{N}}(\vartheta), l) \geq r,$$
$$\Pi'(p_k(\vartheta) - p_{\mathcal{N}}(\vartheta), l) \geq r \text{ for each } \vartheta \in Y\Big\}\Big) = 0.$$

For this, we define for each $\vartheta \in Y$,

$$\mathcal{B}_\vartheta(\sigma, l) = \Big\{k \in \mathbb{N} : \Lambda'(p_k(\vartheta) - p_{\mathcal{N}}(\vartheta), l) \leq 1 - r \text{ or }$$
$$\Gamma'(p_k(\vartheta) - p_{\mathcal{N}}(\vartheta), l) \geq r, \Pi'(p_k(\vartheta) - p_{\mathcal{N}}(\vartheta), l) \geq r\Big\}.$$

Now, we will show that,

$$\mathcal{B}_\vartheta(\sigma, l) \subset \mathcal{A}_\vartheta(\sigma, l).$$

Let us suppose that,

$$\mathcal{B}_{\vartheta}(\sigma, l) \nsubseteq \mathcal{A}_{\vartheta}(\sigma, l)$$

In this case, $\mathcal{B}_{\vartheta}(\sigma, l)$ has at least one element that does not belong to $\mathcal{A}_{\vartheta}(\sigma, l)$. Let $k \in \mathcal{B}_{\vartheta}(\sigma, l) \backslash \mathcal{A}_{\vartheta}(\sigma, l)$. Then we have

$$\Lambda'\left(p_k(\vartheta) - p_{\mathcal{N}}(\vartheta), l\right) \leq 1 - r, \qquad \Lambda'\left(p_k(\vartheta) - p(\vartheta), \frac{l}{2}\right) > 1 - \sigma.$$

In particular, $\Lambda'\left(p_{\mathcal{N}}(\vartheta) - p(\vartheta), \frac{l}{2}\right) > 1 - \sigma$. In this case,

$$\begin{aligned} 1 - r &\geq \Lambda'\left(p_k(\vartheta) - p_{\mathcal{N}}(\vartheta), l\right) \\ &\geq \Lambda'\left(p_k(\vartheta) - p(\vartheta), \frac{l}{2}\right) * \Lambda'\left(p_{\mathcal{N}}(\vartheta) - p(\vartheta), \frac{l}{2}\right) \\ &> (1 - \sigma) * (1 - \sigma) \\ &> 1 - r, \end{aligned}$$

which can't happen. However, on the other hand,

$$\Gamma'\left(p_k(\vartheta) - p_{\mathcal{N}}(\vartheta), l\right) \geq r, \qquad \Gamma'\left(p_k(\vartheta) - p(\vartheta), \frac{l}{2}\right) < \sigma.$$

In particular, $\Gamma'(p_{\mathcal{N}}(\vartheta) - p(\vartheta), \frac{l}{2}) < \sigma$. In this case,

$$\begin{aligned} r &\leq \Gamma'(p_k(\vartheta) - p_{\mathcal{N}}(\vartheta), l) \\ &\leq \Gamma'\left(p_k(\vartheta) - p(\vartheta), \frac{l}{2}\right) \boxdot \Gamma'\left(p_{\mathcal{N}}(\vartheta) - p(\vartheta), \frac{l}{2}\right) \\ &< \sigma \boxdot \sigma \\ &< r, \end{aligned}$$

which can't happen. However, on the other hand,

$$\Pi'(p_k(\vartheta) - p_{\mathcal{N}}(\vartheta), l) \geq r, \qquad \Pi'\left(p_k(\vartheta) - p(\vartheta), \frac{l}{2}\right) < \sigma.$$

In particular, $\Pi'\left(p_{\mathcal{N}}(\vartheta) - p(\vartheta), \frac{l}{2}\right) < \sigma$. In this case,

$$\begin{aligned} r &\leq \Pi'(p_k(\vartheta) - p_{\mathcal{N}}(\vartheta), l) \\ &\leq \Pi'\left(p_k(\vartheta) - p(\vartheta), \frac{l}{2}\right) \boxdot \Pi'\left(p_{\mathcal{N}}(\vartheta) - p(\vartheta), \frac{l}{2}\right) \\ &< \sigma \boxdot \sigma \\ &< r, \end{aligned}$$

which can't happen. Therefore, $\mathcal{B}_{\vartheta}(\sigma, l) \subset \mathcal{A}_{\vartheta}(\sigma, l)$. Hence, by $\delta_{\lambda}(\mathcal{A}_{\vartheta}(\sigma, l)) = 0$ and $\delta_{\lambda}(\mathcal{B}_{\vartheta}(\sigma, l)) = 0$. That is, (p_k) is pointwise λ-statistical Cauchy sequence of function with respect to NN-$(\mathcal{M})$.

The subsequent part presents uniform λ-statistical convergence in NNS. □

Definition 3.9. Let $(Y, \mathcal{M}, *, \boxdot)$ and $(Z, \mathcal{M}', *, \boxdot)$ be two NNS and let $p_k : (Y, \mathcal{M}, *, \boxdot) \to (Z, \mathcal{M}', *, \boxdot)$ be a sequence of functions. If $\forall\ \vartheta \in Y$ and $\forall\ \sigma > 0, l > 0$,

$$\delta_\lambda\Big(\Big\{k \in \mathbb{N} : \Lambda'(p_k(\vartheta) - p(\vartheta), l) \leq 1 - \sigma \text{ or } \Gamma'(p_k(\vartheta) - p(\vartheta), l) \geq \sigma,$$
$$\Pi'(p_k(\vartheta) - p(\vartheta), l) \geq \sigma\Big\}\Big) = 0$$

or equivalently,

$$\delta_\lambda\Big(\Big\{k \in \mathbb{N} : \Lambda'(p_k(\vartheta) - p(\vartheta), l) > 1 - \sigma, \Gamma'(p_k(\vartheta) - p(\vartheta), l) < \sigma$$
$$\text{andr } \Pi'(p_k(\vartheta) - p(\vartheta), l) < \sigma\Big\}\Big) = 1.$$

Then we say that the sequence of function (p_k) is uniform λ-statistical convergence with respect to NN-$(\mathcal{M})$.
We denote it by : $st^\lambda_{\mathcal{M}} - p_k \rightrightarrows p$.

Definition 3.10. Let $p_k : (Y, \mathcal{M}, *, \boxdot) \to (Z, \mathcal{M}', *, \boxdot)$ be a sequence of functions. The sequence of function (p_k) is a uniformly λ- statistical Cauchy sequence in NNS, provided that $\forall\ \sigma > 0$ and $l > 0$, there exists $\mathcal{N} = \mathcal{N}(\sigma, l)$ such that

$$\delta_\lambda\Big(\Big\{k \in \mathbb{N} : \Lambda'(p_k(\vartheta) - p_{\mathcal{N}}(\vartheta), l) \leq 1 - \sigma \text{ or } \Gamma'(p_k(\vartheta) - p_{\mathcal{N}}(\vartheta), l) \geq \sigma,$$
$$\Pi'(p_k(\vartheta) - p_{\mathcal{N}}(\vartheta), l) \geq \sigma\ \ \forall \vartheta \in Y\Big\}\Big) = 0.$$

Remark 3.11. $st^\lambda_{\mathcal{M}} - p_k \rightrightarrows p \Rightarrow st^\lambda_{\mathcal{M}} - p_k \to p$.

Lemma 3.12. *Let $p_k : (Y, \mathcal{M}, *, \boxdot) \to (Z, \mathcal{M}', *, \boxdot)$ be a sequence of functions. Then, for every $\sigma > 0$ and $l > 0$, the following statements are equivalent:*

(1) $st^\lambda_{\mathcal{M}} - p_k \rightrightarrows p$,

(2)

$$\delta_\lambda\Big(\Big\{k \in \mathbb{N} : \Lambda'(p_k(\vartheta) - p(\vartheta), l) \leq 1 - \sigma$$
$$\textit{or } \Gamma'(p_k(\vartheta) - p(\vartheta), l) \geq \sigma, \Pi'(p_k(\vartheta) - p(\vartheta), l) \geq \sigma\Big\}\Big) = 0$$

for all $\vartheta \in Y$,

(3)

$$\delta_\lambda\Big(\Big\{k \in \mathbb{N} : \Lambda'(p_k(\vartheta) - p(\vartheta), l) > 1 - \sigma,$$
$$\Gamma'(p_k(\vartheta) - p(\vartheta), l) < \sigma \textit{ and } \Pi'(p_k(\vartheta) - p(\vartheta), l) < \sigma\Big\}\Big) = 1$$

for all $\vartheta \in Y$,

(4) *for all* $\vartheta \in Y$, *we have*

$$st_\lambda - \lim\Lambda'(p_k(\vartheta) - p(\vartheta), l) = 1,$$
$$st_\lambda - \lim\Gamma'(p_k(\vartheta) - p(\vartheta), l) = 0,$$
$$st_\lambda - \lim\Pi'(p_k(\vartheta) - p(\vartheta), l) = 0.$$

Theorem 3.13. *Let* $(Y, \mathcal{M}, *, \boxdot)$ *and* $(Z, \mathcal{M}', *, \boxdot)$ *be two NNS and let* $p_k : (Y, \mathcal{M}, *, \boxdot) \to (Z, \mathcal{M}', *, \boxdot)$ *be a sequence of functions. If the* (p_k) *is uniformly neutrosophic convergent on* Y *to a function* $p(\vartheta)$ *with respect to NN-*$(\mathcal{M})$*, the* (p_k) *is uniformly* λ*-statistically neutrosophic convergent with respect to NN-*$(\mathcal{M})$*.*

Proof. Since the argument is identical to Theorem 3.5's proof, it is left out. □

Theorem 3.14. *Let* (p_k) *and* (q_k) *be two sequence of function from* $(Y, \mathcal{M}, *, \boxdot)$ *to* $(Z, \mathcal{M}', *, \boxdot)$*. If* $st^\lambda_{\mathcal{M}} - \ p_k \rightrightarrows p$ *and* $st^\lambda_{\mathcal{M}} - \ q_k \rightrightarrows q$ *then* $st^\lambda_{\mathcal{M}} - (\alpha p_k + \beta q_k) \rightrightarrows (\alpha p + \beta q)$ *where* $\alpha, \beta \in \mathbb{R}$ *or* $\mathbb{C}$*.*

Proof. Since the argument is identical to Theorem 3.6's proof, it is left out. □

Example 3.15. Let $(\mathbb{R}, \mathcal{M}, *, \boxdot)$ be NNS defined as in Example 3.3. Let $p_k : [0, 1] \to \mathbb{R}$ be sequence of functions whose terms are given as,

$$p_k(\vartheta) = \begin{cases} \vartheta^k + 1, & \text{if } k \in \left[n - \sqrt{\lambda_n} + 1, n\right], \\ 0, & \text{otherwise .} \end{cases}$$

Since $\mathcal{K}(\sigma, l) = \{k \in I_n : \Lambda(p_k(\vartheta) - p(\vartheta), l) \leq 1 - \sigma \ \text{ or } \ \Gamma(p_k(\vartheta) - p(\vartheta), l) \geq \sigma, \Pi(p_k(\vartheta) - p(\vartheta), l) \geq \sigma\}$.
Hence, we have,

$$\begin{aligned} \mathcal{K}(\sigma, l) &= \left\{ k \in I_n : \frac{l}{l + \|p_k(\vartheta) - 0\|} \leq 1 - \sigma \text{ or } \frac{\|p_k(\vartheta) - 0\|}{l + \|p_k(\vartheta) - 0\|} \geq \sigma, \right. \\ &\qquad \left. \frac{\|p_k(\vartheta) - 0\|}{l} \geq \sigma \right\} \\ &= \left\{ k \in I_n : \|p_k(\vartheta)\| \geq \frac{\sigma l}{1 - \sigma} \text{ or } \|p_k(\vartheta)\| \geq \sigma l \right\} \\ &= \left\{ k \in I_n : \|p_k(\vartheta)\| = \vartheta^k + 1 \right\} \end{aligned}$$

$|\mathcal{K}(\sigma, l)| \leq \sqrt{\lambda_n}$.

Thus, for all $\vartheta \in [0, 1]$

$$\delta_\lambda(\mathcal{K}(\sigma, l)) = \lim_{n\to\infty} \frac{|\mathcal{K}(\sigma, l)|}{\lambda_n} = \lim_{n\to\infty} \frac{\sqrt{\lambda_n}}{\lambda_n} = 0.$$

So, (p_k) is uniformly λ-statistically convergent 0 with respect to NN-$(\mathcal{M})$.

Let S be the collection of all uniformly bounded sequences of functions in Y i.e.,

$$S = \{p_k(\vartheta) \in Y : |p_k(\vartheta)| < M \ \forall\ k \in \mathbb{N}, \forall\ \vartheta \in Y, \text{ for some } M > 0\}.$$

Theorem 3.16. *Let $p_k(\vartheta) \in S$ and let (p_k) be sequence of function from $(Y, \mathcal{M}, *, \boxdot)$ to $(Z, \mathcal{M}', *, \boxdot)$. If $st^{\lambda}_{\mathcal{M}} - \ p_k \rightrightarrows p$ and $p(\vartheta)$ be bounded function $\forall\ \vartheta \in Y$, then $st^{\lambda}_{\mathcal{M}} - \ p_k^2 \rightrightarrows p^2$.*

Proof. Since $p_k(\vartheta) \in S$ then $\exists\ M > 0$ such that $|p_k(\vartheta)| < M$ and $st^{\lambda}_{\mathcal{M}} - \ p_k \rightrightarrows p$ then $\forall\ \vartheta \in Y$ and $\forall\ \sigma > 0, l > 0$. Then, by the definition of λ- statistical uniform convergence in NN, we have,

$$\delta_\lambda(\mathfrak{A}_1) = \delta_\lambda\Big(\Big\{k \in \mathbb{N} : \Lambda'(p_k(\vartheta) - p(\vartheta), l) \le 1 - \sigma \ \text{ or } \\ \Gamma'(p_k(\vartheta) - p(\vartheta), l) \ge \sigma, \Pi'(p_k(\vartheta) - p(\vartheta), l) \ge \sigma\Big\}\Big) = 0,$$

and so,

$$\delta_\lambda\left(\mathfrak{A}_1^{\complement}\right) = \delta_\lambda(\{k \in \mathbb{N} : \Lambda'(p_k(\vartheta) - p(\vartheta), l) > 1 - \sigma,\ \Gamma'(p_k(\vartheta) - p(\vartheta), l) < \sigma \\ \text{and } \Pi'(p_k(\vartheta) - p(\vartheta), l) < \sigma\}) = 1.$$

Also, as $p(\vartheta)$ be bounded function, then $\exists\ M_2 > 0$ such that $|p(\vartheta)| < M_2$ $\forall\ \vartheta \in Y$. Now, for $l > 0$ let

$$\mathfrak{A}_2 = \Big\{k \in \mathbb{N} : \Lambda'(p_k^2(\vartheta) - p^2(\vartheta), l) \le 1 - \sigma \ \text{ or } \Gamma'(p_k^2(\vartheta) - p^2(\vartheta), l) \ge \sigma \\ \Pi'(p_k^2(\vartheta) - p^2(\vartheta), l) \ge \sigma\Big\},$$

$$\mathfrak{A}_2^{\complement} = \Big\{k \in \mathbb{N} : \Lambda'(p_k^2(\vartheta) - p^2(\vartheta), l) > 1 - \sigma,\ \Gamma'(p_k^2(\vartheta) - p^2(\vartheta), l) < \sigma, \\ \text{and} \qquad \Pi'(p_k^2(\vartheta) - p^2(\vartheta), l) < \sigma\Big\}.$$

We want to prove $\delta_\lambda(\mathfrak{A}_2) = 0$ or $\delta_\lambda(\mathfrak{A}_2^{\complement}) = 1$. For this, it is sufficient to show $\mathfrak{A}_1^{\complement} \subset \mathfrak{A}_2^{\complement}$. Let $k_0 \in \mathfrak{A}_1^{\complement}$, then we can write,

$$\Lambda'(p_{k_0}(\vartheta) - p(\vartheta), l) > 1 - \sigma,$$
$$\Gamma'(p_{k_0}(\vartheta) - p(\vartheta), l) < \sigma,$$
$$\Pi'(p_{k_0}(\vartheta) - p(\vartheta), l) < \sigma.$$

Using the above definition of NNS, we have,

$$\Lambda'\left(p_{k_0}^2(\vartheta) - p^2(\vartheta), l\right) = \Lambda'((p_{k_0}(\vartheta) - p(\vartheta))(p_{k_0}(\vartheta) + p(\vartheta)), l) \\ = \Lambda'\left((p_{k_0}(\vartheta) - p(\vartheta)), \frac{l}{|p_{k_0}(\vartheta) + p(\vartheta)|}\right)$$

$$\begin{aligned}
&> \Lambda'\left((p_{k_0}(\vartheta)-p(\vartheta)), \frac{l}{M_1+M_2}\right)\\
&= \Lambda'\left((p_{k_0}(\vartheta)-p(\vartheta)), l_1\right)\\
&> 1-\sigma,
\end{aligned}$$

$$\begin{aligned}
\Gamma'\left(p_{k_0}^2(\vartheta)-p^2(\vartheta), l\right) &= \Gamma'((p_{k_0}(\vartheta)-p(\vartheta))(p_{k_0}(\vartheta)+p(\vartheta)), l)\\
&= \Gamma'\left((p_{k_0}(\vartheta)-p(\vartheta)), \frac{l}{|p_{k_0}(\vartheta)+p(\vartheta)|}\right)\\
&< \Gamma'\left((p_{k_0}(\vartheta)-p(\vartheta)), \frac{l}{M_1+M_2}\right)\\
&= \Gamma'\left((p_{k_0}(\vartheta)-p(\vartheta)), l_1\right)\\
&< \sigma,
\end{aligned}$$

$$\begin{aligned}
\Pi'\left(p_{k_0}^2(\vartheta)-p^2(\vartheta), l\right) &= \Pi'((p_{k_0}(\vartheta)-p(\vartheta))(p_{k_0}(\vartheta)+p(\vartheta)), l)\\
&= \Pi'\left((p_{k_0}(\vartheta)-p(\vartheta)), \frac{l}{|p_{k_0}(\vartheta)+p(\vartheta)|}\right)\\
&< \Pi'\left((p_{k_0}(\vartheta)-p(\vartheta)), \frac{l}{M_1+M_2}\right)\\
&= \Pi'\left((p_{k_0}(\vartheta)-p(\vartheta)), l_1\right)\\
&< \sigma,
\end{aligned}$$

where $l_1 = \frac{l}{M_1+M_2} > 0$. Hence, we get $k_0 \in \mathfrak{A}_2^{\complement}$. So, $\delta_\lambda(\mathfrak{A}_2^{\complement}) = 1$, which implies that $\delta_\lambda(\mathfrak{A}_2) = 0$. Hence, the proof is over. □

Theorem 3.17. *Let $p_k, q_k \in S$ be two sequence of function in NNS $(Y, \mathcal{M}, *, \boxdot)$. If $st^\lambda_{\mathcal{M}} - \ p_k \rightrightarrows p$ and $st^\lambda_{\mathcal{M}} - \ q_k \rightrightarrows q$ and $p(\vartheta)$ and $q(\vartheta)$ bounded function in Y, then $st^\lambda_{\mathcal{M}} - \ p_k q_k \rightrightarrows pq$.*

Proof. We can write $p_k q_k = \frac{(p_k+q_k)^2-(p_k-q_k)^2}{4}$, then using Theorem 3.14 and Theorem 3.16, we can deduce that $st^\lambda_{\mathcal{M}} - p_k q_k \rightrightarrows pq$. □

Theorem 3.18. *Let $0 \neq p_k(\vartheta) \in S$ be the sequence of functions in NNS $(Y, \mathcal{M}, *, \boxdot)$. If $st^\lambda_{\mathcal{M}} - p_k \rightrightarrows p$, $p(\vartheta)$ bounded in Y and $0 \neq p(\vartheta)$, $\forall\ \vartheta \in Y$ then $st^\lambda_{\mathcal{M}} - \ \frac{1}{p_k} \rightrightarrows \frac{1}{p}$.*

Proof. Since $p_k(\vartheta) \in S$ then $\exists\ M_1 > 0$ such that $|p_k(\vartheta)| < M_1$ for each $\vartheta \in Y$ and $st^\lambda_{\mathcal{M}} - \ p_k \rightrightarrows p$ then for every $\sigma > 0$ and $l > 0$ for each $\vartheta \in Y$ the following condition is holds:

$$\begin{aligned}
\delta_\lambda(\mathfrak{A}_1) = \delta_\lambda\Big(\Big\{k \in \mathbb{N} : \Lambda'(p_k(\vartheta)-p(\vartheta), l) \leq 1-\sigma\\
\text{or } \Gamma'(p_k(\vartheta)-p(\vartheta), l) \geq \sigma, \Pi'(p_k(\vartheta)-p(\vartheta), l) \geq \sigma\Big\}\Big) = 0,
\end{aligned}$$

and so,

$$\delta_\lambda\left(\mathfrak{A}_1^{\complement}\right) = \delta_\lambda\Big(\Big\{k \in \mathbb{N} : \Lambda'(p_k(\vartheta) - p(\vartheta), l) > 1-\sigma,\ \Gamma'(p_k(\vartheta) - p(\vartheta), l) < \sigma \text{ and } \Pi'(p_k(\vartheta) - p(\vartheta), l) < \sigma\Big\}\Big) = 1$$

Also, as $p(\vartheta)$ be bounded function, then $\exists\ M_2 > 0$ such that $|p(\vartheta)| < M_2$ $\forall\ \vartheta \in Y$. Now for $l > 0$, let

$$\mathfrak{A}_2 = \left\{k \in \mathbb{N} : \Lambda'\left(\frac{1}{p_k(\vartheta)} - \frac{1}{p(\vartheta)}, l\right) > 1-\sigma, \Gamma'\left(\frac{1}{p_k(\vartheta)} - \frac{1}{p(\vartheta)}, l\right) < \sigma \text{ and } \Pi'\left(\frac{1}{p_k(\vartheta)} - \frac{1}{p(\vartheta)}, l\right) < \sigma\right\}.$$

We want to prove $\delta_\lambda(\mathfrak{A}_2) = 1$. For this, it is sufficient to show $\mathfrak{A}_1^{\complement} \subset \mathfrak{A}_2$. Let $k_0 \in \mathfrak{A}_1^{\complement}$, then we can write,

$$\begin{aligned}
&\Lambda'(p_{k_0}(\vartheta) - p(\vartheta), l) > 1-\sigma,\\
&\Gamma'(p_{k_0}(\vartheta) - p(\vartheta), l) < \sigma,\\
&\Pi'(p_{k_0}(\vartheta) - p(\vartheta), l) < \sigma.
\end{aligned}$$

Using the above definition of NNS, we have

$$\begin{aligned}
\Lambda'\left(\frac{1}{p_{k_0}(\vartheta)} - \frac{1}{p(\vartheta)}, l\right) &= \Lambda'\left(\frac{p_{k_0}(\vartheta) - p(\vartheta)}{p_{k_0}(\vartheta)p(\vartheta)}, l\right)\\
&= \Lambda'\left((p_{k_0}(\vartheta) - p(\vartheta)), l|p_{k_0}(\vartheta)p(\vartheta)|\right)\\
&= \Lambda'\left((p_{k_0}(\vartheta) - p(\vartheta)), lM_1M_2\right)\\
&= \Lambda'\left((p_{k_0}(\vartheta) - p(\vartheta)), l_1\right)\\
&> 1-\sigma,
\end{aligned}$$

$$\begin{aligned}
\Gamma'\left(\frac{1}{p_{k_0}(\vartheta)} - \frac{1}{p(\vartheta)}, l\right) &= \Gamma'\left(\frac{p_{k_0}(\vartheta) - p(\vartheta)}{p_{k_0}(\vartheta)p(\vartheta)}, l\right)\\
&= \Gamma'\left((p_{k_0}(\vartheta) - p(\vartheta)), l|p_{k_0}(\vartheta)p(\vartheta)|\right)\\
&= \Gamma'\left((p_{k_0}(\vartheta) - p(\vartheta)), lM_1M_2\right)\\
&= \Gamma'\left((p_{k_0}(\vartheta) - p(\vartheta)), l_1\right)\\
&< \sigma,
\end{aligned}$$

$$\begin{aligned}
\Pi'\left(\frac{1}{p_{k_0}(\vartheta)} - \frac{1}{p(\vartheta)}, l\right) &= \Pi'\left(\frac{p_{k_0}(\vartheta) - p(\vartheta)}{p_{k_0}(\vartheta)p(\vartheta)}, l\right)\\
&= \Pi'\left((p_{k_0}(\vartheta) - p(\vartheta)), l|p_{k_0}(\vartheta)p(\vartheta)|\right)\\
&= \Pi'\left((p_{k_0}(\vartheta) - p(\vartheta)), lM_1M_2\right)\\
&= \Pi'\left((p_{k_0}(\vartheta) - p(\vartheta)), l_1\right)
\end{aligned}$$

$$< \sigma,$$

where $l_1 = lM_1M_2 > 0$. Hence, we get $k_0 \in \mathfrak{A}_2$ which implies that $\delta_\lambda(\mathfrak{A}_2) = 1$. □

Definition 3.19. Let $(Y, \mathcal{M}, *, \boxdot)$ and $(Z, \mathcal{M}', *, \boxdot)$ be two NNS and let $\mathcal{G}$ be a family of functions from Y to Z. The family $\mathcal{G}$ is Neutrosophic equicontinuous at a point $\vartheta_0 \in Y$ if for every $\sigma > 0$ and $l > 0$, there exists a $\delta > 0$ such that $\Lambda'(p_k(\vartheta_0) - p(\vartheta), l) > 1 - \sigma$, $\Gamma'(p_k(\vartheta_0) - p(\vartheta), l) < \sigma$ and $\Pi'(p_k(\vartheta_0) - p(\vartheta), l) < \sigma$ for all $p \in \mathcal{G}$ and all ϑ such that $\Lambda'(\vartheta_0 - \vartheta, l) > 1 - \delta$, $\Gamma'(\vartheta_0 - \vartheta, l) < \delta$ and $\Pi'(\vartheta_0 - \vartheta, l) < \delta$. The family is neutrosophic equicontinuous if it is equicontinuous at each point of Y.

Remark 3.20. δ may be dependent on σ, Y andr p for continuity ; but, δ needs to be independent of p for equicontinuity.

Theorem 3.21. *Let $(Y, \mathcal{M}, *, \boxdot)$ and $(Z, \mathcal{M}', *, \boxdot)$ be two NNS. Let $st^{\lambda}_{\mathcal{M}} - p_k \to p$ on Y where function $p_k : (Y, \mathcal{M}, *, \boxdot) \to (Z, \mathcal{M}', *, \boxdot)$, $k \in \mathbb{N}$ are neutrosophic equicontinuous on Y and $p : Y \to Z$. The function p is continuous on Y.*

Proof. Let $\vartheta_0 \in Y$. By the neutrosophic equicontinuity of $p_k(\vartheta)$'s, $\forall$ $\sigma > 0$ and $l > 0$, $\exists$ $\delta = \delta(\vartheta_0, \sigma, \frac{l}{3}) > 0$ such that

$$\Lambda'\left(p_k(\vartheta_0) - p_k(\vartheta), \frac{l}{3}\right) > 1 - \sigma, \qquad \Gamma'\left(p_k(\vartheta_0) - p_k(\vartheta), \frac{l}{3}\right) < \sigma$$

$\forall$ $k \in \mathbb{N}$ andr $\forall \vartheta \in Y$ such that $\Lambda'(\vartheta_0 - \vartheta, l) > 1 - \delta$, $\Gamma'(\vartheta_0 - \vartheta, l) < \delta$ andr $\Pi'(\vartheta_0 - \vartheta, l) < \delta$. Let $\vartheta \in \mathfrak{B}(\vartheta_0, \delta, l)$ denotes for an open ball in NNS-$(Y, \mathcal{M}, *, \boxdot)$ with center ϑ_0 and radius δ be fixed. Since $st^{\lambda}_{\mathcal{M}} - p_k \to p$ on Y, for each $\vartheta \in Y$, if we state $\mathfrak{A}_\vartheta(\sigma, l)$ and $\mathfrak{B}_\vartheta(\sigma, l)$ by the sets

$$\left\{k \in \mathbb{N} : \Lambda'\left(p_k(\vartheta) - p(\vartheta), \frac{l}{3}\right) \le 1 - \sigma \text{ or } \Gamma'\left(p_k(\vartheta) - p(\vartheta), \frac{l}{3}\right) \ge \sigma, \right.$$
$$\left. \Pi'\left(p_k(\vartheta) - p(\vartheta), \frac{l}{3}\right) \ge \sigma\right\}$$

$$\left\{k \in \mathbb{N} : \Lambda'\left(p_k(\vartheta_0) - p(\vartheta_0), \frac{l}{3}\right) \le 1 - \sigma \text{ or } \Gamma'\left(p_k(\vartheta_0) - p(\vartheta_0), \frac{l}{3}\right) \ge \sigma \right.$$
$$\left. \Pi'\left(p_k(\vartheta_0) - p(\vartheta_0), \frac{l}{3}\right) \ge \sigma\right\},$$

respectively, then we have $\delta_\lambda(\mathfrak{A}_\vartheta(\sigma, l)) = 0$ and $\delta_\lambda(\mathfrak{B}_\vartheta(\sigma, l)) = 0$; hence $\delta_\lambda(\mathfrak{A}_\vartheta(\sigma, l) \cup \mathfrak{B}_\vartheta(\sigma, l)) = 0$ and $\mathfrak{A}_\vartheta(\sigma, l) \cup \mathfrak{B}_\vartheta(\sigma, l)$ is different from $\mathbb{N}$. Thus, $\exists$ $n \in \mathbb{N}$ such that,

$$\Lambda'\left(p_n(\vartheta) - p(\vartheta), \frac{l}{3}\right) > 1 - \sigma,$$

$$\Gamma'\left(p_n(\vartheta)-p(\vartheta)\frac{l}{3}\right)<\sigma,$$
$$\Pi'\left(p_n(\vartheta)-p(\vartheta),\frac{l}{3}\right)<\sigma,$$
$$\Lambda'\left(p_n(\vartheta_0)-p(\vartheta_0),\frac{l}{3}\right)>1-\sigma,$$
$$\Gamma'\left(p_n(\vartheta_0)-p(\vartheta_0),\frac{l}{3}\right)<\sigma,$$
$$\Pi'\left(p_n(\vartheta_0)-p(\vartheta_0),\frac{l}{3}\right)<\sigma,$$

Now we will show that $p(\vartheta)$ is neutrosophic continuous at ϑ_0. Since $st^{\lambda}_{\mathcal{M}}-\ p_k\to p$ on Y and $\forall\ k\in\mathbb{N}$, $p'_k s$ are continuous and p_n is also continuous for $n\in\mathbb{N}$, we have,

$$\begin{aligned}
&\Lambda'(p(\vartheta)-p(\vartheta_0),l)\\
&\quad=\Lambda'(p(\vartheta)-p_n(\vartheta)+p_n(\vartheta)-p_n(\vartheta_0)+p_n(\vartheta_0)-p(\vartheta_0),l)\\
&\quad\geq\Lambda'\left(p(\vartheta)-p_n(\vartheta),\frac{l}{3}\right)*\Lambda'\left(p_n(\vartheta)-p_n(\vartheta_0),\frac{l}{3}\right)*\Lambda'\left(p_n(\vartheta_0)-p(\vartheta_0),\frac{l}{3}\right)\\
&\quad>(1-\sigma)*(1-\sigma)*(1-\sigma)\\
&\quad=(1-\sigma),
\end{aligned}$$

$$\begin{aligned}
&\Gamma'(p(\vartheta)-p(\vartheta_0),l)\\
&\quad=\Gamma'(p(\vartheta)-p_n(\vartheta)+p_n(\vartheta)-p_n(\vartheta_0)+p_n(\vartheta_0)-p(\vartheta_0),l)\\
&\quad\leq\Gamma'\left(p(\vartheta)-p_n(\vartheta),\frac{l}{3}\right)\boxdot\Gamma'\left(p_n(\vartheta)-p_n(\vartheta_0),\frac{l}{3}\right)\boxdot\Gamma'\left(p_n(\vartheta_0)-p(\vartheta_0),\frac{l}{3}\right)\\
&\quad<\sigma\boxdot\sigma\boxdot\sigma\\
&\quad=\sigma,
\end{aligned}$$

$$\begin{aligned}
&\Pi'(p(\vartheta)-p(\vartheta_0),l)\\
&\quad=\Pi'(p(\vartheta)-p_n(\vartheta)+p_n(\vartheta)-p_n(\vartheta_0)+p_n(\vartheta_0)-p(\vartheta_0),l)\\
&\quad\leq\Pi'\left(p(\vartheta)-p_n(\vartheta),\frac{l}{3}\right)\boxdot\Pi'\left(p_n(\vartheta)-p_n(\vartheta_0),\frac{l}{3}\right)\boxdot\Pi'\left(p_n(\vartheta_0)-p(\vartheta_0),\frac{l}{3}\right)\\
&\quad<\sigma\boxdot\sigma\boxdot\sigma\\
&\quad=\sigma.
\end{aligned}$$

Hence, the proof is over. □

4. Conclusion

In this article, a comprehensive examination of the intricate concept of λ-statistical convergence of a sequence of functions in neutrosophic normed spaces has been conducted. By thoroughly investigating λ-statistical pointwise convergence and λ-statistical uniform convergence, the fundamental properties of these novel concepts have been clarified. The study has offered valuable insights into the practical application of statistical convergence within the realm of neutrosophic norm spaces, thereby laying the groundwork for future research and advancements in this mathematical domain. Overall, the research findings presented in this article enhance our comprehension of λ-statistical convergence and its relevance in the field of neutrosophic normed spaces, presenting new opportunities for exploration and analysis in this captivating area of study.

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