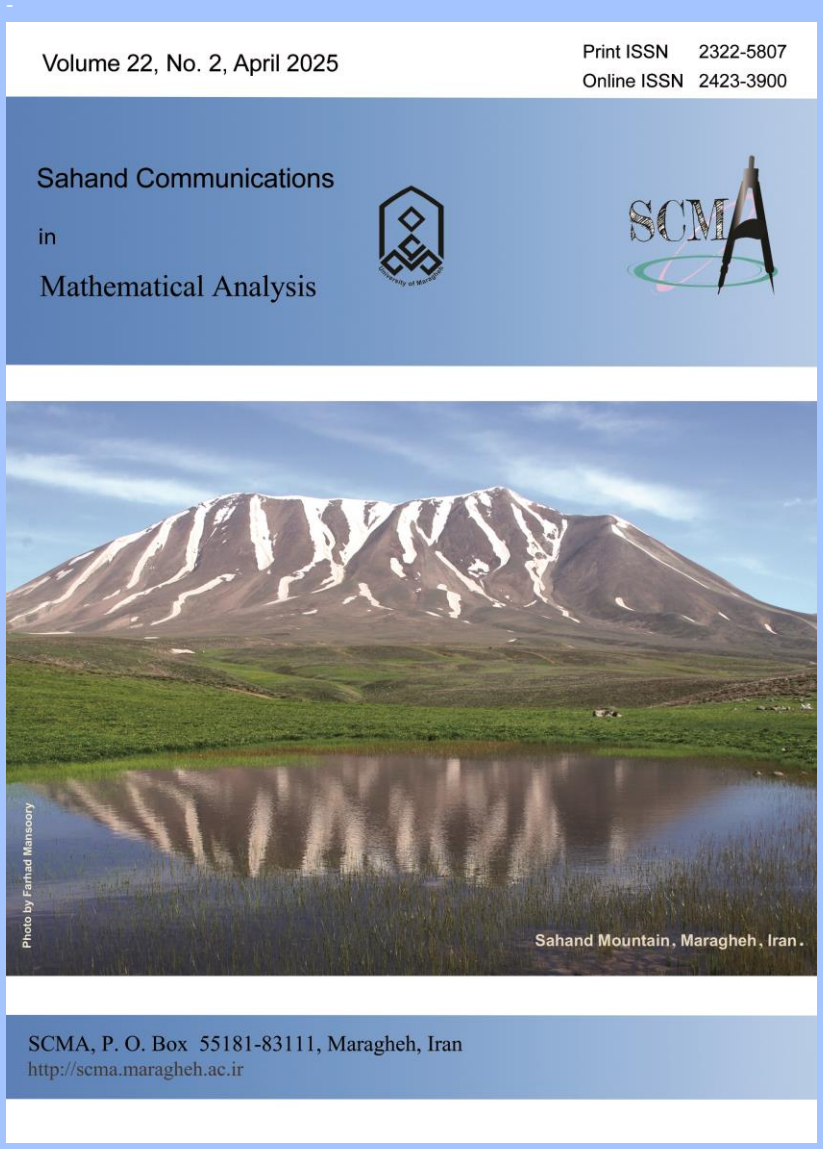
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Characteristics of $\alpha g_{(\gamma,\delta)}$-Irresolute Maps

Justin Jasmine Mershia Rabuni[1*] and Nataraj Balamani[2]

Abstract. The intention of this paper is to bring forth new maps namely $\alpha g_{(\gamma,\delta)}$- irresolute and $\alpha g_{(\gamma,\delta)}$-homeomorphism using αg_{γ}-open sets and study their properties. This new concept enables us to determine the conditions under which maps and inverse maps, preserve αg_{γ}-open sets. We derive the relation between $\alpha g_{(\gamma,\delta)}$- irresolute, αg-(γ,δ)-open, αg-(γ,δ)-closed maps and $\alpha g_{(\gamma,\delta)}$-homeomorphism in topological spaces.

1. Introduction

In 1965, Njastad [29] introduced the concept of α-open sets. The definition of generalised closed sets in topological spaces was first proposed by Levine [22] in 1970. Maki [23] presented αg-closed sets in topological spaces later in 1994. The theory of operation on topological spaces and α-closed graphs of maps in topological spaces were presented by Kasahara [20]. Thereafter, Jankovic [19] explored the maps with α-closed graphs. Ogata [30] advanced on his work by introducing γ-open sets and derived the characteristics of (γ,β)-continuous, (γ,β)-homeomorphism and (γ,β)-open maps from operation on open sets in topological spaces. Following his work, numerous authors have investigated the generalisations of various continuous and irresolute maps via operation approach in topological spaces. Ahmad and Rehman [1, 31] studied the characterizations of (γ,β)-open and (γ,β)-continuous maps. In 2006, Sai Sundara Krishnan and Balachandran [32] brought forth the concepts of (γ,β)-precontinuous maps and (γ,β)-prehomeomorphism and examined some of their fundamental properties. In the year 2007, Sai Sundara

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Krishnan et al.[33] defined semi-(γ,β)-continuous, semi-(γ,β) maps and semi-(γ,β)-homeomorphisms and derived numerous theorems on it. various forms of maps such as γ-continuous, γ-β-continuous, γ-β-closed, γ-β-open, γ-β-generalized maps, etc was investigated by Basu et al. [7]. They also studied a decomposition theorem for γ-continuous maps and a characterization of continuous maps in terms of γ-β-continuous maps. Hussain and Ahmad [13] defined and examined the properties of strongly γ-semi-continuous maps.

Tahiliani [35] defined a new type of map called β-(γ, b)-continuous maps and investigated their properties. He also investigated a general operation approach of β-closed graphs of maps. Carpintero et al. [8] introduced new type of continuous maps called b-(γ, β)-continuous, b-(γ, β)-closed, b-γ-closed and studied some characterization theorems. Carpintero [9] initiated the study on somewhat (γ, β)-continuous maps using γ-semi open sets and investigated the relations between (γ, β)-continuous and somewhat (γ, β)-continuous maps. Carpintero [10] defined the notion of almost-(γ, β) continuous and derived some characterization theorems for almost-(γ, β) continuous maps.

Ibrahim [14] introduced $\alpha_{(\gamma,\gamma')}$-continuous maps and studied some characteristics of those maps. Ibrahim [15] introduced some forms of maps named γ-b-continuous maps and a new concept of the graph of a map named γ-b-closed graph and investigated some of their basic characteristics. Ibrahim [16] established two new concepts namely almost contra-γ-continuous multimaps and contra-γ-continuous and studied various characteristics of them. Ibrahim [17] introduced various maps such as γ-semi-continuous, τ-γ-semi-continuous, γ-γ-semi-continuous, β-semi-g.closed, (γ, β)-semi-irresolute, weakly γ-semi-continuous and analysed its characteristics.

Asaad et al. [4] investigated a new type of maps called γ-P_S-g-continuous, γ-P_S-g-closed and γ-P_S-g-open and studied some theorems and results using the obtained maps. Ibrahim [18] introduced a special class of maps called almost β-γ-continuous maps which are contained in the class of almost β-continuous maps and contains the class of β-γ-continuous maps and obtained fundamental characteristics of almost β-γ-continuous maps.

Asaad [2] defined g-(γ, β)-continuous map and g-β-closed graphs using g-open sets and derived several characterization theorems for them. Asaad and Ahmad [6] introduced a new class of maps called sg-(γ, β)-irresolute and studied some characterizations and properties of sg-(γ, β) map. They also derived some properties of sg-(γ, β)-closed, sg-β-closed,

sg-(γ,β)-homeomorphism maps and sg-β-closed graphs. In 2018, Hussain [12] investigated the properties and characterizations of γ-b-irresolute map, γ-b-continuous map, γ-b-closed and γ-b-open maps and provided sufficient examples for the existence of defined concepts. Saravanakumar and Mohamed Riyazdeen [34] defined $(\tilde{\gamma},\tilde{\beta})$-continuous maps and obtained its properties in topological space in 2023.

Mershia Rabuni and Balamani [24, 25] recently developed the theory of αg_γ-open sets using the operation γ on $\varsigma_{\alpha g}$ and analysed its characteristics in topological spaces. With the help of αg_γ-open sets, Mershia Rabuni and Balamani [26, 27] constructed a variety of spaces, αg_γ-T_i (i = 0, 1, 2, 1/2), αg_γ- T_i $'$ (i = 0, 1, 2), αg-γ-regular, αg-γ-normal, and explored their characterizations. The definitions of several new continuous maps, including αg-(γ,δ)-continuous, αg-δ-continuous, αg_γ-δ-continuous were proposed in their study [28]. They also established the interconnections between these continuities and derived the properties αg-(γ,δ)-homeomorphism and αg-(γ,δ)-open maps.

In our present work, we propose the definition of new maps such as $\alpha g_{(\gamma,\delta)}$-irresolute and $\alpha g_{(\gamma,\delta)}$-homeomorphism using αg_γ-open sets. Also, we establish the characterizations of $\alpha g_{(\gamma,\delta)}$- irresolute maps and $\alpha g_{(\gamma,\delta)}$-homeomorphism. Further, we prove that the composition of any two $\alpha g_{(\gamma,\delta)}$-irresolute maps is $\alpha g_{(\gamma,\delta)}$- irresolute.

2. Preliminaries

Throughout this paper, $(\mathfrak{L},\varsigma,\gamma)$ and $(\mathfrak{K},\xi,\delta)$ denote topological spaces.

Definition 2.1. A subset J of $(\mathfrak{L},\varsigma,\gamma)$ is called

(i) α-open [29] if $J \subseteq int(cl\,(int\,(J)))$. The collection of all α-open sets in $\mathfrak{L}$ is denoted by ς_α. The complement of an α-open set is called α-closed.

(ii) α-generalized closed (briefly αg-closed) [23] if $\alpha cl(J) \subseteq H$ whenever $J \subseteq H$ and H is open in $(\mathfrak{L},\varsigma)$. The complement of an αg-closed set is called αg-open.

Definition 2.2 ([20])**.** Let $(\mathfrak{L},\varsigma,\gamma)$ be a topological space. An operation γ on the topology ς is a mapping from the topology ς into the power set $P(\mathfrak{L})$ of $\mathfrak{L}$ such that $H \subseteq \gamma(H)$ for any $H \in \varsigma$, where $\gamma(H)$ denotes the value of H under the operation γ.

Definition 2.3 ([24])**.** A mapping γ from $\varsigma_{\alpha g}$ into the power set $P(\mathfrak{L})$ of $\mathfrak{L}$ is termed as an operation if $H \subseteq \gamma(H)$ for every $H \in \varsigma_{\alpha g}$, where $\gamma(H)$ is the value of H under the operation γ.

Definition 2.4 ([24])**.** A non-empty subset J of $(\mathfrak{L},\varsigma,\gamma)$ with an operation γ on $\varsigma_{\alpha g}$ is termed as an αg_γ-open set if $\forall\ l \in J$, there exists

an αg-open set H such that $l \in H$ and $\gamma(H) \subseteq J$. The collection of all αg_γ-open sets in $(\mathfrak{L}, \varsigma, \gamma)$ is denoted by $\varsigma_{\alpha g_\gamma}$. The complement of an αg_γ-open set is termed as αg_γ-closed.

Definition 2.5 ([24]). A topological space $(\mathfrak{L}, \varsigma, \gamma)$ is αg_γ-regular if for every $l \in \mathfrak{L}$ and $\forall$ αg-open set H_1 containing l, there exists an αg-open set H_2 containing l such that $\gamma(H_2) \subseteq H_1$.

Definition 2.6 ([24]). An operation γ on $\varsigma_{\alpha g}$ is an αg-open operation if $\forall$ αg-open set H_1 containing $l \in \mathfrak{L}$, there exists an αg_γ-open set H_2 containing l such that $H_2 \subseteq \gamma(H_1)$.

Definition 2.7 ([24]). $\alpha g_\gamma cl(J) = \cap\{K \subseteq \mathfrak{L} : J \subseteq K$ where $\mathfrak{L} \setminus K \in \varsigma_{\alpha g_\gamma}\}$

Definition 2.8 ([25]). A subset J of $(\mathfrak{L}, \varsigma, \gamma)$ is termed as an αg_γ-generalized closed (concisely αg_γ-g.closed) set if $\alpha g cl_\gamma(J) \subseteq H$ $\forall$ $J \subseteq H$ and H is αg_γ-open in $(\mathfrak{L}, \varsigma, \gamma)$.

Definition 2.9 ([26]). A topological space $(\mathfrak{L}, \varsigma, \gamma)$ is

(i) αg_γ-$T_0{}'$ if for $l \neq n$ in $\mathfrak{L}$, there exists an αg_γ-open set H such that either $l \in H$; $n \notin H$ or $n \in H$; $l \notin H$.
(ii) αg_γ-$T_1{}'$ if for $l \neq n$ in $\mathfrak{L}$, there exist αg_γ-open sets H_1 and H_2 such that $l \in H_1; n \notin H_1$ and $n \in H_2; l \notin H_2$.
(iii) αg_γ-$T_2{}'$ if for $l \neq n$ in $\mathfrak{L}$, there exist αg_γ-open sets H_1 and H_2 such that $l \in H_1; n \in H_2$ and $H_1 \cap H_2 = \phi$.
(iv) αg_γ-$T_{1/2}$ space if every αg_γ-g.closed set of $(\mathfrak{L}, \varsigma, \gamma)$ is αg_γ-closed.

Definition 2.10 ([27]). A topological space $(\mathfrak{L}, \varsigma, \gamma)$ is

(i) αg-γ-regular if for each αg_γ-closed set J of $\mathfrak{L}$ not containing l, there exist disjoint αg_γ-open sets H_1 and H_2 containing l and J, respectively.
(ii) αg-γ-normal if for any pair of disjoint αg_γ-closed sets J and G of $\mathfrak{L}$, there exist disjoint αg_γ-open sets H_1 and H_2 containing J and G respectively.

Definition 2.11 ([28]). A map $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ is termed as αg-(γ, δ) continuous if $\forall$ $l \in \mathfrak{L}$ and $\forall$ αg-open set H_2 in $(\mathfrak{K}, \xi, \delta)$ containing $\hbar(l)$, there exists an αg-open set H_1 in $(\mathfrak{L}, \varsigma, \gamma)$ $\ni l \in H_1$ and $\hbar(\gamma(H_1)) \subseteq \delta(H_2)$.

Definition 2.12 ([28]). A mapping $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ is termed as

(i) αg-(γ, δ)- closed if $\hbar(J)$ is αg_δ-closed in $(\mathfrak{K}, \xi, \delta)$ $\forall$ αg_γ-closed set J of $(\mathfrak{L}, \varsigma, \gamma)$.
(ii) αg-(γ, δ)- open if $\hbar(J)$ is αg_δ-open in $(\mathfrak{K}, \xi, \delta)$ $\forall$ αg_γ-open set J of $(\mathfrak{L}, \varsigma, \gamma)$.

(iii) αg-(id, δ)- closed if $\hbar(J)$ is αg_δ-closed in $(\Re, \xi, \delta)$ $\forall$ αg-closed set J of $(\mathfrak{L}, \varsigma, \gamma)$.

Result 2.13. *Let $\gamma : \varsigma_{\alpha g} \to P(\mathfrak{L})$ be an operation on $\varsigma_{\alpha g}$ and J be a subset of $\mathfrak{L}$. Then the following statements are true.*

(a) J is αg_γ-closed if and only if $J = \alpha g_\gamma cl(J)$. [24]
(b) If γ is an αg-open operation on $\varsigma_{\alpha g}$, then $\alpha gcl_\gamma(J)$ is αg_γ-closed in $\mathfrak{L}$. [24]
(c) If J is αg_γ-g.closed then $\alpha gcl_\gamma(J) \setminus J$ does not contain any non-empty αg_γ-closed set. [25]

Theorem 2.14 ([26])**.** *A topological space $(\mathfrak{L}, \varsigma, \gamma)$ is αg_γ-$T_{1/2}$ with an operation γ if and only if $\{l\}$ is αg_γ-closed or αg_γ-open $\forall$ $l \in \mathfrak{L}$.*

Theorem 2.15 ([28])**.** *If a map $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\Re, \xi, \delta)$ is αg-(γ, δ)-continuous, then $\hbar^{-1}(J)$ is αg_γ-open in $(\mathfrak{L}, \varsigma, \gamma)$ for any αg_δ-open set J of $(\Re, \xi, \delta)$.*

Theorem 2.16 ([28])**.** *If $(\Re, \xi, \delta)$ is an αg_δ-regular space, a map $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\Re, \xi, \delta)$ is αg-(γ, δ)-continuous if and only if $\hbar^{-1}(J)$ is αg_γ-open in $(\mathfrak{L}, \varsigma, \gamma)$ for any αg_δ-open set J of $(\Re, \xi, \delta)$.*

3. Main Results

Definition 3.1. A map $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\Re, \xi, \delta)$ is termed as $\alpha g_{(\gamma,\delta)}$-irresolute if for every $l \in \mathfrak{L}$ and for every αg_δ-open set H_2 containing $\hbar(l)$, there exists an αg_γ-open set H_1 containing l such that $\hbar(H_1) \subseteq H_2$

Example 3.2. Consider $\mathfrak{L} = \{\ell, o, p\}$ with $\varsigma = \xi = \{\phi, \{\ell\}, \{o, p\}, \mathfrak{L}\}$. Then $\varsigma_{\alpha g} = \xi_{\alpha g} = P(\mathfrak{L})$. Let $\gamma : \varsigma_{\alpha g} \to P(\mathfrak{L})$ and $\delta : \xi_{\alpha g} \to P(\mathfrak{L})$ be two operations defined as

$$\gamma(J) = \begin{cases} J, & \ell \in J \\ J \cup \{p\}, & \ell \notin J \end{cases}, \quad \forall\ J \in \varsigma_{\alpha g}$$

and

$$\delta(G) = \begin{cases} G, & o \in G \\ \alpha cl(G), & o \notin G \end{cases}, \quad \forall\ G \in \varsigma_{\alpha g}\ .$$

Here $\varsigma_{\alpha g_\gamma} = P(\mathfrak{L}) \setminus \{o\}$ and $\xi_{\alpha g_\delta} = P(\mathfrak{L}) \setminus [\{p\}, \{\ell, p\}]$. Then the map $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{L}, \xi, \delta)$ defined as $\hbar(\ell) = o$; $\hbar(o) = p$; $\hbar(p) = \ell$ is $\alpha g_{(\gamma,\delta)}$- irresolute.

Theorem 3.3. *Let $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\Re, \xi, \delta)$ be a map.*

(a) $\hbar$ is $\alpha g_{(\gamma,\delta)}$-irresolute;
(b) $\hbar(\alpha g_\gamma cl(J)) \subseteq \alpha g_\delta cl(\hbar(J))$, $\forall$ $J \subseteq \mathfrak{L}$;
(c) $\alpha g_\gamma cl(\hbar^{-1}(J)) \subseteq \hbar^{-1}(\alpha g_\delta cl(J))$, $\forall$ $J \subseteq \Re$;

(d) for any αg_δ-closed set J of $(\mathfrak{K}, \xi, \delta)$, $\hbar^{-1}(J)$ is αg_γ-closed in $(\mathfrak{L}, \varsigma, \gamma)$;

(e) for any αg_δ-open set J of $(\mathfrak{K}, \xi, \delta)$, $\hbar^{-1}(J)$ is αg_γ-open in $(\mathfrak{L}, \varsigma, \gamma)$.

Then (a), (b), (c), (d), (e) are equivalent.

Proof. (a) $\Rightarrow$ (b) Consider $n \in \hbar(\alpha g_\gamma cl(J))$ and H_2 to be any αg_δ-open set in $(\mathfrak{K}, \xi, \delta)$ containing n. By Definition 3.1 of $\alpha g_{(\gamma,\delta)}$-irresolute, for every $l \in \mathfrak{L}$ and for every αg_δ-open set H_2 containing $\hbar(l)$, there exists an αg_γ-open set H_1 containing l such that $\hbar(H_1) \subseteq H_2$. Since $l \in \alpha g_\gamma cl(J)$, $H_1 \cap J \neq \phi$. Hence $\hbar(H_1 \cap J) \subseteq \hbar(H_1) \cap \hbar(J) \subseteq H_2 \cap \hbar(J) \neq \phi$. This implies $n \in \alpha g_\delta cl(\hbar(J))$, for H_2 be an αg_δ-open set in $(\mathfrak{K}, \xi, \delta)$ containing n. Therefore, $\hbar\left(\alpha g_\gamma cl(J)\right) \subseteq \alpha g_\delta cl(\hbar(J))$.

(b) $\Rightarrow$ (c) Consider J to be a subset of $\mathfrak{K}$. By (b), $\hbar\left(\alpha g_\gamma cl\left(\hbar^{-1}(J)\right)\right) \subseteq \alpha g_\delta cl\left(\hbar\left(\hbar^{-1}(J)\right)\right) \subseteq \alpha g_\delta cl(J)$. Hence, $\alpha g_\gamma cl\left(\hbar^{-1}(J)\right) \subseteq \hbar^{-1}(\alpha g_\delta cl(J))$.

(c) $\Rightarrow$ (d) Consider J to be an αg_δ-closed set in $(\mathfrak{K}, \xi, \delta)$. Then $J = \alpha g_\delta cl(J)$ [by Result 2.13 (a)]. By (c), $\hbar\left(\alpha g_\gamma cl\left(\hbar^{-1}(J)\right)\right) \subseteq \alpha g_\delta cl\left(\hbar\left(\hbar^{-1}(J)\right)\right) \subseteq \alpha g_\delta cl(J) = J$. Hence, $\alpha g_\gamma cl\left(\hbar^{-1}(J)\right) \subseteq \hbar^{-1}(J)$. Also, $\hbar^{-1}(J) \subseteq \alpha g_\gamma cl\left(\hbar^{-1}(J)\right)$. Consequently, $\hbar^{-1}(J) = \alpha g_\gamma cl\left(\hbar^{-1}(J)\right)$. Then, $\hbar^{-1}(J)$ is αg_γ-closed in $(\mathfrak{L}, \varsigma, \gamma)$ [Result 2.13 (a)].

(d) $\Rightarrow$ (e) Consider J to be an αg_δ-open set in $(\mathfrak{K}, \xi, \delta)$. Then, $\mathfrak{K}\backslash J$ is an αg_δ-closed set in $(\mathfrak{K}, \xi, \delta)$. By (d), $\hbar^{-1}(\mathfrak{K}\backslash J)$ is αg_γ-closed in $(\mathfrak{L}, \varsigma, \gamma)$. Since $\hbar^{-1}(\mathfrak{K}\backslash J) = \mathfrak{K}\backslash\hbar^{-1}(J)$, $\hbar^{-1}(J)$ is αg_γ-open in $(\mathfrak{L}, \varsigma, \gamma)$.

(e) $\Rightarrow$ (a) Let $l \in \mathfrak{L}$ and H_2 be an αg_δ-open set in $(\mathfrak{K}, \xi, \delta)$ containing $\hbar(l)$. Then, $l \in \hbar^{-1}(H_2)$. By (e), $\hbar^{-1}(H_2)$ is an αg_γ-open set in $(\mathfrak{L}, \varsigma, \gamma)$ containing l. Consequently, $\hbar(\hbar^{-1}(H_2)) \subseteq H_2$. Therefore, $\hbar$ is $\alpha g_{(\gamma,\delta)}$-irresolute. □

Remark 3.4. If a map $\hbar$ is bijective and $\hbar^{-1} : (\mathfrak{K}, \xi, \delta) \to (\mathfrak{L}, \varsigma, \gamma)$ is $\alpha g_{(\delta,id)}$-irresolute, then $\hbar$ is αg-(id, δ)-closed.

Proof. Consider $\hbar$ to be bijective and $\hbar^{-1}$ to be $\alpha g_{(\delta,id)}$-irresolute. By Theorem 3.3, for any αg-closed set J of $(\mathfrak{L}, \varsigma, \gamma)$, $\left(\hbar^{-1}\right)^{-1}(J)$ is αg_δ-closed in $(\mathfrak{K}, \xi, \delta)$. This implies $\hbar(J)$ is αg_δ-closed in $(\mathfrak{K}, \xi, \delta)$. Therefore, $\hbar$ is αg-(id, δ)-closed. □

Theorem 3.5. *Suppose that a map $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ is $\alpha g_{(\gamma,\delta)}$-irresolute, αg-(γ, δ)-closed and the operation γ is αg-open then:*

(i) for every αg_γ-g.closed set J of $(\mathfrak{L},\varsigma,\gamma)$, $\hbar(J)$ is αg_δ-g.closed in $(\mathfrak{K},\xi,\delta)$.

(ii) for every αg_δ-g.closed set G of $(\mathfrak{K},\xi,\delta)$, $\hbar^{-1}(G)$ is αg_γ-g.closed in $(\mathfrak{L},\varsigma,\gamma)$.

Proof. i) Consider H to be any αg_δ-open set in $(\mathfrak{K},\xi,\delta)$ such that $\hbar(J) \subseteq H$, where J is an αg_γ-g.closed set of $(\mathfrak{L},\varsigma,\gamma)$. As $\hbar$ is $\alpha g_{(\gamma,\delta)}$-irresolute map, by Theorem 3.3, $\hbar^{-1}(H)$ is an αg_γ-open set in $(\mathfrak{L},\varsigma,\gamma)$. Since J is αg_γ-g.closed and $J \subseteq \hbar^{-1}(H)$, $\alpha gcl_\gamma(J) \subseteq \hbar^{-1}(H)$ implies $\hbar(\alpha gcl_\gamma(J)) \subseteq H$. Since the operation γ is αg-open, $\alpha gcl_\gamma(J)$ is αg_γ-closed [Result 2.13 (b)]. $\hbar$ is αg-(γ,δ)-closed implies $\hbar(\alpha gcl_\gamma(J))$ is αg_δ-closed in $(\mathfrak{K},\xi,\delta)$. Then, $\alpha gcl_\delta(\hbar(J)) \subseteq \alpha gcl_\delta(\hbar(\alpha gcl_\gamma(J))) = \hbar(\alpha gcl_\gamma(J)) \subseteq H$. This implies $\hbar(J)$ is αg_δ-g.closed in $(\mathfrak{K},\xi,\delta)$.

ii) Consider H to be any αg_γ-open set of $(\mathfrak{L},\varsigma,\gamma)$ such that $\hbar^{-1}(G) \subseteq H$, where G is an αg_γ-g.closed set of $(\mathfrak{K},\xi,\delta)$. Let $C = \alpha gcl_\gamma(\hbar^{-1}(G)) \cap (\mathfrak{L}\backslash H)$, then C is αg_γ-closed in $(\mathfrak{L},\varsigma,\gamma)$. Then $\hbar(C)$ is αg_δ-closed in $(\mathfrak{K},\xi,\delta)$, as $\hbar$ is αg-(γ,δ)-closed. Since $\hbar$ is an $\alpha g_{(\gamma,\delta)}$-irresolute map, by Theorem 3.3, $\hbar(C) = \hbar\left(\alpha gcl_\gamma\left(\hbar^{-1}(G)\right)\right) \cap \hbar(\mathfrak{L}\backslash H) \subseteq \alpha gcl_\delta(G) \cap \hbar(\mathfrak{L}\backslash H) \subseteq \alpha gcl_\delta(G) \cap (\mathfrak{K}\backslash G) \subseteq \alpha gcl_\delta(G)\backslash G$. This implies $\hbar(C) = \phi$ and by Result 2.13 (c), $C = \phi$. So $\alpha gcl_\gamma\left(\hbar^{-1}(G)\right) \subseteq H$. Resulting $\hbar^{-1}(G)$ to be αg_γ-g.closed in $(\mathfrak{L},\varsigma,\gamma)$. □

Theorem 3.6. *Let $\hbar : (\mathfrak{L},\varsigma,\gamma) \to (\mathfrak{K},\xi,\delta)$ be an injective, $\alpha g_{(\gamma,\delta)}$-irresolute, αg-(γ,δ)-closed map and the operation γ be αg-open. If $(\mathfrak{K},\xi,\delta)$ is αg_δ-$T_{1/2}$, then $(\mathfrak{L},\varsigma,\gamma)$ is αg_γ-$T_{1/2}$.*

Proof. Consider J to be any αg_γ-g.closed set of $(\mathfrak{L},\varsigma,\gamma)$. As $\hbar$ is $\alpha g_{(\gamma,\delta)}$-irresolute and αg-(γ,δ)-closed. By Theorem 3.5 (i), $\hbar(J)$ is αg_δ-g.closed in $(\mathfrak{K},\xi,\delta)$. Because $(\mathfrak{K},\xi,\delta)$ is αg_δ-$T_{1/2}$, $\hbar(J)$ is αg_δ-closed in $\mathfrak{K}$. As $\hbar$ is $\alpha g_{(\gamma,\delta)}$-irresolute, by Theorem 3.3, $\hbar^{-1}(\hbar(J))$ is αg_γ-closed in $\mathfrak{L}$. Implies J is αg_γ-closed in $\mathfrak{L}$ as $\hbar$ is injective. Concludes that the space $(\mathfrak{L},\varsigma,\gamma)$ is αg_γ-$T_{1/2}$. □

Theorem 3.7. *Let $\hbar : (\mathfrak{L},\varsigma,\gamma) \to (\mathfrak{K},\xi,\delta)$ be surjective, $\alpha g_{(\gamma,\delta)}$-irresolute, αg-(γ,δ)-closed and the operation γ be αg-open. If $(\mathfrak{L},\varsigma,\gamma)$ is αg_γ-$T_{1/2}$, then $(\mathfrak{K},\xi,\delta)$ is αg_δ-$T_{1/2}$.*

Proof. Consider J to be any αg_δ-g.closed set of $(\mathfrak{K},\xi,\delta)$. As $\hbar$ is $\alpha g_{(\gamma,\delta)}$-irresolute and αg-(γ,δ)-closed. By Theorem 3.5 (ii), $\hbar^{-1}(J)$ is αg_γ-g.closed in $(\mathfrak{L},\varsigma,\gamma)$. As $(\mathfrak{L},\varsigma,\gamma)$ is αg_γ-$T_{1/2}$, then $\hbar^{-1}(J)$ is αg_γ-closed in $\mathfrak{L}$. As $\hbar$ is αg-(γ,δ)-closed, $\hbar(\hbar^{-1}(J))$ is αg_δ-closed in $\mathfrak{K}$. Then J

is αg_δ-closed in $\mathfrak{K}$ as $\hbar$ is surjective. Therefore, the space $(\mathfrak{K}, \xi, \delta\ )$ is αg_δ-$T_{1/2}$. □

Definition 3.8. A map $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ is termed as $\alpha g_{(\gamma,\delta)}$-homeomorphism, if $\hbar$ is bijective, $\alpha g_{(\gamma,\delta)}$-irresolute and $\hbar^{-1}$ is $\alpha g_{(\gamma,\delta)}$-irresolute.

Theorem 3.9. *For a bijective map $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$.*

(i) $\hbar^{-1}$ is $\alpha g_{(\gamma,\delta)}$-irresolute;
(ii) $\hbar$ is αg-(γ, δ)-open;
(iii) $\hbar$ is αg-(γ, δ)-closed.

Then (i), (ii), (iii) are equivalent.

Proof. (i) ⇒ (ii) $\hbar^{-1}$ is $\alpha g_{(\gamma,\delta)}$-irresolute implies for any αg_γ-open set J of $(\mathfrak{L}, \varsigma, \gamma)$, $\left(\hbar^{-1}\right)^{-1}(J) = \hbar(J)$ is αg_δ-open in $(\mathfrak{K}, \xi, \delta)$, by Theorem 3.3. Therefore, $\hbar$ is αg-(γ, δ)-open.

(ii) ⇒ (iii) Consider J to be an αg_γ-closed set. Then, $(\mathfrak{L}\backslash J)$ is an αg_γ-open set. Since $\hbar$ is αg-(γ, δ)-open, for any αg_γ-open set $(\mathfrak{L}\backslash J)$ of $(\mathfrak{L}, \varsigma, \gamma)$, $\hbar\,(\mathfrak{L}\backslash J) = \mathfrak{K}\backslash\hbar(J)$ is αg_δ-open in $(\mathfrak{K}, \xi, \delta)$. Implies $\hbar(J)$ is αg_δ-closed in $(\mathfrak{K}, \xi, \delta)$. Hence, $\hbar$ is αg-(γ, δ)-closed.

(iii) ⇒ (i). Consider H to be an αg_γ-open set in $(\mathfrak{L}, \varsigma, \gamma)$. Then, $\mathfrak{L}\backslash H$ is an αg_γ-closed set. Since $\hbar$ is αg-(γ, δ)-closed, $\hbar(\mathfrak{L}\backslash H)$ is αg_δ-closed in $(\mathfrak{K}, \xi, \delta)$. Consider $\hbar$ to be a map such that $\hbar^{-1} = \hbar$ and $\hbar^{-1} = \hbar$. Since $\hbar$ is bijective, $\hbar$ is also bijective. To prove that $\hbar = \hbar^{-1}$ is $\alpha g_{(\gamma,\delta)}$-irresolute. Now

$$\hbar(\mathfrak{L}\backslash H) = \hbar^{-1}(\mathfrak{L}\backslash H) = \mathfrak{K}\backslash\hbar^{-1}(H)$$

is αg_δ-closed in $(\mathfrak{K}, \xi, \delta)$ implies $\hbar^{-1}(H)$ is αg_δ-open. By Theorem 3.3, $\hbar^{-1}$ is $\alpha g_{(\gamma,\delta)}$-irresolute. □

Theorem 3.10. *For a bijective map $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$.*

(i) $\hbar$ is $\alpha g_{(\gamma,\delta)}$-homeomorphism;
(ii) $\hbar$ is $\alpha g_{(\gamma,\delta)}$-irresolute and αg-(γ, δ)-open;
(iii) $\hbar$ is $\alpha g_{(\gamma,\delta)}$-irresolute and αg-(γ, δ)-closed.

Then (i), (ii), (iii) are equivalent.

Proof. (i) ⇒ (ii) Assume that J is αg_γ-open in $(\mathfrak{L}, \varsigma, \gamma)$. Since $\hbar$ is an $\alpha g_{(\gamma,\delta)}$-homeomorphism, $\hbar^{-1}$ is $\alpha g_{(\gamma,\delta)}$-irresolute, by Theorem 3.3, $\left(\hbar^{-1}\right)^{-1}(J) = \hbar(J)$ is αg_δ-open in $(\mathfrak{K}, \xi, \delta)$, which implies that $\hbar$ is an αg-(γ, δ)-open map.

(ii) ⇒ (iii) Consider J to be an αg_γ-closed set of $(\mathfrak{L}, \varsigma, \gamma)$. Then, $\mathfrak{L}\backslash J$ is αg_γ-open. Since $\hbar$ is αg-(γ, δ)-open, then $\mathfrak{K}\backslash\hbar(J) =$

$\hbar(\mathfrak{L}\setminus J)$ is αg_δ-open in $(\mathfrak{K},\xi,\delta)$ implies $\hbar(J)$ is an αg_δ-closed set of $(\mathfrak{K},\xi,\delta)$. Therefore, $\hbar$ is an αg-(γ,δ)-closed map.

(iii) $\Rightarrow$ (i) Consider H to be an αg_γ-open set in $(\mathfrak{L},\varsigma,\gamma)$. Then, $\mathfrak{L}\setminus H$ is an αg_γ-closed set. Since $\hbar$ is αg-(γ,δ)-closed, $\hbar(\mathfrak{L}\setminus H)$ is αg_δ-closed in $(\mathfrak{K},\xi,\delta)$. Consider $\hbar$ to be a map such that $\hbar^{-1} = \hbar$ and $\hbar^{-1} = \hbar$. Since $\hbar$ is bijective, $\hbar$ is also bijective. To prove that $\hbar = \hbar^{-1}$ is $\alpha g_{(\gamma,\delta)}$-irresolute. Now $\hbar(\mathfrak{L}\setminus H) = \hbar^{-1}(\mathfrak{L}\setminus H) = \mathfrak{K}\setminus\hbar^{-1}(H)$ is αg_δ-closed in $(\mathfrak{K},\xi,\delta)$ implies $\hbar^{-1}(H)$ is αg_δ-open. By Theorem 3.3, $\hbar^{-1}$ is $\alpha g_{(\gamma,\delta)}$-irresolute. Therefore, $\hbar$ is $\alpha g_{(\gamma,\delta)}$-homeomorphism. □

Theorem 3.11. *If $\hbar : (\mathfrak{L},\varsigma,\gamma) \to (\mathfrak{K},\xi,\delta)$ is $\alpha g_{(\gamma,\delta)}$-irresolute and $\hbar : (\mathfrak{K},\xi,\delta) \to (\mathfrak{P},\zeta,\mu)$ is $\alpha g_{(\delta,\mu)}$-irresolute, then $\hbar \circ \hbar : (\mathfrak{L},\varsigma,\gamma) \to (\mathfrak{P},\zeta,\mu)$ is $\alpha g_{(\gamma,\mu)}$-irresolute.*

Proof. Consider any point l in $\mathfrak{L}$ and any αg_μ-open set H_3 of $\mathfrak{P}$ containing $\hbar(\hbar(l))$. Since $\hbar$ is $\alpha g_{(\delta,\mu)}$-irresolute, there exists an αg_δ-open set H_2 of $\mathfrak{K}$ such that $\hbar(l) \in H_2$ and $\hbar(H_2) \subseteq H_3$. Also since $\hbar$ is $\alpha g_{(\gamma,\delta)}$-irresolute, there exists an αg_γ-open set H_1 of $\mathfrak{L}$ containing l such that $\hbar(H_1) \subseteq H_2$. Consequently, this implies $\hbar(\hbar(H_1)) \subseteq \hbar(H_2) \subseteq H_3$. Concludes $\hbar \circ \hbar$ is $\alpha g_{(\gamma,\mu)}$-irresolute. □

Theorem 3.12. *Let $\hbar : (\mathfrak{L},\varsigma,\gamma) \to (\mathfrak{K},\xi,\delta)$ be $\alpha g_{(\gamma,\delta)}$-irresolute and $\hbar : (\mathfrak{K},\xi,\delta) \to (\mathfrak{P},\zeta,\mu)$ be αg-(δ,μ)-continuous. If $(\mathfrak{P},\zeta,\mu)$ is αg_μ-regular, then $\hbar \circ \hbar : (\mathfrak{L},\varsigma,\gamma) \to (\mathfrak{P},\zeta,\mu)$ is αg-(γ,μ)-continuous.*

Proof. Consider G to be αg_μ-open in $\mathfrak{P}$. Since $\hbar$ is αg-(δ,μ)-continuous [Theorem 2.15], $\hbar^{-1}(G)$ is αg_δ-open in $\mathfrak{K}$. As $\hbar$ is $\alpha g_{(\gamma,\delta)}$-irresolute, by Theorem 3.3, $\hbar^{-1}\left(\hbar^{-1}(G)\right) = (\hbar\circ\hbar)^{-1}(G)$ is αg_γ-open in $\mathfrak{L}$. Then, $\hbar \circ \hbar$ is αg-(γ,μ)-continuous [Theorem 2.16]. □

Theorem 3.13. *Let $(\mathfrak{L},\varsigma,\gamma)$ be an αg_γ-$T_{1/2}$ space and $\hbar : (\mathfrak{L},\varsigma,\gamma) \to (\mathfrak{K},\xi,\delta)$ be an $\alpha g_{(\gamma,\delta)}$-homeomorphism. Then, $(\mathfrak{K},\xi,\delta)$ is αg_δ-$T_{1/2}$.*

Proof. Consider $\{n\}$ to be a singleton set of $(\mathfrak{K},\xi,\delta)$ and l be any point in $\mathfrak{L}$ such that $n = \hbar(l) \Rightarrow l = \hbar^{-1}(n)$. By Theorem 2.14, $\{l\}$ is either αg_γ-closed or αg_γ-open. As $\hbar^{-1}$ is $\alpha g_{(\gamma,\delta)}$-irresolute, $\{n\} = \{\hbar(l)\}$ is αg_δ-closed or αg_δ-open. By Theorem 2.14, we get $(\mathfrak{K},\xi,\delta)$ is αg_δ-$T_{1/2}$. □

Proposition 3.14. *If $\hbar : (\mathfrak{L},\varsigma,\gamma) \to (\mathfrak{K},\xi,\delta)$ is any map, $\hbar : (\mathfrak{K},\xi,\delta) \to (\mathfrak{P},\zeta,\mu)$ is αg-(δ,μ)-open and injective and $(\hbar \circ \hbar) : (\mathfrak{L},\varsigma,\gamma) \to (\mathfrak{P},\zeta,\mu)$ is $\alpha g_{(\gamma,\mu)}$-irresolute. Then, $\hbar$ is $\alpha g_{(\gamma,\delta)}$-irresolute.*

Proof. Consider G to be an αg_δ-open set of $\mathfrak{K}$. Since $\hbar$ is αg-(δ,μ)-open, $\hbar(G)$ is an αg_μ-open set of $\mathfrak{P}$. Since $(\hbar \circ \hbar)$ is $\alpha g_{(\gamma,\mu)}$-irresolute and

$\hbar$ is injective, then $(\hbar \circ \hbar)^{-1}(\hbar(G)) = \hbar^{-1}\ (\hbar^{-1}(\hbar(G))) = \hbar^{-1}(G)$ is αg_γ-open in $\mathfrak{L}$. Therefore, $\hbar$ is $\alpha g_{(\gamma,\delta)}$-irresolute, by Theorem 3.3. □

Proposition 3.15. *If $\hbar : (\mathfrak{K}, \xi, \delta) \to (\mathfrak{P}, \zeta, \mu)$ is any map, $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ is αg-(γ, δ)-open and surjective and $(\hbar \circ \hbar) : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{P}, \zeta, \mu)$ is $\alpha g_{(\gamma,\mu)}$-irresolute, then $\hbar$ is $\alpha g_{(\gamma,\delta)}$-irresolute.*

Proof. Consider H to be an αg_μ-open set of $\mathfrak{P}$. Since $(\hbar \circ \hbar)$ is $\alpha g_{(\gamma,\mu)}$-irresolute, $(\hbar \circ \hbar)^{-1}(H)$ is αg_γ-open in $\mathfrak{L}$ which implies, $\hbar^{-1}(H) = \hbar\left(\hbar^{-1}\left(\hbar^{-1}(H)\right)\right) = \hbar\left((\hbar \circ \hbar)^{-1}(H)\right)$ is an αg_δ-open set in $\mathfrak{K}$ as $\hbar$ is an αg-(γ, δ)-open map. Therefore, $\hbar$ is $\alpha g_{(\gamma,\delta)}$-irresolute. □

Proposition 3.16. *If $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ is $\alpha g_{(\gamma,\delta)}$-irresolute and surjective and $(\hbar \circ \hbar) : (\mathfrak{L}, \varsigma, \gamma\ ) \to (\mathfrak{P}, \zeta, \mu)$ is αg-(γ, μ)-open, then $\hbar : (\mathfrak{K}, \xi, \delta) \to (\mathfrak{P}, \zeta, \mu)$ is αg-(δ, μ)-open.*

Proof. Assume H to be an αg_δ-open set of $\mathfrak{K}$. Since $\hbar$ is surjective, $\hbar(H) = \hbar\left(\hbar\left(\hbar^{-1}(H)\right)\right) = (\hbar \circ \hbar)(\hbar^{-1}(H))$. Since $\hbar$ is $\alpha g_{(\gamma,\delta)}$-irresolute, $\hbar^{-1}(H)$ is an αg_γ-open set in $\mathfrak{L}$. As $(\hbar \circ \hbar)$ is αg-(γ, μ)-open, $(\hbar \circ \hbar)(\hbar^{-1}(H))$ is an αg_μ-open set in $\mathfrak{P}$. Therefore, $\hbar$ is αg-(δ, μ)-open. □

Proposition 3.17. *Let $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ be any map. If $\hbar(\alpha g Int_\gamma(J)) \subseteq \alpha g Int_\delta(\hbar(J))$ for every $J \subset \mathfrak{L}$, then $\hbar$ is αg-(γ, δ)-open.*

Proof. Consider J to be any αg_γ-open set of $\mathfrak{L}$. Then, $J = \alpha g Int_\gamma(J)$ implies $\hbar(J) = \hbar(\alpha g Int_\gamma(J) \subseteq \alpha g Int_\delta(\hbar(J))$, by the given hypothesis. But always $\alpha g Int_\delta(\hbar(J)) \subseteq \hbar(J)$. Thus, $\hbar(J) = \alpha g Int_\delta(\hbar(J))$ implies $\hbar(J)$ is αg_δ-open set in $\mathfrak{K}$. Therefore, $\hbar$ is αg-(γ, δ)-open. □

Remark 3.18. But the converse i.e., if $\hbar$ is αg-(γ, δ)-open, $(\alpha g Int_\gamma(J))$ is not necessarily contained in $\alpha g Int_\delta(\hbar(J))$ where $J \subseteq \mathfrak{L}$.

Example 3.19. Consider $\mathfrak{L} = \mathfrak{K} = \{\ell, o, p\}$ with $\varsigma = \{\phi, \{\ell\}, \{o\}, \{\ell, o\}, \mathfrak{L}\}$ and $\xi = \{\phi, \{p\}, \mathfrak{K}\}$. Then $\varsigma_{\alpha g} = \{\phi, \{\ell\}, \{o\}, \{\ell, o\}, \mathfrak{L}\}$ and $\xi_{\alpha g} = P(\mathfrak{L}) \setminus \{\ell, o\}$. Let $\gamma : \varsigma_{\alpha g} \to P(\mathfrak{L})$ and $\delta : \xi_{\alpha g} \to P(\mathfrak{K})$ be two operations defined as

$$\gamma(J) = \begin{cases} J, & \ell \in J \\ J \cup \{p\}, & \text{otherwise} \end{cases}, \quad \forall\ J \in \varsigma_{\alpha g}$$

and

$$\delta(G) = \begin{cases} G, & G = \{\ell\} \text{ or } \{\ell, p\} \\ cl(G), & \text{otherwise} \end{cases}, \quad \forall\ G \in \xi_{\alpha g}\ .$$

Here, the map $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ defined as $\hbar(\ell) = \ell$, $\hbar(\sigma) = \rho, \hbar(\rho) = \sigma$ is αg-(γ, δ)-open. For $J = \{\sigma, \rho\}$, $\alpha gInt_\gamma(J) = \{\sigma\} \Rightarrow \hbar(\alpha gInt_\gamma(J)) = \{\rho\}$ and $\alpha gInt_\delta(\hbar(J)) = \alpha gInt_\delta(\{\sigma, \rho\}) = \phi$. Thus, $\hbar(\alpha gInt_\gamma(J)) \nsubseteq \alpha gInt_\delta(\hbar(J))$.

Proposition 3.20. *Let $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ be any map. Then $\hbar(\alpha g_\gamma Int(J)) \subseteq \alpha g_\delta Int(\hbar(J))$ for every subset J of $(\mathfrak{L}, \varsigma, \delta)$ if and only if $\hbar$ is αg-(γ, δ)-open.*

Proof. Consider J to be any αg_γ-open set of $\mathfrak{L}$. Then, $J = \alpha g_\gamma Int(J)$ implies

$$\hbar(J) = \hbar(\alpha g_\gamma Int(J) \subseteq \alpha g_\delta Int(\hbar(J)).$$

Always $\alpha g_\delta Int(\hbar(J)) \subseteq \hbar(J)$. Thus, $\hbar(J) = \alpha g_\delta Int(\hbar(J))$ implies $\hbar(J)$ is αg_δ-open set in $\mathfrak{K}$. Therefore, $\hbar$ is αg-(γ, δ)-open. Conversely, assume $\hbar$ is αg-(γ, δ)-open. As $\alpha g_\gamma Int(J)$ is αg_γ-open in $\mathfrak{L}$, $\hbar(\alpha g_\gamma Int(J))$ is αg_δ-open in $\mathfrak{K}$. This implies $\alpha g_\delta Int(\hbar(\alpha g_\gamma Int(J))) = \hbar(\alpha g_\gamma Int(J))$. Since $\alpha g_\gamma Int(J) \subseteq J$, $\hbar(\alpha g_\gamma Int(J)) \subseteq \hbar(J)$. Therefore, $\hbar(\alpha g_\gamma Int(J)) \subseteq \alpha g_\delta Int(\hbar(J))$. □

Proposition 3.21. *Let $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ be any map. If $\alpha gcl_\delta(\hbar(J)) \subseteq \hbar(\alpha gcl_\gamma(J))$ where $J \subset \mathfrak{L}$, then $\hbar$ is αg-(γ, δ)-closed.*

Proof. Consider J to be any αg_γ-closed set of $\mathfrak{L}$. Then, $J = \alpha gcl_\gamma(J) \Rightarrow \hbar(J) = \hbar(\alpha gcl_\gamma(J))$. By assumption, $\alpha gcl_\delta(\hbar(J)) \subseteq \hbar(\alpha gcl_\gamma(J)) = \hbar(J)$. Since $\hbar(J) \subseteq \alpha gcl_\delta(\hbar(J))$, $\hbar(J) = \alpha gcl_\delta(\hbar(J))$. Therefore, $\hbar(J)$ is an αg_δ-closed set in $\mathfrak{K}$. Therefore, $\hbar$ is αg-(γ, δ)-closed. □

Remark 3.22. But the converse i.e., if $\hbar$ is αg-(γ, δ)-closed, $\alpha gcl_\delta(\hbar(J))$ is not necessarily contained in $\hbar(\alpha gcl_\gamma(J))$ where $J \subseteq \mathfrak{L}$.

Example 3.23. Consider $\mathfrak{L} = \mathfrak{K} = \{\ell, \sigma, \rho\}$ with $\varsigma = \{\phi, \{\ell\}, \{\ell, \sigma\}, \mathfrak{L}\}$ and $\xi = \{\phi, \{\rho\}, \mathfrak{K}\}$. Then $\varsigma_{\alpha g} = \{\phi, \{\ell\}, \{\sigma\}, \{\ell, \sigma\}, \{\ell, \rho\}, \mathfrak{L}\}$ and $\xi_{\alpha g} = P(\mathfrak{L}) \setminus \{\ell, \sigma\}$. Let $\gamma : \varsigma_{\alpha g} \to P(\mathfrak{L})$ and $\delta : \xi_{\alpha g} \to P(\mathfrak{K})$ be two operations defined as

$$\gamma(J) = \begin{cases} J, & \ell \in J \\ cl(J), & \ell \notin J \end{cases}$$

and

$$\delta(G) = \begin{cases} G, & G = \{\ell\} \text{ or } \{\ell, \rho\} \\ cl(G), & \text{otherwise.} \end{cases}$$

Here, the map $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ defined as $\hbar(\ell) = \ell$, $\hbar(\sigma) = \sigma, \hbar(\rho) = \rho$ is αg-(γ, δ)-closed. For $J = \{\ell\}$, $\hbar(\alpha gcl_\gamma(J)) = \{\ell, \rho\}$ and $\alpha gcl_\delta(\hbar(J)) = \mathfrak{L}$. Thus, $\alpha gcl_\delta(\hbar(J)) \nsubseteq \hbar(\alpha gcl_\gamma(J))$.

Proposition 3.24. *Let $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ be any map. Then $\alpha g_\delta cl(\hbar(J)) \subseteq \hbar(\alpha g_\gamma cl(J))$, for every subset J of $(\mathfrak{L}, \varsigma, \gamma)$ if and only if $\hbar$ is αg-(γ, δ)-closed.*

Proof. Consider J to be any αg_γ-closed set of $\mathfrak{L}$. Then, $J = \alpha g_\gamma cl(J) \Rightarrow \hbar(J) = \hbar(\alpha g_\gamma cl(J))$. By assumption, $\alpha g_\delta cl(\hbar(J)) \subseteq \hbar(\alpha g_\gamma cl(J)) = \hbar(J)$. Since $\hbar(J) \subseteq \alpha g_\delta cl(\hbar(J))$, $\hbar(J) = \alpha g_\delta cl(\hbar(J))$. Therefore, $\hbar(J)$ is an αg_δ-closed set in $\mathfrak{K}$. Therefore, $\hbar$ is αg-(γ, δ)-closed.

Conversely, Assume that $\hbar$ is αg-(γ, δ)-closed. Since $\alpha g_\gamma cl(J)$ is αg_γ-closed in $\mathfrak{L}$ [Result 2.13 (a)], $\hbar(\alpha g_\gamma cl(J)$ is αg_δ-closed in $\mathfrak{K}$, which implies that $\alpha g_\delta cl(\hbar(\alpha g_\gamma cl(J))) = \hbar(\alpha g_\gamma cl(J))$. Now $J \subseteq \alpha g_\gamma cl(J) \Rightarrow \hbar(J) \subseteq \hbar(\alpha g_\gamma cl(J))$. Therefore, $\alpha g_\delta cl(\hbar(J)) \subseteq \hbar(\alpha g_\gamma cl(J))$. □

Theorem 3.25. *A map $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ is αg-(γ, δ)-closed if and only if for each $J \subseteq \mathfrak{K}$ and any αg_γ-open set $G \subseteq \mathfrak{L}$ such that $\hbar^{-1}(J) \subseteq G$, there exists an αg_δ-open set H in $\mathfrak{K}$, such that $J \subseteq H$ and $\hbar^{-1}(H) \subseteq G$.*

Proof. Consider J to be any subset of $\mathfrak{K}$ and G to be any αg_γ-open set in $\mathfrak{L}$, such that $\hbar^{-1}(J) \subseteq G$. Then, $\hbar(\mathfrak{L}\backslash G)$ is an αg_δ-closed set in $\mathfrak{K}$, because $\mathfrak{L}\backslash G$ is αg_γ-closed in $\mathfrak{L}$ and $\hbar$ is αg-(γ, δ)-closed. Consider $H = \mathfrak{K}\backslash\hbar(\mathfrak{L}\backslash G)$ is αg_δ-open in $\mathfrak{K}$. Since $\hbar^{-1}(J) \subseteq G \Rightarrow \mathfrak{L}\backslash G \subseteq \mathfrak{L}\backslash\hbar^{-1}(J) = \hbar^{-1}(\mathfrak{K}\backslash J) \Rightarrow \hbar(\mathfrak{L}\backslash G) \subseteq \hbar(\hbar^{-1}(\mathfrak{K}\backslash J)) \subseteq \mathfrak{K}\backslash J \Rightarrow \mathfrak{K}\backslash\hbar(\mathfrak{L}\backslash G) \supseteq \mathfrak{K}\backslash(\mathfrak{K}\backslash J) = J$, which implies that $J \subseteq H$. As well $\hbar^{-1}(H) = \hbar^{-1}(\mathfrak{K}\backslash\hbar(\mathfrak{L}\backslash G)) = \mathfrak{L}\backslash\hbar^{-1}(\hbar(\mathfrak{L}\backslash G)) \subseteq \mathfrak{L}\backslash(\mathfrak{L}\backslash G) = G$. This implies $\hbar^{-1}(H) \subseteq G$ and $J \subseteq H$.

Conversely, F be any αg_γ-closed set in $\mathfrak{L}$. To prove that $\hbar(F)$ is αg_δ-closed in $\mathfrak{K}$. For $J = \mathfrak{K}\backslash\hbar(F)$, $\hbar^{-1}(J) = \hbar^{-1}(\mathfrak{K}\backslash\hbar(F)) = \mathfrak{L}\backslash\hbar^{-1}(\hbar(F)) \subseteq \mathfrak{L}\backslash F$. Suppose that $\mathfrak{L}\backslash F = G$. G is αg_γ-open in $\mathfrak{L}$ such that $\hbar^{-1}(J) \subseteq G$. By assumption, there exists an αg_δ-open set H in $\mathfrak{K}$, such that $J \subseteq H$ and $\hbar^{-1}(H) \subseteq G$. This implies $J = \mathfrak{K}\backslash\hbar(F) \subseteq H$ and $\hbar^{-1}(H) \cap (\mathfrak{L}\backslash G) = \phi \Rightarrow \hbar(\hbar^{-1}(H) \cap (\mathfrak{L}\backslash G)) = \hbar(\phi) = \phi \Rightarrow H \cap \hbar(\mathfrak{L}\backslash G) = \phi$. That is, $H \subseteq \mathfrak{K}\backslash\hbar(\mathfrak{L}\backslash G) = \mathfrak{K}\backslash\hbar(F)$. So $\mathfrak{K}\backslash\hbar(F) \subseteq H \subseteq \mathfrak{K}\backslash\hbar(F)$. That is, $H = \mathfrak{K}\backslash\hbar(F) \Rightarrow \hbar(F) = \mathfrak{K}\backslash H$ is an αg_δ-closed set in $\mathfrak{K}$, because H is αg_δ-open in $\mathfrak{K}$. Thus, $\hbar$ is αg-(γ, δ)-closed. □

Theorem 3.26. *A map $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ is αg-(γ, δ)-open if and only if for each $J \subseteq \mathfrak{K}$ and any αg_γ-open set $H \subseteq \mathfrak{L}$ such that $\hbar^{-1}(J) \subseteq H$, there exists an αg_δ-closed set S in $\mathfrak{K}$, such that $J \subseteq S$ and $\hbar^{-1}(S) \subseteq H$.*

Proof. Similar to Theorem 3.25 □

Theorem 3.27. *Let $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ be an injective and an $\alpha g_{(\gamma,\delta)}$-irresolute map. If $(\mathfrak{K}, \xi, \delta)$ is an αg_δ-$T_2{}'$ space, then $(\mathfrak{L}, \varsigma, \gamma)$ is an αg_γ-$T_2{}'$ space.*

Proof. Assume that l and n are any two distinct points in $\mathfrak{L}$. Since $(\mathfrak{K}, \xi, \delta)$ is an αg_δ-$T_2{}'$ space, there exist αg_δ-open sets H_1 and H_2 such that $l \in H_1; n \in H_2$ and $H_1 \cap H_2 = \phi$. Since $\hbar$ is $\alpha g_{(\gamma,\delta)}$-irresolute, for every αg_δ-open sets H_1 and H_2 in $(\mathfrak{K}, \xi, \delta)$ there exist αg_γ-open sets P and Q in $(\mathfrak{L}, \varsigma, \gamma)$ containing l and n respectively $\ni \hbar(P) \subseteq H_1$ and $\hbar(Q) \subseteq H_2$. Hence, $\hbar(P) \cap \hbar(Q) = \phi$ implies $\hbar(P \cap Q) = \phi$. Since $\hbar$ is injective, $\hbar^{-1}(\hbar(P \cap Q)) = \phi$. Thus, $(P \cap Q) = \phi$. Therefore, $(\mathfrak{L}, \varsigma, \gamma)$ is an αg_γ-$T_2{}'$ space. □

Theorem 3.28. *Let $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ be an injective and an $\alpha g_{(\gamma,\delta)}$-irresolute map. If $(\mathfrak{K}, \xi, \delta)$ is an αg_δ-$T_1{}'$ (resp. αg_δ-$T_0{}'$) space, then $(\mathfrak{L}, \varsigma, \gamma)$ is an αg_γ-$T_1{}'$ (resp. αg_δ-$T_0{}'$) space.*

Proof. By Theorem 3.27. □

Theorem 3.29. *Let $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ be a bijective, $\alpha g_{(\gamma,\delta)}$-homeomorphism. If $(\mathfrak{L}, \varsigma, \gamma)$ is αg-γ-regular, then $(\mathfrak{K}, \xi, \delta)$ is also αg-δ-regular.*

Proof. Consider J to be an αg_δ-closed set of $(\mathfrak{K}, \xi, \delta)$ such that $n \notin J$. Since $\hbar$ is $\alpha g_{(\gamma,\delta)}$-irresolute, by Theorem 3.3, $\hbar^{-1}(J)$ is αg_γ-closed in $(\mathfrak{L}, \varsigma, \gamma)$. Since $\hbar$ is bijective, suppose that $\hbar(l) = n$, then $l \notin \hbar^{-1}(J)$. Since $(\mathfrak{L}, \varsigma, \gamma)$ is an αg-γ-regular space, then there exist disjoint αg_γ-open sets G and F in $\mathfrak{L}$ such that $l \in G$ and $\hbar^{-1}(J) \subseteq F$. Since $\hbar$ is αg-(γ, δ)-closed, by Theorem 3.25, there exists an αg_δ-open set H in $\mathfrak{K}$ containing J such that $\hbar^{-1}(H) \subseteq F$. Since $\hbar$ is a bijective αg-(γ, δ)-closed map. By Theorem 3.9, $n \in \hbar(G)$ and $\hbar(G)$ is an αg_δ-open set in $\mathfrak{K}$. Thus, $G \cap \hbar^{-1}(H) = \phi$ implies $\hbar(G) \cap H = \phi$. Therefore, $(\mathfrak{K}, \xi, \delta)$ is αg-δ-regular. □

Theorem 3.30. *Let $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ be a bijective, $\alpha g_{(\gamma,\delta)}$-irresolute and αg-(γ, δ)-closed. If $(\mathfrak{L}, \varsigma, \gamma)$ is αg-γ-regular, then $(\mathfrak{K}, \xi, \delta)$ is also αg-δ-regular.*

Proof. Similar to Theorem 3.29. □

Theorem 3.31. *Let $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ be injective, $\alpha g_{(\gamma,\delta)}$-irresolute and αg-(γ, δ)-closed. If $(\mathfrak{K}, \xi, \delta)$ is αg-δ-regular, then $(\mathfrak{L}, \varsigma, \gamma)$ is also αg-γ-regular.*

Proof. Suppose that J is any αg_γ-closed set of $(\mathfrak{L}, \varsigma, \gamma)$ such that $l \notin J$. Since $\hbar$ is αg-(γ, δ)-closed, $\hbar(J)$ is αg_δ-closed in $\mathfrak{K}$ and $\hbar(l) \notin \hbar(J)$. As $(\mathfrak{K}, \xi, \delta)$ is αg-δ-regular, there exist disjoint αg_δ-open sets H_1 and H_2 in $\mathfrak{K}$ such that $\hbar(l) \in H_1$ and $\hbar(J) \subseteq H_2$. As $\hbar$ is injective, $l \in \hbar^{-1}(H_1), J \subseteq \hbar^{-1}(H_2)$ and $\hbar^{-1}(H_1) \cap \hbar^{-1}(H_2) = \phi$. Since $\hbar$ is $\alpha g_{(\gamma,\delta)}$-irresolute, by Theorem 3.3, $\hbar^{-1}(H_1)$ and $\hbar^{-1}(H_2)$ are αg_γ-open sets in $\mathfrak{L}$. So, $(\mathfrak{L}, \varsigma, \gamma)$ is αg-γ-regular. □

Theorem 3.32. *Let $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ be surjective, $\alpha g_{(\gamma,\delta)}$-irresolute, αg-(γ, δ)-open and αg-(γ, δ)-closed. If $(\mathfrak{K}, \xi, \delta)$ is αg-δ-regular, then $(\mathfrak{L}, \varsigma, \gamma)$ is also αg-γ-regular.*

Proof. Similar to Theorem 3.31. □

Theorem 3.33. *Let $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ be bijective, $\alpha g_{(\gamma,\delta)}$-irresolute and αg-(γ, δ)-open. If $(\mathfrak{L}, \varsigma, \gamma)$ is αg-γ-normal, then $(\mathfrak{K}, \xi, \delta)$ is also αg-δ-normal.*

Proof. Assume that J_1 and J_2 are disjoint αg_δ-closed sets of $(\mathfrak{K}, \xi, \delta)$ and $\hbar$ is $\alpha g_{(\gamma,\delta)}$-irresolute, by Theorem 3.3, $\hbar^{-1}(J_1)$ and $\hbar^{-1}(J_2)$ are αg_γ-closed sets in $(\mathfrak{L}, \varsigma, \gamma)$. Since $(\mathfrak{L}, \varsigma, \gamma)$ is αg-γ-normal, there exist disjoint αg_γ-open sets G_1 and G_2 in $\mathfrak{L}$ such that $\hbar^{-1}(J_1) \subseteq G_1$ and $\hbar^{-1}(J_2) \subseteq G_2$. Since $\hbar$ is αg-(γ, δ)-open and bijective, then $\hbar(G_1)$ and $\hbar(G_2)$ are αg_δ-open in $(\mathfrak{K}, \xi, \delta)$ such that $J_1 \subseteq \hbar(G_1)$, $J_2 \subseteq \hbar(G_2)$ and $\hbar(G_2) \cap \hbar(G_2) = \phi$. Then, $(\mathfrak{K}, \xi, \delta)$ is αg-δ-normal. □

Theorem 3.34. *Let $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ be a surjective, $\alpha g_{(\gamma,\delta)}$-irresolute, αg-(γ, δ)-open and αg-(γ, δ)-closed. If $(\mathfrak{L}, \varsigma, \gamma)$ is αg-γ-normal, then $(\mathfrak{K}, \xi, \delta)$ is also αg-δ-normal.*

Proof. Similar to Theorem 3.33 □

Theorem 3.35. *Let $\hbar : (\mathfrak{L}, \varsigma, \gamma) \to (\mathfrak{K}, \xi, \delta)$ be an injective, $\alpha g_{(\gamma,\delta)}$-irresolute and αg-(γ, δ)-closed. If $(\mathfrak{K}, \xi, \delta)$ is αg-δ-normal, then $(\mathfrak{L}, \varsigma, \gamma)$ is also αg-γ-normal.*

Proof. Assume that $\hbar$ is αg-(γ, δ)-closed and J_1 and J_2 are disjoint αg_γ-closed sets of $(\mathfrak{L}, \varsigma, \gamma)$. Then, $\hbar(J_1)$ and $\hbar(J_2)$ are αg_δ-closed sets in $(\mathfrak{K}, \xi, \delta)$. Since $(\mathfrak{K}, \xi, \delta)$ is αg-δ-normal, there exist disjoint αg_δ-open sets H_1 and H_2 in $\mathfrak{K}$ such that $\hbar(J_1) \subseteq H_1$ and $\hbar(J_2) \subseteq H_2$. Since $\hbar$ is injective, then $J_1 \subseteq \hbar^{-1}(H_1)$, $J_2 \subseteq \hbar^{-1}(H_2)$ and $\hbar^{-1}(H_1) \cap \hbar^{-1}(H_2) = \phi$. Since $\hbar$ is $\alpha g_{(\gamma,\delta)}$-irresolute, by Theorem 3.3, $\hbar^{-1}(H_1)$ and $\hbar^{-1}(H_2)$ are αg_γ-open sets in $(\mathfrak{L}, \varsigma, \gamma)$. Therefore, $(\mathfrak{L}, \varsigma, \gamma)$ is αg-γ-normal. □

4. Conclusion

In this manuscript, we have introduced some maps using operation approach on αg open sets such as $\alpha g_{(\gamma,\delta)}$- irresolute maps and $\alpha g_{(\gamma,\delta)}$-homeomorphism and derived their characterizations. Also, we have derived some results on composition of maps using underlined sets. By inducing surjection and bijection on newly defined maps, the characteristics of the αg_γ-$T_{1/2}$, αg_γ-$T_i{}'$ $(i = 0, 1, 2)$, αg-γ-regular and αg-γ-normal spaces are examined. Moreover, we have proved that the image of an αg_γ-g.closed set is αg_δ-g.closed and the inverse image of an αg_δ-g.closed set is αg_γ-g.closed whenever the given map is both $\alpha g_{(\gamma,\delta)}$-irresolute

and αg-(γ, δ)-closed, under an αg-open operation. As a future work, contra-irresoluteness, strongly irresoluteness, and almost irresoluteness can be studied using αg_γ-open sets and the properties of each map can be derived.

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