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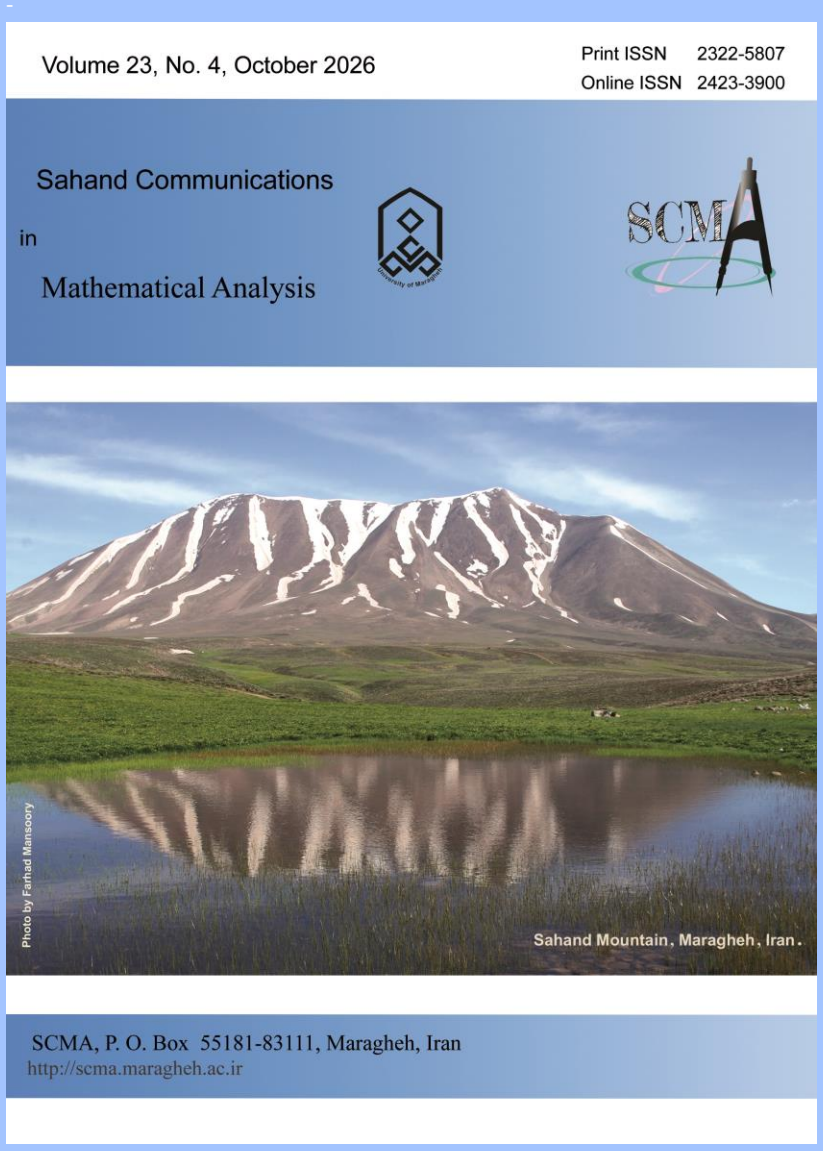

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Abstract. This paper develops a copula-based extension of stochastic frontier analysis (SFA) to account for dependence between inefficiency and random error components, an assumption commonly ignored in classical models. By integrating copula functions into the SFA framework, the proposed approach allows flexible modeling of joint distributions while preserving marginal specifications. A log-linear Cobb-Douglas production frontier is employed, and the likelihood function is derived under dependent error components, with parameters estimated via maximum likelihood.

The empirical application focuses on regional healthcare efficiency across twelve NUTS-1 regions in Turkey. Efficiency scores obtained from Data Envelopment Analysis (DEA), classical SFA, and copula-based SFA models are compared. The results indicate that DEA tends to overestimate efficiency, whereas stochastic frontier models provide more conservative estimates. Importantly, the copula-based specifications reveal a strong dependence structure between inefficiency and noise components, suggesting that the standard independence assumption may lead to biased inference.

The findings demonstrate that incorporating copulas improves both the flexibility and theoretical consistency of stochastic frontier models. Moreover, efficiency rankings remain robust across different copula families, highlighting the stability of the proposed framework. From an applied perspective, the results emphasize that regional performance differences are largely driven by inefficiency rather than random variation.

Overall, the proposed copula-based SFA framework offers a more general and realistic approach for efficiency analysis and provides a foundation for further methodological developments.

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1. Introduction

Evaluating the performance of healthcare systems has become increasingly important for policymakers, particularly in countries characterized by heterogeneous regional structures and unequal distribution of healthcare resources. The efficient utilization of limited resources, such as healthcare personnel and infrastructure, plays a critical role in improving both service delivery and population health outcomes. In this context, measuring and comparing the efficiency of regional healthcare systems provides valuable insights for evidence-based planning and resource allocation.

Various methods have been developed to assess efficiency, among which Data Envelopment Analysis (DEA), Free Disposal Hull (FDH), and Stochastic Frontier Analysis (SFA) are the most commonly used approaches. Nonparametric frontier methods, including DEA and FDH, are characterized by their reliance on minimal assumptions and their flexibility in modeling the input–output relationship without imposing a predefined functional specification. However, these methods do not account for statistical noise and assume that all deviations from the efficiency frontier are due to inefficiency. In contrast, SFA incorporates a composite error term ($\varepsilon = v - u$), where v represents random noise and u captures inefficiency. Common distributional assumptions for the inefficiency term include the half-normal, truncated normal, exponential, and gamma distributions, while the noise term is typically assumed a normal distribution (Aigner et al., 1977; Amsler et al., 2014; Hollingsworth, 2008; Ozcan, 2014).

Traditional SFA models generally assume independence between the inefficiency and noise components. However, this assumption has been questioned in the literature. Smith (2008) was among the first to suggest the potential dependence between these components and proposed the use of copulas to model this relationship. Subsequent studies, including Carta and Steel (2012), El Mehdi and Hafner (2013), Amsler et al. (2014), Najjari et al. (2016), Najjari and Mirzapour (2020), and Kazemirad et al. (2025), have further developed this approach. Although the application of copulas in SFA is both theoretically and computationally challenging, it provides a flexible framework for modeling complex dependence structures. In this study, particular attention is given to Archimedean copula families, including newly developed forms with trigonometric and hyperbolic generators (cot, coth, and csch-copulas), which offer enhanced flexibility in capturing dependence patterns.

In the context of Turkey, several studies have applied DEA to evaluate healthcare efficiency. For example, İlgün et al. (2022) analyzed maternal and child health hospitals using a two-stage DEA approach and found

that structural factors such as hospital size and location significantly influence efficiency levels. These findings highlight the importance of regional disparities and resource allocation in determining healthcare performance. Moreover, maternal mortality remains a key indicator of healthcare system performance in Turkey, despite notable improvements in recent years.

A critical methodological issue in healthcare efficiency analysis is the presence of undesirable outputs, such as mortality rates, where lower values indicate better performance. The DEA literature proposes several transformation techniques to address this issue and ensure consistency with the isotonicity assumption (Seiford & Zhu, 2002). Similarly, SFA models can incorporate such indicators either through transformation or by explicitly modeling inefficiency effects.

Building on this theoretical and empirical background, the present study aims to evaluate the efficiency of healthcare services across Turkish regions using both DEA-BCC and SFA approaches, including copula-based extensions. The analysis incorporates key healthcare inputs, such as the number of nurses, physicians, and general practitioners, and an output representing healthcare utilization. By identifying efficient and inefficient regions, this study provides practical insights for improving resource allocation and enhancing overall system performance.

The remainder of the paper is organized as follows. Section 2 presents the BCC, CCR, SFA, and copula-based models along with the regional data. Section 3 reports the efficiency results for the twelve Turkish regions. Finally, Section 4 concludes the study.

2. Preliminaries

2.1. **CCR and BCC Model (Input-Oriented DEA.** The concept of efficiency measurement was first introduced by Farrell (1957), who defined productive efficiency as the ability to maximize outputs given a set of inputs. This concept was later operationalized by Charnes, Cooper, and Rhodes (1978) through the development of the Data Envelopment Analysis (DEA) method under the assumption of constant returns to scale (CCR model). Recognizing that many real-world systems, especially healthcare systems, operate under variable returns to scale, Banker, Charnes, and Cooper (1984) proposed the BCC model, which allows for a more flexible and realistic assessment of efficiency: The primal form of CCR (CRS) model for the efficiency score of DMU_k is as follows,

$$\max \sum_{r=1}^{s} u_r y_{rk} \tag{2.1}$$

$$
\begin{aligned}
&\texttt{subject to}\\
&\sum_{i=1}^{k} v_i x_{ik} = 1\\
&\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \geq 0;\ j = 1, 2, \ldots, n,\\
&v_i, u_r \geq 0,\ i = 1, 2, \ldots, m,\ r = 1, 2, \ldots, s,
\end{aligned}
$$

where n is number of DMUs with s outputs denoted by $y_{rk},\ r = 1, \ldots, s$ and m inputs denoted by x_{ik}, $i = 1, \cdots, m$. And $u_r,\ v_i$ are the weights on output r and input i, respectively.

In this paper the primal form of input-oriented BCC (VRS) model is used and it is as follows,

$$
\begin{aligned}
&\min\ \theta_o\\
&\texttt{subject to}\\
&\theta_o x_{io} - \sum_{j=1}^{n} \lambda_j x_{ij} \geq 0\\
&\sum_{j=1}^{n} \lambda_j y_{rj} - y_{ro} \geq 0\\
&\sum_{j=1}^{n} \lambda_j = 1\\
&\lambda_j \geq 0\\
&i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n,\ r = 1, 2, \ldots, s
\end{aligned}
$$

where θ_o is efficiency score for DMU_o and nonnegative x_{io}, y_{ro} are i'th input and r'th output for the DMU_o respectively, and λ_j is intensity of DMU_j. DMU_o is called an efficient DMU if the θ_o is equal to one.

2.2. **Copulas and their properties.** A bivariate copula is a function $C : [0, 1]^2 \to [0, 1]$ which satisfies:

(a) for every u, v in [0,1], $C(u, 0) = 0 = C(0, v)$, and $C(u, 1) = u$ and $C(1, v) = v$;
(b) for every u_1, u_2, v_1, v_2 in [0,1] such that $u_1 \leq u_2$ and $v_1 \leq v_2$,

$$C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0.$$

Copula functions are a powerful technique for modeling dependence structures. Copulas allow us to combine univariate distributions to obtain a joint distribution with a particular dependence structure, in the famous Sklar Theorem: Let X and Y be random variables with joint distribution function H and marginal distribution functions F and G, respectively. Then there exists a copula C such that, $H(x, y) = C(F(x), G(y))$, for all x,y in R. If F and G are continuous, then C is unique. Otherwise, the copula C is uniquely determined on $Ran(F) \times Ran(G)$. Conversely, if C is a copula and F and G are distribution functions, then the function H is a joint distribution function with margins F and G. As a result of the Sklar Theorem, copulas link joint distribution functions to their one-dimensional margins (Sklar, 1959).

Archimedean copulas (AC) are one of the main classes of copulas. Originally this copulas were introduced in the study of probabilistic metric spaces, and they are a development of the triangle inequality. These copulas are relatively easy to construct, and many important parametric families belong to this class. Their capacity to accommodate a wide range of dependence structures has attracted substantial attention and motivated extensive efforts toward their application in diverse disciplines. For details see, Baciǵal et al. (2015a, 2015b) and Nelsen (2006). This family is characterized by generator functions. The generator function in these copulas (φ) is strictly decreasing and continuous function from $[0, 1]$ to $[0, \infty]$ such that $\varphi(1) = 0$. An Archimedean copula function $C : [0, 1]^2 \to [0, 1]$ with generator function φ is defined by,

$$C(u, v) = \varphi^{[-1]}(\varphi(u) + \varphi(v)), \tag{2.2}$$

here $\varphi^{[-1]}$ is the pseudo-inverse of φ, and is defined as

$$\varphi^{[-1]}(t) = \begin{cases} \varphi^{(-1)}(t), & 0 \le t \le \varphi(0) \\ 0, & \varphi(0) \le t \le \infty. \end{cases}$$

For this generator function if $\varphi(0) = \infty$, then $\varphi^{[-1]} = \varphi^{(-1)}$. At this case, $C(u, v)$ is strict Archimedean copula.

2.3. **Stochastic Frontier Models.** Aigner et al. (1977) and Meeusen & Van den Broeck (1977) firstly presented stochastic frontier analysis (SFA) as a method of economic modeling. This method is formulated in parametric and nonparametric models, but in the literature and applications, parametric SFA is usually is preferred (El Mehdi and Hafner, 2013). SFA model without a random component is written as,

$$y_i = f(x_i; \beta).TE_i, \quad i = 1, \ldots, I, \tag{2.3}$$

where y_i is the output of the producer i, x_i is a vector of N inputs of producer i, $f(x_i, \beta)$ is the production frontier and β is a vector of technology parameters. TE_i is the technical efficiency. If TE_i is a stochastic variable, then it can be writen as $TE_i = \exp(-u_i)$, where $u_i \geq 0$. If we insert a component of random shocks ($\exp(v_i)$) to (2.3) we get,

$$y_i = f(x_i; \beta).\exp(-u_i).\exp(v_i). \tag{2.4}$$

Now, by using the log-linear Cobb-Douglas form for $f(x_i; \beta)$ takes, the model (2.4) we get,

$$ln\ y_i = \beta_0 + \sum_n \beta_n \texttt{ln} x_{ni} + v_i - u_i. \tag{2.5}$$

2.4. **Copulas in Stochastic Frontier Models.** Studies integrating copula functions within stochastic frontier analysis (SFA) remain relatively limited in the literature. Smith (2008) is among the earliest scholars to propose the application of copula techniques in the SFA framework. Subsequently, Carta and Steel (2012) introduced an innovative methodology for modeling multi-output production frontiers based on copulas. In addition, several related contributions have been made in recent years, including studies by El Mehdi and Hafner (2013), Amsler et al. (2014), Najjari et al. (2016), Najjari and Mirzapour (2020), and Kazemirad et al. (2025).

The fundamental relationship between copula functions and SFA is shown bellow. First we consider the classical stochastic frontier model:

$$\texttt{ln}\ y_i = \beta_0 + \sum_n \beta_n \texttt{ln} x_{ni} + \varepsilon_i. \tag{2.6}$$

where $\varepsilon_i = v_i - u_i$, $i = 1, \ldots, n$ and y_i is a scalar output, x_i is a vector of input, β is a vector of parameters, and v_i and u_i are error components. For each i, the inefficiency terms u_i assumed to be independently distributed across observations (and similarly for v_i). Here the dependence between u and v is modeled by copula functions. v_i is the noise component and non-negative u_i is the technical inefficiency component. Let $u \sim F$ and $v \sim G$, then by the Sklar theorem (Nelsen, 2006), the joint distribution function is $H(u, v) = C(F(u), G(v))$. And its joint density function is as follows,

$$h(u, v) = f(u)g(v)c_\theta(F(u), G(v)). \tag{2.7}$$

for $\varepsilon = v - u$, the marginal function is,

$$h(\varepsilon) = \int_0^{+\infty} f(u)g(u + \varepsilon)c_\theta(F(u), G(u + \varepsilon))du. \tag{2.8}$$

The technical efficiency based on copulas is defined as (Smith, 2008). The use of the maximum likelihood estimator (MLE) provides an efficient approach for estimating the parameters of stochastic frontier models.

Copulas provide a flexible framework by enabling the separate modeling of marginal distributions and their dependence structure, thereby allowing the researcher to construct a more general joint distribution function. Following the estimation of the stochastic frontier model, the next step is to compute the technical efficiency of decision-making units (DMUs), which is defined as follows:

$$TE = \frac{1}{h(\varepsilon)} \int_{\mathbb{R}^+} \exp\{-u\} h(u, \varepsilon) du$$

where $h(u, \varepsilon)$ and $h(\varepsilon)$ are defined in (2.7) and (2.8). Note that $u \sim N^+\left(0, \sigma_u^2\right)$, $u \geq 0$ and $v \sim N(0, \sigma_v^2)$. Also $E(u) = \sigma_u \sqrt{\frac{2}{\pi}}$ and $Var(u) = \frac{(\pi-2)\sigma_u^2}{\pi}$.

MATLAB program was used to estimate the parameters by the maximum likelihood method. Here $\varepsilon = v - u$ and so $Var(\varepsilon) = Var(v) + Var(u) - 2cov(u, v)$. This indicates that a positive dependence between u and v reduces the variance of ϵ, while a negative dependence results in an increase in its variance. Details of the used AC families in this study are shown in Table 1.

TABLE 1. Details of the selected copula families in this study

Family	Generator	τ	λ_L	λ_U	θ interval
Clayton	$\frac{1}{\theta}(\frac{1}{t^\theta} - 1)$	$\frac{\theta}{\theta+2}$	$2^{\frac{-1}{\theta}}$	0	$(0, \infty)$
Gumbel	$(-\ln t)^\theta$	$\frac{\theta-1}{\theta}$	0	$2 - 2^{\frac{1}{\theta}}$	$[1, \infty)$
cot-copula	$\cot^\theta(\frac{\pi t}{2})$	$1 - \frac{8}{\pi^2\theta}$	$2^{\frac{-1}{\theta}}$	$2 - 2^{\frac{1}{\theta}}$	$[1, \infty)$
coth-copula	$\coth(\theta t) - \coth(\theta)$	$1 + \frac{2}{\theta^2} - \frac{2}{\theta}\coth(\theta)$	$\frac{1}{2}$	0	$[1, \infty)$
csch-copula	$\operatorname{csch}(t^\theta) - \operatorname{csch}(1)$	$\frac{\theta}{\theta+2}$	$2^{\frac{-1}{\theta}}$	0	$(0, \infty)$

Note: τ is the Kendall's tau to measure dependence between variables.

3. Results and Discussion

The data in this study are from the General Directorate of Public Health under the Ministry of Health of the Republic of Turkey. All data sets are available through the official website (http://www.saglik.gov.tr/). The most recent data correspond to the year 2024, and therefore the analysis is based on these observations. In 2024, Türkiye's total population was approximately 85.66 million, distributed across 12 NUTS-1 statistical regions, as presented in Table 2.

TABLE 2. Population of Turkey by NUTS-1 Regions (2024)

NUTS-1 Region	Population (million)
Istanbul Region	15.7
Aegean	10.9
Mediterranean	10.9
Southeast Anatolia	9.41
East Marmara	8.6
West Anatolia	8.5
Central Anatolia	4.2
West Black Sea	4.5
East Black Sea	2.6
Central East Anatolia	3.9
Northeast Anatolia	2.1
West Marmara	3.5

To evaluate the efficiency of the twelve Turkish regions, which are considered as decision-making units (DMUs), the sample size requirement of DEA (i.e., n=3(m+s), Cooper et al. (2007)) was satisfied. To avoid over-parameterization and ensure model stability, the number of input variables was restricted to three based on DEA sample size rules and multicollinearity considerations. Accordingly, three input variables and one output variable were selected for the analysis. The input variables included (Table 3):

(X1) number of nurses and midwives,
(X2) Specialist Physicians and
(X3) Population per Emergency Care Ambulance.

The output variable is (O1) Under-5 Mortality Rate (per 1.000 Live Births) at the 28-week threshold and more. Since under-five mortality rate is an undesirable output, it was transformed to a desirable form using an inverse transformation to ensure consistency with the model assumptions ($Y = \frac{1}{\text{Mortality}} \times 1000$).

TABLE 3. Inputs and outputs in BCC, CCR and SFA models

DMU	Nurses and Midwives	Specialist Physicians	Population per Emergency Care Ambulance	$\frac{1}{Mortality} \times 1000$
Istanbul	334	317	58	169
Western Marmara	390	218	62	175
Aegean	380	277	64	152
Eastern Marmara	350	228	60	156
Western Anatolia	464	397	61	159
Mediterranean	381	229	66	115
Central Anatolia	417	224	69	125
Western Blacksea	434	219	71	139
Eastern Blacksea	450	239	74	169
Northeastern Anatolia	382	206	73	103
Mideastern Anatolia	426	206	74	88
Southeastern Anatolia	331	165	64	72

In DEA input-oriented DEA model is considered appropriate in this study. Results are shown in the Table 5. SFA unlike DEA, which assumes that all deviations from the efficiency frontier are due to inefficiency, SFA separates inefficiency effects from random error. In this paper the Cobb Douglas functional form was preferred due to the limited sample size and its parsimonious structure, which reduces the risk of over-parameterization. A Cobb Douglas production frontier was specified as follows:

$$\texttt{ln}\, y_i = \beta_0 + \beta_1 \texttt{ln(Nurses}_i\texttt{)} + \beta_1 \texttt{ln(Physicians}_i\texttt{)} + \beta_2 \texttt{ln(Emergency}_i\texttt{)} + \varepsilon_i.$$

where $\varepsilon_i = v_i - u_i$, $i = 1, \ldots, 12$ and y_i is a scalar output, and v_i and u_i are error components. For each i, u_i's are independent. Here v_i is the noise component and u_i is the technical inefficiency component which is non-negative.

For the aforementioned regions of Turkey, the models were estimated accordingly. The parameters were obtained using maximum likelihood estimation under the assumption of a half-normal distribution for the inefficiency term. The estimated parameters of the stochastic frontier model, under various copula specifications as well as the standard SFA model, are presented in Table 4.

TABLE 4. SFA-Copulas model parameters

Parameters	$\hat{\sigma}_u$	$\hat{\sigma}_v$	θ	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$
Clayton	0.303	0.055	9.411	2.243	1.187	0.190	-1.256
Gumbel	0.275	0.042	8.092	2.274	1.234	0.174	-1.309
cot-copula	0.270	0.054	9.473	2.336	1.234	0.169	-1.318
coth-copula	0.319	0.081	9.806	2.313	1.175	0.178	-1.240
csch-copula	0.324	0.094	9.076	2.356	1.178	0.168	-1.240
SFA	0.089	0.025	-	1.228	0.801	0.160	-0.606

1. Variance Parameters (σ_u and σ_v): The inefficiency variance σ_u is consistently larger than the noise variance (σ_v) across all models. For example: Clayton: σ_u=0.303 while σ_v=0.055 and in Cot-copula σ_u=0.27 while σ_v=0.054. It means a substantial portion of total variation is due to inefficiency rather than random noise. This confirms that managerial and structural inefficiencies play a dominant role in healthcare performance differences. Compared to the classical SFA model, copula-based specifications yield higher inefficiency variance and stronger parameter estimates, indicating that ignoring dependence between error components may lead to biased and underestimated inefficiency levels.

2. Dependence Parameter (θ): The copula parameter θ reflects the dependence between noise and inefficiency: Clayton: 9.411, Gumbel: 8.092, Cot-copula: 9.473, Coth: 9.806 (highest dependence) and Csch:

9.076. This means Higher θ values (e.g., coth-copula) indicate a strong dependence structure, suggesting that ignoring dependence (as in classical SFA) may lead to biased results (Lower values ≈ 1) indicate weak dependence).

3. Input Elasticities (β coefficients): β_1 (Main Input): Positive in all models (0.801 to 1.234) Strongest contributor to output.
β_2 and β_3 (Negative Values): β_2: (0.16 to -0.19) and β_3: clearly negative (-0.606 to -1.318). The negative coefficients for some inputs may reflect multicollinearity effects or inefficiencies in resource allocation, and should therefore be interpreted with caution. The estimated coefficients remain highly consistent across different copula specifications, suggesting that the results are robust to the choice of dependence structure.

The results presented in Table 4 demonstrate that inefficiency effects play a dominant role in explaining variations in healthcare performance across Turkish regions. The variance parameters indicate that inefficiency is substantially larger than random noise in all model specifications. The positive and relatively large coefficient for nurses suggests that increasing nursing staff significantly improves healthcare outcomes. In contrast, the impact of physicians is relatively small and even negative in the classical SFA model, which may reflect multicollinearity or inefficiencies in resource allocation. The negative coefficient for population per emergency care ambulance indicates that higher population pressure reduces system performance. Furthermore, the high dependence parameters across copula models confirm the presence of a strong dependence structure between inefficiency and noise, supporting the use of copula-based SFA. Overall, the consistency of parameter estimates across different copula specifications indicates the robustness of the results.

The results presented in Table 5 demonstrate notable differences between DEA and SFA efficiency estimates. The BCC model identifies several regions as fully efficient, suggesting a potential overestimation of performance due to its deterministic nature. In contrast, the CCR model provides greater discrimination by accounting for scale efficiency. The copula-based SFA models produce highly consistent rankings across different specifications, indicating the robustness of the results. Furthermore, classical SFA results are broadly aligned with copula-based models, although they tend to produce slightly lower efficiency scores. A key finding is the substantial discrepancy in rankings between DEA and SFA approaches. For instance, Southeastern Anatolia is ranked among the most efficient regions under DEA but appears as the least efficient region under SFA. This highlights the importance of methodological choice in efficiency analysis. Overall, the findings suggest that stochastic frontier

approaches provide a more realistic assessment of healthcare efficiency by distinguishing inefficiency from random variation.

TABLE 5. Efficiency scores by DEA and SFA models by their ranks

DMU	BCC		CCR		Clayton		Gumbel	
	T.E.	R	T.E.	R	T.E.	R	T.E.	R
Istanbul	1.00	3	1.00	2	0.99	3	1.00	2
Western Marmara	1.00	1	1.00	1	1.00	1	1.00	3
Aegean	0.93	6	0.84	6	0.89	5	0.89	5
Eastern Marmara	1.00	4	0.96	3	0.96	4	0.96	4
Western Anatolia	0.95	5	0.89	4	0.65	10	0.64	10
Mediterranean	0.92	7	0.66	9	0.72	9	0.72	9
Central Anatolia	0.90	11	0.70	8	0.75	7	0.75	8
Western Blacksea	0.91	8	0.79	7	0.83	6	0.83	6
Eastern Blacksea	0.90	9	0.88	5	1.00	2	1.00	1
Northeastern Anatolia	0.90	10	0.62	10	0.75	8	0.75	7
Mideastern Anatolia	0.86	12	0.53	12	0.57	11	0.57	11
Southeastern Anatolia	1.00	2	0.54	11	0.55	12	0.55	12
DMU	Coth		Csch		Cot		SFA	
	T.E.	R	T.E.	R	T.E.	R	T.E.	R
Istanbul	1.00	3	1.00	1	1.00	2	0.85	4
Western Marmara	1.00	2	1.00	3	0.99	3	1.00	1
Aegean	0.89	5	0.89	5	0.89	5	0.81	6
Eastern Marmara	0.96	4	0.96	4	0.96	4	0.92	3
Western Anatolia	0.65	10	0.65	10	0.64	10	0.57	11
Mediterranean	0.72	9	0.72	9	0.72	9	0.69	9
Central Anatolia	0.75	7	0.75	7	0.75	8	0.74	7
Western Blacksea	0.83	6	0.83	6	0.83	6	0.83	5
Eastern Blacksea	1.00	1	1.00	2	1.00	1	0.97	2
Northeastern Anatolia	0.75	8	0.74	8	0.75	7	0.72	8
Mideastern Anatolia	0.57	11	0.57	11	0.57	11	0.57	10
Southeastern Anatolia	0.55	12	0.54	12	0.55	12	0.56	12

Spearman's rho Correlations among mentioned models are in Table 6.

TABLE 6. Spearman's rho Correlations among models

models		BCC	CCR	Clayton	Gumbel	Coth	Csch	Cot	SFA
BCC	Coe.	1	0.601	0.287	0.224	0.231	0.259	0.224	0.217
	Sig.		0.039	0.366	0.484	0.471	0.417	0.484	0.499
CCR	Coe.		1	0.804	0.755	0.776	0.79	0.755	0.734
	Sig.			0.002	0.005	0.003	0.002	0.005	0.007
Clayton	Coe.			1	0.972	0.993	0.972	0.972	0.979
	Sig.				0.000	0.000	0.000	0.000	0.000
Gumbel	Coe.				1	0.986	0.986	1	0.944
	Sig.					0.000	0.000	0.000	0.000
Coth	Coe.					1	0.979	0.986	0.972
	Sig.						0.000	0.000	0.000
Csch	Coe.						1	0.986	0.937
	Sig.							0.000	0.000
Cot	Coe.							1	0.944
	Sig.								0.000

The CCR model shows strong and statistically significant correlations with SFA and copula-based models, suggesting that constant returns to

scale assumptions provide results more consistent with stochastic frontier approaches. The copula-based SFA models exhibit extremely high correlations (above 0.97), indicating that efficiency rankings are highly robust to the choice of copula specification. The strong correlation between copula-based models and classical SFA confirms that while copulas enhance model flexibility, they do not substantially alter efficiency rankings. The Spearman correlation results presented in Table 6 reveal important differences across efficiency measurement methods. The correlation between the BCC model and SFA-based approaches is weak and statistically insignificant, indicating substantial discrepancies between deterministic and stochastic efficiency rankings. In contrast, the CCR model shows strong and significant correlations with both classical SFA and copula-based models, suggesting that constant returns to scale assumptions yield results more consistent with stochastic frontier methods. Furthermore, the copula-based SFA models exhibit extremely high correlations among themselves, exceeding 0.97 in most cases, which demonstrates the robustness of efficiency rankings to different dependence structures. The strong correlation between copula models and classical SFA also indicates that while copula extensions provide greater flexibility, they do not significantly alter the overall ranking patterns. Overall, these findings highlight the importance of methodological choice in efficiency analysis and suggest that stochastic frontier approaches provide more stable and reliable results.

4. Conclusion

This study evaluates regional healthcare efficiency in Turkey using DEA, classical SFA, and copula-based stochastic frontier models. The results show notable differences between deterministic and stochastic approaches, with DEA tending to overestimate efficiency, while SFA provides more realistic estimates by separating inefficiency from random noise.

A key contribution of this study is the incorporation of copula functions to model the dependence between inefficiency and noise components. The findings indicate a strong dependence structure, suggesting that the conventional independence assumption in standard SFA may lead to biased results. The consistency of rankings across different copula specifications confirms the robustness of the proposed approach.

Overall, the results highlight that regional performance differences are mainly driven by inefficiency rather than random variation. From a methodological perspective, the copula-based framework enhances model flexibility, while from a policy perspective, it emphasizes the importance of efficient resource allocation.

Despite its contributions, the study is limited by a small sample size and cross-sectional data. Future research may extend this framework using panel data and alternative copula structures.

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