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Micro Generalized Star Semi Neighborhood in Micro Topological Spaces

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Abstract. The purpose of this paper is to introduce and investigate a new class of micro neighborhood namely micro generalized star semi neighborhoods (briefly Mic-g*s nhbd), micro generalized star semi limit point (briefly Mic-g*s limit point), micro generalized star semi derived set (briefly Mic-g*s DS) in micro topological spaces. The characteristics of micro generalized star semi interior (briefly Mic-g*s Intr.) and micro generalized star semi closure (briefly Mic-g*s Clr.) are also investigated.

1. Introduction

The notion of semi open sets and generalized closed sets was introduced by Levine [6, 7] in 1963 and 1970, respectively. Lellis Thivagar and Carmel Richard [5] identified a new form of topological space using approximation spaces called nano topological space in which they defined nano interior and nano closure of a set in 2013. Chandrasekar [1] expanded the concept of nano topology to micro topology and defined micro closure, micro interior of a set in 2019. Subsequently, several developments arose in the field of micro topology. Chandrasekar and Swathi [2] introduced the concept of micro-α interior and micro α-closure in 2018. Hariwan Z. Ibrahim [4] introduced the concept of micro neighborhood in the year 2020.

In 2021, Selvaraj Ganesan [3] introduced the concept of micro semi interior and micro semi closure. Sathishmohan et al. [10] introduced

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micro pre neighborhoods, micro semi pre neighborhoods and micro pre operators, micro semi pre operators in 2024. Micro generalized star semi closed set was introduced by Sathishmohan et al. [8] in 2025 where the notions of Mic-g*s interior and Mic-g*s closure are defined. In this paper we define and discuss a new class of micro neighborhood namely micro generalized star semi neighborhood and obtain its basic properties. We also study the properties of Mic-g*s interior, Mic-g*s closure, Mic-g*s limit point and Mic-g*s derived sets.

2. Motivation

In a micro topological space, the concept of a micro neighborhood is defined using micro open sets. Related notions such as micro interior, micro closure, micro limit point, and micro derived set are also defined in the same manner. However, by using micro semi open sets, micro semi neighborhoods and their related notions can be defined. The class of micro generalized star semi open sets is stronger than micro open and micro semi open sets. Therefore, by considering this more general class of sets, broader results can be obtained, which is fundamental for extending these concepts to properties such as compactness, connectedness, and separation axioms in micro topological spaces.

Notations. The following notations are followed throughout this paper.
$\mathcal{M}$.T.S. - micro topological space,
$\mathcal{M}$-Op. - micro-open,
$\mathcal{M}$-Clo. - micro-closed,
$\mathcal{M}$-Intr. - micro interior,
$\mathcal{M}$-Clr. - micro closure,
$\mathcal{M}$.$\mathfrak{nbhd}$ - micro neighborhood,
$\mathcal{M}$-$\mathfrak{lmt.pt}$. - micro limit point,
$\mathcal{M}$-DS - micro derived set,
$\mathcal{M}$-$\mathfrak{S}$.Op. - micro semi open,
$\mathcal{M}$-$\mathfrak{S}$.Clo. - micro semi closed,
$\mathcal{M}$-$\mathfrak{S}$.Intr. - micro semi interior,
$\mathcal{M}$-$\mathfrak{S}$.Clr. - micro semi closure,
$\mathcal{M}$-$\mathfrak{S}$.$\mathfrak{nbhd}$ - micro semi neighborhood,
$\mathcal{M}$-$\mathfrak{P}$.Op. - micro pre-open,
$\mathcal{M}$-$\mathfrak{P}$.Clo. - micro pre-closed,
$\mathcal{M}$-$\mathfrak{P}$.$\mathfrak{nbhd}$ - micro pre-neighborhood,
$\mathcal{M}$-A.Op. - micro α-open,
$\mathcal{M}$-A.Clo. - micro α-closed,
$\mathcal{M}$-A.Intr. - micro α-interior,
$\mathcal{M}$-A.Clr. - micro α-closure,
$\mathcal{M}$-g.Op. - micro g open,

$\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. - micro generalized star semi open,
$\mathcal{M}$-$g^{\star}\mathfrak{S}$.Clo. - micro generalized star semi closed,
$\mathcal{M}$-$g^{\star}\mathfrak{S}$.Clo.Op. - micro generalized star semi clopen,
$\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{nbhd}$ - micro generalized star semi neighborhood,
$\mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr. - micro generalized star semi interior,
$\mathcal{M}$-$g^{\star}\mathfrak{S}$.Clr. - micro generalized star semi closure,
$\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{lmt.pt}$. - micro generalized star semi limit point,
$\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS - micro generalized star semi derived set.

3. Preliminaries

Definition 3.1 ([1])**.** The $\mathcal{M}$-Clr. of a set $\mathscr{A}$ is defined as

$$\mathcal{M}\text{-Clr.}(\mathscr{A}) = \bigcap\{\mathscr{B} : \mathscr{B} \text{ is } \mathcal{M}\text{-Clo. and } \mathscr{A} \subseteq \mathscr{B}\}.$$

Definition 3.2 ([1])**.** The $\mathcal{M}$-Intr. of a set $\mathscr{A}$ is defined as

$$\mathcal{M}\text{-Intr.}(\mathscr{A}) = \bigcup\{\mathscr{B} : \mathscr{B} \text{ is } \mathcal{M}\text{-Op. and } \mathscr{A} \supseteq \mathscr{B}\}.$$

Definition 3.3 ([1])**.** The subset $\mathscr{D}$ is said to be $\mathcal{M}$-$\mathfrak{S}$.Op. if $\mathscr{D} \subseteq \mathcal{M}\text{-Clr.}(\mathcal{M}\text{-Intr.}(\mathscr{D}))$ and $\mathcal{M}$-$\mathfrak{P}$.Clo. if $\mathcal{M}\text{-Intr.}(\mathcal{M}\text{-Clr.}(\mathscr{D})) \subseteq \mathscr{D}$.

Definition 3.4 ([1])**.** The subset $\mathscr{D}$ is said to be $\mathcal{M}$-$\mathfrak{P}$.Op. if $\mathscr{D} \subseteq \mathcal{M}\text{-Intr.}(\mathcal{M}\text{-Clr.}(\mathscr{D}))$ and $\mathcal{M}$-$\mathfrak{P}$.Clo. if $\mathcal{M}\text{-Clr.}(\mathcal{M}\text{-Intr.}(\mathscr{D})) \subseteq \mathscr{D}$.

Definition 3.5 ([4])**.** Let $\mathtt{U}$ be a $\mathcal{M}$.T.S. and $\wp \in \mathtt{U}$. Then a subset $\mathscr{D}$ of $\mathtt{U}$ is said to be $\mathcal{M}$.$\mathfrak{nbhd}$ of $\wp$, if $\exists$ a $\mathcal{M}$-Op. set $\mathcal{L}$ in $\mathtt{U}$ such that $\wp \in \mathcal{L} \subseteq \mathscr{D}$.

Definition 3.6 ([9])**.** Let $\mathtt{U}$ be a $\mathcal{M}$.T.S. and $\wp \in \mathtt{U}$. Then a subset $\mathscr{D}$ of $\mathtt{U}$ is said to be $\mathcal{M}$-$\mathfrak{S}$.$\mathfrak{nbhd}$ of $\wp$, if $\exists$ a $\mathcal{M}$-$\mathfrak{S}$.Op. set $\mathcal{L}$ in $\mathtt{U}$ such that $\wp \in \mathcal{L} \subseteq \mathscr{D}$.

Definition 3.7 ([9])**.** Let $\mathtt{U}$ be a $\mathcal{M}$.T.S. and $\wp \in \mathtt{U}$. Then a subset $\mathscr{D}$ of $\mathtt{U}$ is said to be $\mathcal{M}$-$\mathfrak{P}$.$\mathfrak{nbhd}$ of $\wp$, if $\exists$ a $\mathcal{M}$-$\mathfrak{P}$.Op. set $\mathcal{L}$ in $\mathtt{U}$ such that $\wp \in \mathcal{L} \subseteq \mathscr{D}$.

Definition 3.8 ([3])**.** The $\mathcal{M}$-$\mathfrak{S}$.Clr. of a subset $\mathscr{C}$ of $\mathtt{U}$ is defined to be the intersection of all $\mathcal{M}$-$\mathfrak{S}$.Clo. sets of $\mathtt{U} \supseteq \mathscr{C}$.

Definition 3.9 ([3])**.** The $\mathcal{M}$-$\mathfrak{S}$.Intr. of a subset $\mathscr{C}$ of $\mathtt{U}$ is defined to be the union of all $\mathcal{M}$-$\mathfrak{S}$.Op. of $\mathtt{U} \subseteq \mathscr{C}$.

Definition 3.10 ([2])**.** The $\mathcal{M}$-A.Clr. of a set $\mathscr{A}$ is equal to

$$\bigcap\{\mathscr{K} : \mathscr{K} \text{ is an } \mathcal{M}\text{-A.Clo. set in } \mathtt{U} \text{ and } \mathscr{A} \subseteq \mathscr{K}\}.$$

Definition 3.11 ([2])**.** The $\mathcal{M}$-A.Intr. of a set $\mathscr{A}$ is equal to

$$\bigcup\{\mathscr{K} : \mathscr{K} \text{ is an } \mathcal{M}\text{.A.Op. set in } \mathtt{U} \text{ and } \mathscr{A} \supseteq \mathscr{K}\}.$$

Definition 3.12 ([8])**.** Let $\mathtt{U}$ be a $\mathcal{M}$.T.S. A subset $\mathscr{A}$ of $\mathtt{U}$ is said to be $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Clo. if $\mathcal{M}$-$\mathfrak{S}$.Clr. $(\mathscr{A}) \subseteq \mathtt{V}$, whenever $\mathscr{A} \subseteq \mathtt{V}$, $\mathtt{V}$ is $\mathcal{M}$-g.Op.

Definition 3.13 ([8])**.** Let $\mathtt{U}$ be a $\mathcal{M}$.T.S. A subset $\mathscr{A}$ of $\mathtt{U}$ is $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. if its complement, $\mathscr{A}^{C}$ is $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Clo.

Definition 3.14 ([8])**.** The union of all $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. sets contained in $\mathscr{C}$ is said to be $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr. $(\mathscr{C})$.

Definition 3.15 ([8])**.** The intersection of all $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Clo. sets containing $\mathscr{C}$ is said to be $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Clr. $(\mathscr{C})$.

4. Micro Generalized Star Semi Neighborhoods

In this section, we introduce a new concept of $\mathcal{M}.\mathfrak{nbhd}$ namely $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$ and study few of its characteristics.

Definition 4.1. A subset $\mathscr{A}$ of a $\mathcal{M}$.T.S., $\mathtt{U}$ is said to be $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$ of a point $\wp \in \mathtt{U}$ if $\exists$ a $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. set $\mathtt{V}$ containing the point $\wp$ such that $\wp \in \mathtt{V} \subseteq \mathscr{A}$.

Definition 4.2. The set of all $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhds}$ of the point $\wp \in \mathtt{U}$ is called the $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$ system of the point $\wp$.

Example 4.3. Let $\mathtt{U} = \{\varpi, \varrho, \varsigma, \varphi\}$, $\mathtt{U}/R = \{\{\varpi, \varrho\}, \{\varsigma, \varphi\}\}$, $\mathfrak{X} = \{\varsigma\}$, $\tau_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varsigma, \varphi\}\}$, $\mu = \{\varpi, \varphi\}$.

$$\mu_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varphi\}, \{\varsigma, \varphi\}, \{\varpi, \varphi\}, \{\varpi, \varsigma, \varphi\}\},$$
$$(\mu_R(\mathfrak{X}))^C = \{\mathtt{U}, \emptyset, \{\varrho\}, \{\varrho, \varsigma\}, \{\varpi, \varrho\}, \{\varpi, \varrho, \varsigma\}\},$$
$$\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Op.} = \Big\{\mathtt{U}, \emptyset, \{\varphi\}, \{\varpi, \varphi\}, \{\varrho, \varphi\}, \{\varsigma, \varphi\}, \{\varpi, \varrho, \varphi\}, \{\varrho, \varsigma, \varphi\}, \{\varpi, \varsigma, \varphi\}\Big\}.$$

The following are the $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$ system of each point of $\mathtt{U}$.

$$\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathfrak{nbhd}(\varpi) = \{\mathtt{U}, \{\varpi, \varphi\}, \{\varpi, \varrho, \varphi\}, \{\varpi, \varsigma, \varphi\}\},$$
$$\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathfrak{nbhd}(\varrho) = \{\mathtt{U}, \{\varrho, \varphi\}, \{\varpi, \varrho, \varphi\}, \{\varrho, \varsigma, \varphi\}\},$$
$$\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathfrak{nbhd}(\varsigma) = \{\mathtt{U}, \{\varsigma, \varphi\}, \{\varrho, \varsigma, \varphi\}, \{\varpi, \varsigma, \varphi\}\},$$
$$\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathfrak{nbhd}(\varphi) = \Big\{\mathtt{U}, \{\varphi\}, \{\varpi, \varphi\}, \{\varrho, \varphi\}, \{\varsigma, \varphi\}, \{\varpi, \varrho, \varphi\}, \{\varrho, \varsigma, \varphi\}, \{\varpi, \varsigma, \varphi\}\Big\}.$$

Theorem 4.4. *Every* $\mathcal{M}.\mathfrak{nbhd}$ *of* $\wp \in \mathtt{U}$ *is* $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$.

Proof. Let $\mathscr{A}$ be a $\mathcal{M}.\mathfrak{nbhd}$ of $\wp \in \mathtt{U}$. Then $\exists$ a $\mathcal{M}$-Op. set $\mathtt{V}$ containing a point $\wp$ such that $\wp \in \mathtt{V} \subseteq \mathscr{A}$. Since every $\mathcal{M}$-Op. set is $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. in $\mathcal{M}$.T.S. $\mathtt{U}$. Then $\mathtt{V}$ is $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. set in $\mathcal{M}$.T.S. $\mathtt{U}$. Thus

$\exists$ a $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. set $\mathtt{V}$ containing the point $\wp$ such that $\wp \in \mathtt{V} \subseteq \mathscr{A}$. Thus, $\mathscr{A}$ is $\mathcal{M}$-$g^\star\mathfrak{S}.\mathfrak{nbhd}$ of the point $\wp \in \mathtt{U}$. □

Remark 4.5. The converse part of the above theorem need not be true, as shown in the following example.

Example 4.6. Let $\mathtt{U} = \{\varrho, \varsigma, \varphi\}$, $\mathtt{U}/R = \{\{\varrho\}, \{\varsigma, \varphi\}\}$, $\mathfrak{X} = \{\varrho\}$, $\tau_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varrho\}\}$, $\mu = \{\varsigma\}$.

$$\begin{aligned}
&\mu_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varrho\}, \{\varsigma\}, \{\varrho, \varsigma\}\},\\
&(\mu_R(\mathfrak{X}))^C = \{\mathtt{U}, \emptyset, \{\varphi\}, \{\varrho, \varphi\}, \{\varsigma, \varphi\}\},\\
&\mathcal{M}\text{-}g^\star\mathfrak{S}.\mathrm{Op.} = \{\mathtt{U}, \emptyset, \{\varrho\}, \{\varsigma\}, \{\varrho, \varsigma\}, \{\varrho, \varphi\}, \{\varsigma, \varphi\}\},\\
&\mathcal{M}.\mathfrak{nbhd}(\varsigma) = \{\mathtt{U}, \{\varsigma\}, \{\varrho, \varsigma\}\},\\
&\mathcal{M}\text{-}g^\star\mathfrak{S}.\mathfrak{nbhd}(\varsigma) = \{\mathtt{U}, \{\varsigma\}, \{\varrho, \varsigma\}, \{\varsigma, \varphi\}\}.
\end{aligned}$$

The subset $\{\varsigma, \varphi\}$ is $\mathcal{M}$-$g^\star\mathfrak{S}.\mathfrak{nbhd}(\varsigma)$ but not $\mathcal{M}.\mathfrak{nbhd}(\varsigma)$.

Theorem 4.7. *Every* $\mathcal{M}$-$\mathfrak{S}.\mathfrak{nbhd}$ *of* $\wp \in \mathtt{U}$ *is* $\mathcal{M}$-$g^\star\mathfrak{S}.\mathfrak{nbhd}$.

Proof. Let $\mathscr{A}$ be a $\mathcal{M}$-$\mathfrak{S}.\mathfrak{nbhd}$ of $\wp \in \mathtt{U}$. Then $\exists$ a $\mathcal{M}$-$\mathfrak{S}$.Op. set $\mathtt{V}$ containing a point $\wp$ such that $\wp \in \mathtt{V} \subseteq \mathscr{A}$. Since every $\mathcal{M}$-$\mathfrak{S}$.Op. set is $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. in $\mathcal{M}$.T.S. $\mathtt{U}$, then $\mathtt{V}$ is $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. set in $\mathcal{M}$.T.S. $\mathtt{U}$. Therefore, $\exists$ a $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. set $\mathtt{V}$ containing the point $\wp$ such that $\wp \in \mathtt{V} \subseteq \mathscr{A}$. Thus, $\mathscr{A}$ is $\mathcal{M}$-$g^\star\mathfrak{S}.\mathfrak{nbhd}$ of the point $\wp \in \mathtt{U}$. □

Remark 4.8. The converse part of the above theorem need not be true, as shown in the following example.

Example 4.9. Let $\mathtt{U} = \{\varpi, \varrho, \varsigma, \varphi\}$, $\mathtt{U}/R = \{\{\varpi, \varrho\}, \{\varsigma\}, \{\varphi\}\}$, $\mathfrak{X} = \{\varpi\}$, $\tau_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varpi, \varrho\}\}$, $\mu = \{\varpi\}$.

$$\begin{aligned}
&\mu_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varsigma\}, \{\varpi, \varrho\}, \{\varpi, \varrho, \varsigma\}\},\\
&(\mu_R(\mathfrak{X}))^C = \{\mathtt{U}, \emptyset, \{\varphi\}, \{\varsigma, \varphi\}, \{\varpi, \varrho, \varphi\}\},\\
&\mathcal{M}\text{-}\mathfrak{S}.\mathrm{Op.} = \{\mathtt{U}, \emptyset, \{\varsigma\}, \{\varpi, \varrho\}, \{\varsigma, \varphi\}, \{\varpi, \varrho, \varsigma\}, \{\varpi, \varrho, \varphi\}\}\\
&\mathcal{M}\text{-}g^\star\mathfrak{S}.\mathrm{Op.} = \Big\{\mathtt{U}, \emptyset, \{\varpi\}, \{\varrho\}, \{\varsigma\}, \{\varpi, \varsigma\}, \{\varrho, \varsigma\}, \{\varpi, \varrho\}, \{\varsigma, \varphi\},\\
&\qquad\qquad \{\varpi, \varrho, \varsigma\}, \{\varpi, \varrho, \varphi\}\Big\},\\
&\mathcal{M}\text{-}\mathfrak{S}.\mathfrak{nbhd}(\varsigma) = \{\mathtt{U}, \{\varsigma\}, \{\varsigma, \varphi\}, \{\varpi, \varrho, \varsigma\}\},\\
&\mathcal{M}\text{-}g^\star\mathfrak{S}.\mathfrak{nbhd}(\varsigma) = \{\mathtt{U}, \{\varsigma\}, \{\varrho, \varsigma\}, \{\varpi, \varsigma\}, \{\varsigma, \varphi\}, \{\varpi, \varrho, \varsigma\}\}.
\end{aligned}$$

The subsets $\{\varrho, \varsigma\}$ and $\{\varpi, \varsigma\}$ are $\mathcal{M}$-$g^\star\mathfrak{S}.\mathfrak{nbhd}(\varsigma)$ but not $\mathcal{M}$-$\mathfrak{S}.\mathfrak{nbhd}(\varsigma)$.

Remark 4.10. $\mathcal{M}$-$g^\star\mathfrak{S}.\mathfrak{nbhd}$ and $\mathcal{M}$-$\mathfrak{P}.\mathfrak{nbhd}$ are independent to each other.

Example 4.11. Let $\mathtt{U} = \{\varpi, \varrho, \varsigma, \varphi\}$, $\mathtt{U}/R = \{\{\varpi, \varsigma\}, \{\varrho, \varphi\}\}$, $\mathfrak{X} = \{\varpi, \varsigma\}$, $\tau_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varpi, \varsigma\}\}$, $\mu = \{\varpi, \varrho, \varsigma\}$.

$\mu_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varpi, \varsigma\}, \{\varpi, \varrho, \varsigma\}\}$,

$(\mu_R(\mathfrak{X}))^C = \{\mathtt{U}, \emptyset, \{\varphi\}, \{\varrho, \varphi\}\}$,

$\mathcal{M}\text{-}\mathfrak{P}.\mathrm{Op.} = \Big\{\mathtt{U}, \emptyset, \{\varpi\}, \{\varsigma\}, \{\varpi, \varrho\}, \{\varrho, \varsigma\}, \{\varsigma, \varphi\}, \{\varpi, \varphi\}, \{\varpi, \varsigma\}, \{\varpi, \varrho, \varsigma\}, \{\varpi, \varsigma, \varphi\}, \{\varrho, \varsigma, \varphi\}, \{\varpi, \varrho, \varphi\}\Big\}$,

$\mathcal{M}\text{-}g^\star\mathfrak{S}.\mathrm{Op.} = \Big\{\mathtt{U}, \emptyset, \{\varrho\}, \{\varsigma\}, \{\varphi\}, \{\varpi, \varsigma\}, \{\varrho, \varsigma\}, \{\varpi, \varrho\}, \{\varpi, \varrho, \varsigma\}, \{\varpi, \varsigma, \varphi\}\Big\}$,

$\mathcal{M}\text{-}\mathfrak{P}.\mathfrak{nbhd}(\varrho) = \{\mathtt{U}, \{\varpi, \varrho\}, \{\varrho, \varsigma\}, \{\varpi, \varrho, \varsigma\}, \{\varrho, \varsigma, \varphi\}, \{\varpi, \varrho, \varphi\}\}$,

$\mathcal{M}\text{-}g^\star\mathfrak{S}.\mathfrak{nbhd}(\varrho) = \{\mathtt{U}, \{\varrho\}, \{\varrho, \varsigma\}, \{\varpi, \varrho\}, \{\varpi, \varrho, \varsigma\}\}$.

The subset $\{\varrho\}$ is $\mathcal{M}\text{-}g^\star\mathfrak{S}.\mathfrak{nbhd}(\varrho)$ but not $\mathcal{M}\text{-}\mathfrak{P}.\mathfrak{nbhd}(\varrho)$.
Also, the subsets $\{\varrho, \varsigma, \varphi\}$ and $\{\varpi, \varrho, \varphi\}$ are $\mathcal{M}\text{-}\mathfrak{P}.\mathfrak{nbhd}(\varrho)$ but not $\mathcal{M}\text{-}g^\star\mathfrak{S}.\mathfrak{nbhd}(\varrho)$.

Theorem 4.12. *The arbitrary union of $\mathcal{M}\text{-}g^\star\mathfrak{S}.\mathfrak{nbhds}$ of a point $\wp \in \mathtt{U}$ is $\mathcal{M}\text{-}g^\star\mathfrak{S}.\mathfrak{nbhd}$ of the point $\wp$.*

Proof. Let $\mathscr{D}_1, \mathscr{D}_2, \ldots, \mathscr{D}_i$ where $i \in \mathbb{N}$, be the $\mathcal{M}\text{-}g^\star\mathfrak{S}.\mathfrak{nbhd}$ of a point $\wp \in \mathtt{U}$. We claim that $\bigcup \mathscr{D}_i$ is $\mathcal{M}\text{-}g^\star\mathfrak{S}.\mathfrak{nbhd}$ of $\wp$. Since $\mathscr{D}_i$ is a $\mathcal{M}\text{-}g^\star\mathfrak{S}.\mathfrak{nbhd}$ of $\wp \in \mathtt{U}$ for each i, $\exists$ a $\mathcal{M}\text{-}g^\star\mathfrak{S}.\mathrm{Op.}$ set $\mathscr{H}_i$ in $\mathtt{U}$ such that, $\wp \in \mathscr{H}_i \subseteq \mathscr{D}_i$ for each i. Since each $\mathscr{D}_i \subseteq \bigcup \mathscr{D}_i$, we have $\wp \in \mathscr{H}_i \subseteq \mathscr{D}_i \subseteq \bigcup \mathscr{D}_i$. Therefore, $\wp \in \mathscr{H}_i \subseteq \bigcup \mathscr{D}_i$. As a result $\exists$ a $\mathcal{M}\text{-}g^\star\mathfrak{S}.\mathrm{Op.}$ set $\mathscr{H}_i \subseteq \bigcup \mathscr{D}_i$ containing the point $\wp$. Thus, $\bigcup \mathscr{D}_i$ is $\mathcal{M}\text{-}g^\star\mathfrak{S}.\mathfrak{nbhd}$ of the point $\wp \in \mathtt{U}$. □

Example 4.13. Let $\mathtt{U} = \{\varpi, \varrho, \varsigma, \varphi\}$, $\mathtt{U}/R = \{\{\varpi, \varrho, \varsigma\}, \{\varphi\}\}$,
$\mathfrak{X} = \{\varpi, \varsigma\}$, $\tau_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varpi, \varrho, \varsigma\}\}$, $\mu = \{\varpi, \varphi\}$.

$\mu_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varpi\}, \{\varpi, \varphi\}, \{\varpi, \varrho, \varsigma\}\}$,

$(\mu_R(\mathfrak{X}))^C = \{\mathtt{U}, \emptyset, \{\varphi\}, \{\varrho, \varsigma\}, \{\varrho, \varsigma, \varphi\}\}$,

$\mathcal{M}\text{-}g^\star\mathfrak{S}.\mathrm{Op.} = \Big\{\mathtt{U}, \emptyset, \{\varpi\}, \{\varrho\}, \{\varsigma\}, \{\varpi, \varsigma\}, \{\varpi, \varrho\}, \{\varpi, \varphi\}, \{\varpi, \varrho, \varsigma\}, \{\varpi, \varrho, \varphi\}, \{\varpi, \varsigma, \varphi\}\Big\}$.

$\mathcal{M}\text{-}g^\star\mathfrak{S}.\mathfrak{nbhd}(\varpi) = \Big\{\mathtt{U}, \{\varpi\}, \{\varpi, \varsigma\}, \{\varpi, \varrho\}, \{\varpi, \varphi\}, \{\varpi, \varrho, \varsigma\}, \{\varpi, \varrho, \varphi\}, \{\varpi, \varsigma, \varphi\}\Big\}$.

$\bigcup(\mathcal{M}\text{-}g^\star\mathfrak{S}.\mathfrak{nbhd}\,(\varpi)) = \{\mathtt{U}\}$ which is again a $\mathcal{M}\text{-}g^\star\mathfrak{S}.\mathfrak{nbhd}(\varpi)$.

Remark 4.14. The intersection of $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$ system of a point $\wp \in \mathtt{U}$ need not be a $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$ of $\wp$.

Example 4.15. Let $\mathtt{U} = \{\varpi, \varrho, \varsigma, \varphi\}$, $\mathtt{U}/R = \{\{\varpi, \varphi\}, \{\varrho, \varsigma\}\}$, $\mathfrak{X} = \{\varrho\}$, $\tau_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varrho, \varsigma\}\}$, $\mu = \{\varphi\}$.

$\mu_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varphi\}, \{\varrho, \varsigma\}, \{\varrho, \varsigma, \varphi\}\}$,

$(\mu_R(\mathfrak{X}))^C = \{\mathtt{U}, \emptyset, \{\varpi\}, \{\varpi, \varphi\}, \{\varpi, \varrho, \varsigma\}\}$,

$\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathrm{Op.} = \Big\{\mathtt{U}, \emptyset, \{\varrho\}, \{\varsigma\}, \{\varphi\}, \{\varpi, \varphi\}, \{\varrho, \varsigma\}, \{\varrho, \varphi\}, \{\varsigma, \varphi\}, \{\varpi, \varrho, \varsigma\}, \{\varrho, \varsigma, \varphi\}\Big\}$,

$\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}(\varpi) = \{\mathtt{U}, \{\varpi, \varphi\}, \{\varpi, \varrho, \varsigma\}\}$.

$\bigcap(\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}\,(\varpi)) = \{\varpi\}$ which is not $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}(\varpi)$.

Theorem 4.16. *A subset in* $\mathtt{U}$ *is* $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathrm{Op.}$ *set iff it is* $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$ *of each of its points.*

Proof. **Necessary Part:** Let $\mathtt{V}$ be a $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathrm{Op.}$ set of $\mathtt{U}$. Then for each point $\wp \in \mathtt{V}$, we have $\wp \in \mathtt{V}_{\wp} \subseteq \mathtt{V}$. Thus, $\mathtt{V}$ is a $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$ for each $\wp \in \mathtt{V}$

Sufficient Part: Let $\mathtt{V}$ be the $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$ of each of its points. If $\mathtt{V} = \emptyset$, then $\mathtt{V}$ is $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathrm{Op.}$ If $\mathtt{V} \neq \emptyset$, then for each point $\wp \in \mathtt{V}$, $\exists$ a $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathrm{Op.}$ set $\mathtt{V}_{\wp}$ containing the point $\wp$ such that $\wp \in \mathtt{V}_{\wp} \subseteq \mathtt{V}$. Then we have $\mathtt{V} = \bigcup\{\mathtt{V}_{\wp} : \wp \in \mathtt{V}\}$. Since each $\mathtt{V}_{\wp}$ is $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathrm{Op.}$, arbitrary union of $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathrm{Op.}$ is also $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathrm{Op.}$ Hence, $\mathtt{V} = \bigcup\{\mathtt{V}_{\wp} : \wp \in \mathtt{V}\}$ is $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathrm{Op.}$ □

Remark 4.17. A $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$ of a point $\wp \in \mathtt{U}$ need not be a $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathrm{Op.}$

Example 4.18. Let $\mathtt{U} = \{\varpi, \varrho, \varsigma, \varphi\}$, $\mathtt{U}/R = \{\{\varpi, \varrho\}, \{\varsigma, \varphi\}\}$, $\mathfrak{X} = \{\varsigma\}$, $\tau_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varsigma, \varphi\}\}$, $\mu = \{\varpi, \varsigma\}$.

$\mu_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varsigma\}, \{\varpi, \varsigma\}, \{\varsigma, \varphi\}, \{\varpi, \varsigma, \varphi\}\}$,

$(\mu_R(\mathfrak{X}))^C = \{\mathtt{U}, \emptyset, \{\varrho\}, \{\varpi, \varrho\}, \{\varrho, \varphi\}, \{\varpi, \varrho, \varphi\}\}$,

$\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathrm{Op.} = \Big\{\mathtt{U}, \emptyset, \{\varpi\}, \{\varphi\}, \{\varsigma\}, \{\varpi, \varphi\}, \{\varpi, \varsigma\}, \{\varrho, \varsigma\}, \{\varsigma, \varphi\}, \{\varpi, \varrho, \varsigma\}, \{\varrho, \varsigma, \varphi\}, \{\varpi, \varsigma, \varphi\}\Big\}$.

The subset $\{\varpi, \varrho, \varphi\}$ of $\mathtt{U}$ is $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$ of the point $\varphi \in \mathtt{U}$, Since $\exists$ a $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathrm{Op.}$ subset $\{\varpi, \varphi\}$ of $\{\varpi, \varrho, \varphi\}$ such that $\varphi \in \{\varpi, \varphi\} \subseteq \{\varpi, \varrho, \varphi\}$. But $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$, $\{\varpi, \varrho, \varphi\}$ of the point φ is not $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathrm{Op.}$

Theorem 4.19. *Let* $\mathscr{K}$ *be a* $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathrm{Clo.}$ *subset of* $\mathtt{U}$. *Let* $\wp \in \mathscr{K}^C$. *Then* $\exists$ *a* $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$, $\mathtt{V}$ *of* $\wp$ *such that* $\mathtt{V} \cap \mathscr{K} = \emptyset$.

Proof. Let $\mathscr{K}$ be a $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Clo. subset of $\mathtt{U}$. Then $\mathscr{K}^{C}$ is $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. subset of $\mathtt{U}$ which contains the point $\wp$. Since each $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. set is a $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$ of each of its points, $\mathscr{K}^{C}$ is a $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$ of the point $\wp \in \mathtt{U}$. Set $\mathscr{K}^{C} = \mathtt{V}$, which then follows that $\wp \in \mathscr{K}^{C} \subseteq \mathtt{V}$. Hence, $\mathtt{V}$ is a $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$ of the point $\wp \in \mathtt{U}$ and $\mathtt{V} \cap \mathscr{K} = \mathscr{K}^{C} \cap \mathscr{K}$ which equals $\emptyset$. i.e., $\mathtt{V} \cap \mathscr{K} = \emptyset$. □

Theorem 4.20. *Let* $\mathtt{U}$ *be a* $\mathcal{M}$.T.S. *and* $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}(\wp)$ *be the system of* $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhds}$ *for each* $\wp \in \mathtt{U}$. *Then the following result holds.*

(i) *For each* $\wp \in \mathtt{U}$, $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}(\wp) \neq \emptyset$.
(ii) *If* $\mathscr{A} \in \mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}(\wp)$, *then* $\wp \in \mathscr{A}$.
(iii) *If* $\mathscr{A} \in \mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}(\wp)$ *and* $\mathscr{A}$ *is a subset of* $\mathscr{B}$, *then* $\mathscr{B} \in \mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}(\wp)$.
(iv) *If* $\mathscr{A} \in \mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}(\wp)$ *then* $\exists\, \mathscr{B} \in \mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}(\wp)$ *such that* $\mathscr{B}$ *is the subset of* $\mathscr{A}$ *and* $\mathscr{B} \in \mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}(\ell)$ *for every* $\ell \in \mathscr{B}$.

Proof.
(i) Given that $\wp \in \mathtt{U}$, then it follows that $\mathtt{U}$ is the $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. set containing the point $\wp$. Since every $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. set is $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$ for each of its points, we have $\mathtt{U}$ is the $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$ of $\wp$. Thus, $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}(\wp) \neq \emptyset$ for each $\wp \in \mathtt{U}$.
(ii) Considering $\mathscr{A} \in \mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}(\wp)$ implies $\exists$ a $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. set $\mathscr{B}$ containing the point $\wp$ such that $\wp \in \mathscr{B} \subseteq \mathscr{A}$. Hence, $\wp \in \mathscr{A}$.
(iii) Since $\mathscr{A} \in \mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}(\wp)$, then $\exists$ a $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. set $\mathtt{V}$ containing the point $\wp$ such that $\wp \in \mathtt{V} \subseteq \mathscr{A}$. Since $\mathscr{A}$ is the subset of $\mathscr{B}$, we have $\wp \in \mathtt{V} \subseteq \mathscr{A} \subseteq \mathscr{B}$ which further implies that $\wp \in \mathtt{V} \subseteq \mathscr{B}$. Therefore, by Definition 4.1, $\mathscr{B}$ is the $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$ of the point $\wp \in \mathtt{U}$ (i.e.,) $\mathscr{B} \in \mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}(\wp)$.
(iv) Let $\mathscr{A} \in \mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}(\wp)$. By Definition4.1 $\exists$ a $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. set $\mathscr{B}$ containing the point $\wp$ such that $\wp \in \mathscr{B} \subseteq \mathscr{A}$. Since $\mathscr{B}$ is $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op., $\mathscr{B}$ is $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$ of the point $\wp$. Since every $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. set is $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$ of each of its points, we have $\mathscr{B} \in \mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}(\ell)$ for every $\ell \in \mathscr{B}$. □

5. Micro Generalized Star Semi Interior

In this part, we discuss the properties of $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.

Definition 5.1. The subset $\mathscr{C}$ of the $\mathtt{U}$ is called $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Clo.Op. if it is both $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Clo. and $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op.

Theorem 5.2. *Let* $\mathtt{U}$ *be an* $\mathcal{M}$.T.S. *and let* $\mathscr{C}$ *be a subset of* $\mathtt{U}$. *Then the following conditions hold.*

(i) $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\emptyset) = \emptyset$
(ii) $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathtt{U}) = \mathtt{U}$
(iii) *If* $\mathscr{C}$ *is* $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. *then* $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathscr{C}) = \mathscr{C}$

Proof. (i) The largest $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. subset of $\emptyset$ is $\emptyset$. Therefore by Definition3.14 $\mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\emptyset) = \emptyset$.

(ii) The largest $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. subset of $\mathtt{U}$ is $\mathtt{U}$. Therefore by Definition3.14 $\mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathtt{U}) = \mathtt{U}$.

(iii) Since $\mathscr{C}$ is $\mathcal{M}$-$g^\star\mathfrak{S}$.Op., the union of all $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. subsets of $\mathscr{C}$ is $\mathscr{C}$. Therefore by Definition3.14, $\mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{C}) = \mathscr{C}$. □

Theorem 5.3. *Let $\mathscr{C}$ be a subset in the $\mathcal{M}$.*T.S. $\mathtt{U}$. *A point $\wp \in \mathtt{U}$ is in the $\mathcal{M}$-$g^\star\mathfrak{S}$.*Intr.$(\mathscr{C})$ *iff $\exists$ a $\mathcal{M}$-$g^\star\mathfrak{S}$.*Op. *set $\mathscr{O}$ containing $\wp$ such that $\mathscr{O} \subseteq \mathscr{C}$.*

Proof. Assume that the point $\wp \in \mathtt{U}$ is in the $\mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{C})$. By the definition of $\mathcal{M}$-$g^\star\mathfrak{S}$.Intr., $\exists$ a $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. set containing $\wp$ such that $\mathscr{O} \subseteq \mathscr{C}$. Conversely, Suppose that $\exists$ a $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. set $\mathscr{O}$ containing $\wp$ such that $\mathscr{O} \subseteq \mathscr{C}$. Then we have $\wp \in \mathscr{O} \subseteq \mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{C})$. Hence $\wp \in \mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{C})$. □

Theorem 5.4. *Let $\mathtt{U}$ be a $\mathcal{M}$.*T.S. *containing the subsets $\mathscr{C}$ and $\mathscr{D}$. Then $\mathscr{C} \subseteq \mathscr{D} \implies \mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{C}) \subseteq \mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{D})$.*

Proof. Let $\wp$ be a point in $\mathtt{U}$ and $\wp \in \mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{C})$. Therefore by Theorem 5.3, $\exists$ a $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. set $\mathscr{O}$ contained in $\mathscr{C}$ such that $\wp \in \mathscr{O} \subseteq \mathscr{C}$. From hypothesis, we obtain $\wp \in \mathscr{O} \subseteq \mathscr{C} \subseteq \mathscr{D}$. Hence, $\wp \in \mathscr{O} \subseteq \mathscr{D}$. Again by Theorem 5.3, we have $\wp \in \mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{D})$. Thus, $\mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{C}) \subseteq \mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{D})$. □

Remark 5.5. The equality in Theorem 5.4 holds when $\mathscr{C}$ is $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. and $\mathscr{D}$ is $\mathcal{M}$-$g^\star\mathfrak{S}$.Clo.

Example 5.6. Let $\mathtt{U} = \{\varpi, \varrho, \varsigma, \varphi\}$, $\mathtt{U}/R = \{\{\varpi\}, \{\varrho, \varsigma, \varphi\}\}$, $\mathfrak{X} = \{\varrho\}$. $\tau_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varrho, \varsigma, \varphi\}\}$, $\mu = \{\varrho\}$,

$$\mu_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varrho\}, \{\varrho, \varsigma, \varphi\}\},$$

$$(\mu_R(\mathfrak{X}))^C = \{\mathtt{U}, \emptyset, \{\varpi\}, \{\varpi, \varsigma, \varphi\}\},$$

$$\mathcal{M}\text{-}g^\star\mathfrak{S}.\text{Op.} = \Big\{\mathtt{U}, \emptyset, \{\varrho\}, \{\varsigma\}, \{\varphi\}, \{\varrho, \varphi\}, \{\varrho, \varsigma\}, \{\varsigma, \varphi\}, \{\varpi, \varrho\}, \{\varrho, \varsigma, \varphi\}, \{\varpi, \varrho, \varphi\}, \{\varpi, \varrho, \varsigma\}\Big\}.$$

Let $\mathscr{C} = \{\varsigma, \varphi\}$ and $\mathscr{D} = \{\varpi, \varsigma, \varphi\}$, then
$\mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{C}) = \mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{D}) = \{\varsigma, \varphi\}$.

From the above example, we have $\mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{C}) = \mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{D})$ but $\mathscr{C} \neq \mathscr{D}$.

Theorem 5.7. *Let $\mathtt{U}$ be a $\mathcal{M}$.*T.S. *and let $\mathscr{C}$ and $\mathscr{D}$ be the subsets of $\mathtt{U}$. Then the following statements are true.*

(i) $\mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathtt{U} - \mathscr{C}) \subseteq \mathtt{U} - [\mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{C})]$

(ii) $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathscr{C} - \mathscr{D}) \subseteq \mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathscr{C}) - \mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathscr{D})$

Proof. (i) Let $\wp$ be a point in $\mathtt{U}$ such that $\wp \in \mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathtt{U} - \mathscr{C})$. We know that $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathscr{A}) \subseteq \mathscr{A}$, then $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathtt{U} - \mathscr{C}) \subseteq \mathtt{U} - \mathscr{C}$. Therefore $\wp \in \mathtt{U} - \mathscr{C}$. (i.e.,) $\wp \in \mathtt{U}$ and $\wp \notin \mathscr{C}$. Also from $\wp \notin \mathscr{C}$, we obtain $\wp \notin \mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathscr{C})$. Thus $\wp \in \mathtt{U}$ and $\wp \notin \mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathscr{C})$ implies $\wp \in \mathtt{U} - [\mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathscr{C})]$. Therefore $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathtt{U} - \mathscr{C}) \subseteq \mathtt{U} - [\mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathscr{C})]$

(ii) By using the fact of set theory, we have $\mathscr{C} - \mathscr{D} = \mathscr{C} \cap (\mathtt{U} - \mathscr{D})$. Therefore $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathscr{C} - \mathscr{D}) = \mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathscr{C} \cap (\mathtt{U} - \mathscr{D}))$, then $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathscr{C} - \mathscr{D} = \mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathscr{C}) \cap \mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathtt{U} - \mathscr{D})$. By using Theorem 5.7 (1), we have $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathscr{C} - \mathscr{D}) \subseteq \mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathscr{C}) \cap [\mathtt{U} - \mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathscr{D})]$. Therefore,

$$\mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{C} - \mathscr{D}) \subseteq \mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{C}) - \mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{D}). \qquad \square$$

Remark 5.8. The equality condition in Theorem 5.7 (i) holds when the subset $\mathscr{C}$ is $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Clo.Op.

Example 5.9. Let $\mathtt{U} = \{\varpi, \varrho, \varsigma, \varphi\}$, $\mathtt{U}/R = \{\{\varpi, \varrho\}, \{\varsigma, \varphi\}\}$, $\mathfrak{X} = \{\varsigma\}$, $\tau_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varsigma, \varphi\}\}$, $\mu = \{\varpi\}$.

$\mu_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varpi\}, \{\varsigma, \varphi\}, \{\varpi, \varsigma, \varphi\}\}$,

$(\mu_R(\mathfrak{X}))^C = \{\mathtt{U}, \emptyset, \{\varrho\}, \{\varpi, \varrho\}, \{\varrho, \varsigma, \varphi\}\}$,

$\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. $= \Big\{\mathtt{U}, \emptyset, \{\varpi\}, \{\varsigma\}, \{\varphi\}, \{\varpi, \varrho\}, \{\varpi, \varsigma\}, \{\varpi, \varphi\}, \{\varsigma, \varphi\}, \{\varpi, \varsigma, \varphi\}, \{\varrho, \varsigma, \varphi\}\Big\}$,

$\mathcal{M}$-$g^{\star}\mathfrak{S}$.Clo. $= \Big\{\mathtt{U}, \emptyset, \{\varpi\}, \{\varrho\}, \{\varpi, \varrho\}, \{\varrho, \varsigma\}, \{\varrho, \varphi\}, \{\varsigma, \varphi\}, \{\varpi, \varrho, \varphi\}, \{\varpi, \varrho, \varsigma\}, \{\varrho, \varsigma, \varphi\}\Big\}$,

$\mathcal{M}$-$g^{\star}\mathfrak{S}$.Clo.Op. $= \{\mathtt{U}, \emptyset, \{\varpi\}, \{\varpi, \varrho\}, \{\varsigma, \varphi\}, \{\varrho, \varsigma, \varphi\}\}$.

For every subset $\mathscr{C}$ in $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Clo.Op., we have
$\mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr. $(\mathtt{U} - \mathscr{C}) = \mathtt{U} - [\mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr. $(\mathscr{C})]$

Theorem 5.10. *Let* $\mathtt{U}$ *be a* $\mathcal{M}$.T.S. *and* $\mathscr{C}$ *is a subset of* $\mathtt{U}$. *Then the following statements are true.*

(i) $\mathcal{M}$-Intr.$(\mathscr{C}) \subseteq \mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathscr{C})$
(ii) $\mathcal{M}$-$\mathfrak{S}$.Intr.$(\mathscr{C}) \subseteq \mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathscr{C})$
(iii) $\mathcal{M}$-A.Intr.$(\mathscr{C}) \subseteq \mathcal{M}$-$g^{\star}\mathfrak{S}$.Intr.$(\mathscr{C})$

Proof. (i) Let $\wp$ be a point in $\mathtt{U}$ for which $\wp \in \mathcal{M}$-Intr.$(\mathscr{C})$. Then $\wp \in \mathscr{D} \subseteq \mathscr{C}$, where $\mathscr{D}$ is the $\mathcal{M}$-Op. subset of $\mathtt{U}$. Since every $\mathcal{M}$-Op. is $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op., $\mathscr{D}$ is $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. in $\mathtt{U}$. Therefore $\wp \in \mathscr{D} \subseteq \mathscr{C}$, where $\mathscr{D}$ is the $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. subset of $\mathtt{U}$. Therefore

by Theorem 5.3, $\wp \in \mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{C})$. Hence, $\mathcal{M}$-Intr.$(\mathscr{C}) \subseteq \mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{C})$.

(ii) Let $\wp$ be a point in $\mathtt{U}$ where $\wp \in \mathcal{M}$-$\mathfrak{S}$.Intr.$(\mathscr{C})$. Then $\wp \in \mathscr{D} \subseteq \mathscr{C}$, where $\mathscr{D}$ is the $\mathcal{M}$-$\mathfrak{S}$.Op. subset of $\mathtt{U}$. Since every $\mathcal{M}$-$\mathfrak{S}$.Op. is $\mathcal{M}$-$g^\star\mathfrak{S}$.Op., $\mathscr{D}$ is $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. in $\mathtt{U}$. Therefore $\wp \in \mathscr{D} \subseteq \mathscr{C}$, where $\mathscr{D}$ is the $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. subset of $\mathtt{U}$. Therefore by Theorem 5.3, $\wp \in \mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{C})$. Hence, $\mathcal{M}$-$\mathfrak{S}$.Intr.$(\mathscr{C}) \subseteq \mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{C})$.

(iii) Let $\wp$ be a point in $\mathtt{U}$ satisfying $\wp \in \mathcal{M}$-A.Intr.$(\mathscr{C})$. Then $\wp \in \mathscr{D} \subseteq \mathscr{C}$, where $\mathscr{D}$ is the $\mathcal{M}$-A.Op. subset of $\mathtt{U}$. Since every $\mathcal{M}$-A.Op. set is $\mathcal{M}$-$g^\star\mathfrak{S}$.Op., $\mathscr{D}$ is $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. in $\mathtt{U}$. Therefore $\wp \in \mathscr{D} \subseteq \mathscr{C}$, where $\mathscr{D}$ is the $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. subset of $\mathtt{U}$. Therefore by Theorem 5.3, $\wp \in \mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{C})$. Hence, $\mathcal{M}$-A.Intr.$(\mathscr{C}) \subseteq \mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{C})$. □

Remark 5.11. (i) The reverse inequality in the statements of Theorem 5.10 does not hold which is shown in the following example.

(ii) The equality in Theorem 5.10 ((i), (ii), (iii)) holds when $\mathscr{C}$ is $\mathcal{M}$-Op., $\mathcal{M}$-$\mathfrak{S}$.Op. and $\mathcal{M}$-A.Op. respectively.

Example 5.12. (i) Let $\mathtt{U} = \{\varpi, \varrho, \varsigma, \varphi\}$, $\mathtt{U}/R = \{\{\varpi, \varrho\}, \{\varsigma, \varphi\}\}$, $\mathfrak{X} = \{\varpi\}$, $\tau_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varpi, \varrho\}\}$, $\mu = \{\varpi\}$.

$$\mu_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varpi\}, \{\varpi, \varrho\}\},$$
$$(\mu_R(\mathfrak{X}))^C = \{\mathtt{U}, \emptyset, \{\varsigma, \varphi\}, \{\varrho, \varsigma, \varphi\}\},$$
$$\mathcal{M}\text{-}g^\star\mathfrak{S}.\text{Op.} = \Big\{\mathtt{U}, \emptyset, \{\varpi\}, \{\varrho\}, \{\varphi\}, \{\varpi, \varrho\}, \{\varpi, \varsigma\}, \{\varpi, \varphi\}, \{\varsigma, \varphi\}, \{\varpi, \varrho, \varsigma\}, \{\varpi, \varrho, \varphi\}, \{\varpi, \varsigma, \varphi\}\Big\}.$$

Let $\mathscr{C} = \{\varrho, \varphi\}$, $\mathcal{M}$-Intr.$(\mathscr{C}) = \mathcal{M}$-$\mathfrak{S}$.Intr.$(\mathscr{C}) = \mathcal{M}$-A.Intr.$(\mathscr{C}) = \{\emptyset\}$, $\mathcal{M}$-$g^\star\mathfrak{S}$.Intr.$(\mathscr{C}) = \{\varrho\}$. As a result,

$$[\mathcal{M}\text{-Intr.}(\mathscr{C}), \mathcal{M}\text{-}\mathfrak{S}\text{.Intr.}(\mathscr{C}), \mathcal{M}\text{-A.Intr.}(\mathscr{C})] \not\supseteq \mathcal{M}\text{-}g^\star\mathfrak{S}\text{.Intr.}(\mathscr{C}).$$

(ii) Let $\mathtt{U} = \{\varpi, \varrho, \varsigma, \varphi\}$, $\mathtt{U}/R = \{\{\varpi, \varphi\}, \{\varrho, \varsigma\}\}$, $\mathfrak{X} = \{\varrho\}$, $\tau_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varrho, \varsigma\}\}$, $\mu = \{\varpi\}$.

$$\mu_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varpi\}, \{\varrho, \varsigma\}, \{\varpi, \varrho, \varsigma\}\},$$
$$(\mu_R(\mathfrak{X}))^C = \{\mathtt{U}, \emptyset, \{\varphi\}, \{\varpi, \varphi\}, \{\varrho, \varsigma, \varphi\}\},$$
$$\mathcal{M}\text{-}g^\star\mathfrak{S}.\text{Op.} = \Big\{\mathtt{U}, \emptyset, \{\varpi\}, \{\varrho\}, \{\varsigma\}, \{\varpi, \varrho\}, \{\varpi, \varsigma\}, \{\varrho, \varsigma\}, \{\varpi, \varrho, \varsigma\}, \{\varrho, \varsigma, \varphi\}\Big\}.$$

Let $\mathscr{C} = \{\varrho, \varsigma\}$ which is $\mathcal{M}$-Op., $\mathcal{M}$-$\mathfrak{S}$.Op., $\mathcal{M}$-A.Op, then the equality holds. i.e.,

$$\mathcal{M}\text{-Intr.}(\mathscr{C}) = \mathcal{M}\text{-}\mathfrak{S}\text{.Intr.}(\mathscr{C}) = \mathcal{M}\text{-A.Intr.}(\mathscr{C}) = \mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{C}).$$

Theorem 5.13. *For any two subsets $\mathscr{C}$ and $\mathscr{D}$ in a $\mathcal{M}$.T.S., $\mathtt{U}$, the following condition holds.*

(i) $\mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{C}) \cup \mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{D}) \subseteq \mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{C} \cup \mathscr{D})$.
(ii) $\mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{C}) \cap \mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{D}) \supseteq \mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{C} \cap \mathscr{D})$.

Proof. (i) By the property of set theory, we have $\mathscr{C} \subseteq \mathscr{C} \cup \mathscr{D}$. Therefore by using Theorem 5.4,

$$\mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{C}) \subseteq \mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{C} \cup \mathscr{D}).$$

Similarly, $\mathscr{D} \subseteq \mathscr{C} \cup \mathscr{D}$ shows that

$$\mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{D}) \subseteq \mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{C} \cup \mathscr{D}).$$

Therefore, the union of the above inclusions results that,

$$\mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{C}) \cup \mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{D}) \subseteq \mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{C} \cup \mathscr{D}).$$

(ii) By the property of set theory, we have

$$\mathscr{C} \cap \mathscr{D} \subseteq \mathscr{C}.$$

Therefore using Theorem 5.4, we have $\mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{C} \cap \mathscr{D}) \subseteq \mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{C})$. Similarly, $\mathscr{C} \cap \mathscr{D} \subseteq \mathscr{D}$ implies

$$\mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{C} \cap \mathscr{D}) \subseteq \mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{D}).$$

Therefore $\mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{C} \cap \mathscr{D})$ is $\subseteq$ in both $\mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{C})$ and $\mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{D})$. Hence,
$\mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{C}) \cap \mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{D}) \supseteq \mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Intr.}(\mathscr{C} \cap \mathscr{D})$. □

6. Micro Generalized Star Semi Closure

In this section, we discuss some basic properties of $\mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Clr.}$

Theorem 6.1. *Let $\mathtt{U}$ be an $\mathcal{M}$.T.S. and let $\mathscr{C}$ is a subset of $\mathtt{U}$. Then the following condition holds.*

(i) $\mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Clr.}(\emptyset) = \emptyset$
(ii) $\mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Clr.}(\mathtt{U}) = \mathtt{U}$
(iii) *If $\mathscr{C}$ is $\mathcal{M}\text{-}g^{\star}\mathfrak{S}$.Clo. then $\mathscr{C} = \mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Clr.}(\mathscr{C})$*

Proof. (i) The smallest $\mathcal{M}\text{-}g^{\star}\mathfrak{S}$.Clo. subset $\supseteq \emptyset$ is $\emptyset$. Therefore by Definition3.15 $\mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Clr.}(\emptyset) = \emptyset$.
(ii) The smallest $\mathcal{M}\text{-}g^{\star}\mathfrak{S}$.Clo. subset $\supseteq \mathtt{U}$ is $\mathtt{U}$. Therefore by Definition3.15 $\mathcal{M}\text{-}g^{\star}\mathfrak{S}\text{.Clr.}(\mathtt{U}) = \mathtt{U}$.

(iii) Since $\mathscr{C}$ is $\mathcal{M}$-$g^\star\mathfrak{S}$.Clo., the intersection of all $\mathcal{M}$-$g^\star\mathfrak{S}$.Clo. sets containing $\mathscr{C} = \mathscr{C}$. Therefore by Definition 3.15,

$$\mathcal{M}\text{-}g^\star\mathfrak{S}.\mathrm{Clr}.(\mathscr{C}) = \mathscr{C}. \qquad \square$$

Theorem 6.2. *Let* $\mathtt{U}$ *be a* $\mathcal{M}$.T.S. *&* $\mathscr{C} \subseteq \mathtt{U}$*, then* $\mathscr{C} \subseteq \mathcal{M}$-$g^\star\mathfrak{S}$.Clr.$(\mathscr{C}) \subseteq \mathcal{M}$-Clr.$(\mathscr{C})$.

Proof. By the Definition3.15, $\mathcal{M}$-$g^\star\mathfrak{S}$.Clr.$(\mathscr{C})$ is the intersection of all $\mathcal{M}$-$g^\star\mathfrak{S}$.Clo. subsets containing $\mathscr{C}$. Therefore $\mathscr{C} \subseteq \mathcal{M}$-$g^\star\mathfrak{S}$.Clr.$(\mathscr{C})$. Since every $\mathcal{M}$-Clo. set is $\mathcal{M}$-$g^\star\mathfrak{S}$.Clo., we have $\mathcal{M}$-$g^\star\mathfrak{S}$.Clr.$(\mathscr{C}) \subseteq \mathcal{M}$-Clr.$(\mathscr{C})$. Therefore $\mathscr{C} \subseteq \mathcal{M}$-$g^\star\mathfrak{S}$.Clr.$(\mathscr{C}) \subseteq \mathcal{M}$-Clr. $\square$

Theorem 6.3. *Let* $\mathscr{C}$ *be a subset of the* $\mathcal{M}$.T.S.,$\mathtt{U}$ *and* $\wp$ *be a point in* $\mathtt{U}$ *satisfying* $\wp \in \mathcal{M}$-$g^\star\mathfrak{S}$.Clr.$(\mathscr{C})$ *iff for every* $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. *set* $\mathscr{D}$ *containing '*$\wp$*', such that* $\mathscr{D} \cap \mathscr{C} \neq \emptyset$.

Proof. **Necessary Part:** Assume that $\wp \in \mathcal{M}$-$g^\star\mathfrak{S}$.Clr.$(\mathscr{C})$. We claim that for every $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. set $\mathscr{D}$ containing '$\wp$', $\mathscr{D} \cap \mathscr{C} \neq \emptyset$. Suppose that $\exists$ a $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. set $\mathscr{D}$ containing '$\wp$' such that $\mathscr{D} \cap \mathscr{C} = \emptyset$. Since $\mathscr{D}$ is $\mathcal{M}$-$g^\star\mathfrak{S}$.Op., $\mathtt{U} - \mathscr{D}$ is $\mathcal{M}$-$g^\star\mathfrak{S}$.Clo. $\mathscr{D} \cap \mathscr{C} = \emptyset$ results that $\mathscr{C} \subseteq \mathtt{U} - \mathscr{D}$. Consequently, $\mathcal{M}$-$g^\star\mathfrak{S}$.Clr.$(\mathscr{C}) \subseteq \mathcal{M}$-$g^\star\mathfrak{S}$.Clr.$(\mathtt{U} - \mathscr{D}) = \mathtt{U} - \mathscr{D}$. Since $\wp \in \mathscr{D}$ and $\mathcal{M}$-$g^\star\mathfrak{S}$.Clr.$(\mathscr{C}) \subseteq \mathtt{U} - \mathscr{D}$, we have $\wp \notin \mathcal{M}$-$g^\star\mathfrak{S}$.Clr.$(\mathscr{C})$ which is a contradiction. Hence, for every $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. set $\mathscr{D}$ containing '$\wp$', $\mathscr{D} \cap \mathscr{C} \neq \emptyset$.

Sufficient Part: Conversely, assume that for every $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. set $\mathscr{D}$ containing '$\wp$', $\mathscr{D} \cap \mathscr{C} \neq \emptyset$. We claim that $\wp \in \mathcal{M}$-$g^\star\mathfrak{S}$.Clr.$(\mathscr{C})$. Suppose that $\wp \notin \mathcal{M}$-$g^\star\mathfrak{S}$.Clr.$(\mathscr{C})$, $\exists$ a $\mathcal{M}$-$g^\star\mathfrak{S}$.Clo. set $\mathscr{D}$ for which $\wp \notin \mathscr{D}$. Then $\wp \in \mathtt{U} - \mathscr{D}$ which is a $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. set in $\mathtt{U}$. From our assumption, $(\mathtt{U} - \mathscr{D}) \cap \mathscr{C} \neq \emptyset$ which is again a contradiction. Therefore $\wp \in \mathcal{M}$-$g^\star\mathfrak{S}$.Clr.$(\mathscr{C})$. $\square$

Theorem 6.4. *Let* $\mathtt{U}$ *be a* $\mathcal{M}$.T.S. *containing the subsets* $\mathscr{C}$ *and* $\mathscr{D}$. *Then* $\mathscr{C} \subseteq \mathscr{D} \implies \mathcal{M}$-$g^\star\mathfrak{S}$.Clr.$(\mathscr{C}) \subseteq \mathcal{M}$-$g^\star\mathfrak{S}$.Clr.$(\mathscr{D})$.

Proof. The proof of the above theorem is similar to Theorem 5.4. $\square$

Remark 6.5. The equality in the Theorem 6.4 holds when $\mathscr{C}$ is $\mathcal{M}$-$g^\star\mathfrak{S}$.Op. and $\mathscr{D}$ is $\mathcal{M}$-$g^\star\mathfrak{S}$.Clo.

Example 6.6. Let $\mathtt{U} = \{\varpi, \varrho, \varsigma, \varphi\}$, $\mathtt{U}/R = \{\{\varpi, \varsigma\}, \{\varrho, \varphi\}\}$, $\mathfrak{X} = \{\varpi, \varsigma\}$, $\tau_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varpi, \varsigma\}\}$, $\mu = \{\varpi, \varrho, \varsigma\}$.

$\mu_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varpi, \varsigma\}, \{\varpi, \varrho, \varsigma\}\}$,

$(\mu_R(\mathfrak{X}))^C = \{\mathtt{U}, \emptyset, \{\varphi\}, \{\varrho, \varphi\}\}$,

$\mathcal{M}$-$g^\star\mathfrak{S}$.Clo. $= \Big\{\mathtt{U}, \emptyset, \{\varrho\}, \{\varphi\}, \{\varpi, \varphi\}, \{\varrho, \varphi\}, \{\varsigma, \varphi\}, \{\varpi, \varrho, \varphi\}, \{\varrho, \varsigma, \varphi\},$

$$\{\varpi, \varsigma, \varphi\}\Big\},$$

$$\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Op.} = \Big\{\mathtt{U}, \emptyset, \{\varpi\}, \{\varrho\}, \{\varsigma\}, \{\varphi\}, \{\varpi, \varrho\}, \{\varrho, \varsigma\}, \{\varpi, \varphi\}, \{\varrho, \varphi\},$$

$$\{\varsigma, \varphi\}, \{\varrho, \varsigma, \varphi\}, \{\varpi, \varrho, \varphi\}\Big\}.$$

Let $\mathscr{C} = \{\varsigma\}$ and $\mathscr{D} = \{\varsigma, \varphi\}$. Then,

$$\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Clr.}(\mathscr{C}) = \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Clr.}(\mathscr{D}) = \{\varsigma, \varphi\}.$$

From the above example, $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Clr.}(\mathscr{C}) = \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Clr.}(\mathscr{D})$. But $\mathscr{C} \neq \mathscr{D}$.

Theorem 6.7. *For any two subsets $\mathscr{C}$ and $\mathscr{D}$ in a $\mathcal{M}$.T.S., $\mathtt{U}$, the following condition holds.*

(i) $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Clr.}(\mathscr{C}) \cup \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Clr.}(\mathscr{D}) \subseteq \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Clr.}(\mathscr{C} \cup \mathscr{D})$.
(ii) $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Clr.}(\mathscr{C}) \cap \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Clr.}(\mathscr{D}) \supseteq \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Clr.}(\mathscr{C} \cap \mathscr{D})$.

Proof. The proof of the above theorem is similar to Theorem 5.13. □

Theorem 6.8. *Let $\mathtt{U}$ be an $\mathcal{M}$.T.S. and let $\mathscr{C}$ is a subset of $\mathtt{U}$, then the following condition holds.*

(i) $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Intr.}(\mathscr{C}) \subseteq \mathscr{C} \subseteq \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Clr.}(\mathscr{C})$.
(ii) $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Clr.}(\mathtt{U} - \mathscr{C}) = \mathtt{U} - \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Intr.}(\mathscr{C})$.
(iii) $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Intr.}(\mathtt{U} - \mathscr{C}) = \mathtt{U} - \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Clr.}(\mathscr{C})$.

Proof. (i) From Definition 3.14, $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Intr.}(\mathscr{C})$ is the union of all $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Op.}$ subsets $\subseteq \mathscr{C}$. Therefore $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Intr.}(\mathscr{C}) \subseteq \mathscr{C}$. Also, from Definition3.15, $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Clr.}(\mathscr{C})$ is the intersection of all $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Clo.}$ subsets $\supseteq \mathscr{C}$. Therefore $\mathscr{C} \subseteq \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Clr.}(\mathscr{C})$. Therefore, $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Intr.}(\mathscr{C}) \subseteq \mathscr{C} \subseteq \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Clr.}(\mathscr{C})$.

(ii) Assume that $\wp \in \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Clr.}(\mathtt{U} - \mathscr{C})$. Then $\wp$ belongs to every $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Clo.}$ set containing $(\mathtt{U} - \mathscr{C})$. Therefore $\wp \notin \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Op.}$ set which results that $\wp \notin \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Intr.}(\mathtt{U} - \mathscr{C})$. Therefore, $\wp \in \mathtt{U} - \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Intr.}(\mathscr{C})$.
Conversely, suppose that $\wp \in \mathtt{U} - \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Intr.}(\mathscr{C})$, then $\wp \notin \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Intr.}(\mathscr{C})$. Therefore none of the $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Op.}$ subset of $\mathscr{C}$ contains the point $\wp$. Let $\mathscr{D}$ be a $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Clo.}$ set containing $(\mathtt{U} - \mathscr{C})$. Then $(\mathtt{U} - \mathscr{D})$ is a $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Op.}$ set contained in $\mathscr{C}$. Therefore $\wp \notin (\mathtt{U} - \mathscr{D})$ which shows that $\wp \in \mathscr{D}$. Hence, $\wp \in \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\text{Clr.}(\mathtt{U} - \mathscr{C})$.

(iii) The proof of this result is obtained by replacing $\mathscr{C}$ by $(\mathtt{U} - \mathscr{C})$ in proof of Theorem 6.8 (ii). □

7. Micro Generalized Star Semi Limit points and Derived Sets

The notions of $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{lmt.pts}$. and $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS are introduced and its basic properties are explored in this part.

Definition 7.1. Let $\mathtt{U}$ be a $\mathcal{M}$.T.S. The point $\wp \in \mathtt{U}$ is said to be $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{lmt.pt}$. of the subset $\mathscr{C} \subseteq \mathtt{U}$ iff for every $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{nbhd}$ of $\wp$ intersects $\mathscr{C}$ at some point other than $\wp$. i.e., for each $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{nbhd}$ $\mathscr{D}$ of $\wp$, we have $\mathscr{D} \cap (\mathscr{C} - \{\wp\}) \neq \emptyset$.

Definition 7.2. The set of all $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{lmt.pts}$. of the subset $\mathscr{C}$ in $\mathtt{U}$ is called $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C}$).

Example 7.3. Let $\mathtt{U} = \{\varpi, \varrho, \varsigma, \varphi\}$, $\mathtt{U}/R = \{\{\varpi, \varrho\}, \{\varsigma, \varphi\}\}$, $\mathfrak{X} = \{\varsigma\}$, $\tau_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varsigma, \varphi\}\}$, $\mu = \{\varpi\}$.

$\mu_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varpi\}, \{\varsigma, \varphi\}, \{\varpi, \varsigma, \varphi\}\}$,

$(\mu_R(\mathfrak{X}))^C = \{\mathtt{U}, \emptyset, \{\varrho\}, \{\varpi, \varrho\}, \{\varrho, \varsigma, \varphi\}\}$,

$\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. $= \Big\{\mathtt{U}, \emptyset, \{\varpi\}, \{\varsigma\}, \{\varphi\}, \{\varpi, \varrho\}, \{\varpi, \varsigma\}, \{\varpi, \varphi\}, \{\varsigma, \varphi\}, \{\varpi, \varsigma, \varphi\}, \{\varrho, \varsigma, \varphi\}\Big\}$.

Let $\mathscr{C} = \{\varpi, \varrho, \varphi\}$, the $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{lmt.pt}$. of $\mathscr{C}$ is $\{\varrho\}$. $\because \{\varrho\}$ is the only $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{lmt.pt}$. of $\mathscr{C}$, $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C}$) $= \{\varrho\}$.

Theorem 7.4. *Every* $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{lmt.pt}$. *is* $\mathcal{M}$-$\mathfrak{lmt.pt}$.

Proof. Let $\mathtt{U}$ be a $\mathcal{M}$.T.S. and $\mathscr{C} \subseteq \mathtt{U}$. Assume that $\wp$ is a $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{lmt.pt}$. of $\mathscr{C}$. By the Definition7.1, $\exists$ a $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{nbhd}$ $\mathscr{D}$ for which $\mathscr{D} \cap (\mathscr{C} - \{\wp\}) \neq \emptyset$. Since every $\mathcal{M}$.$\mathfrak{nbhd}$ is $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{nbhd}$ (by Theorem 4.4 (i)), $\exists$ a $\mathcal{M}$.$\mathfrak{nbhd}$, $\mathscr{D}$ containing a point $\wp$ satisfying $\mathscr{D} \cap (\mathscr{C} - \{\wp\}) \neq \emptyset$. Therefore, $\wp$ is the $\mathcal{M}$-$\mathfrak{lmt.pt}$. of the subset $\mathscr{C} \in \mathtt{U}$. □

Remark 7.5. The converse of the Theorem 7.4 is not true, as shown in the following example.

Example 7.6. Let $\mathtt{U} = \{\varpi, \varrho, \varsigma, \varphi\}$, $\mathtt{U}/R = \{\{\varpi\}, \{\varrho\}, \{\varsigma\}, \{\varphi\}\}$, $\mathfrak{X} = \{\varrho, \varsigma\}$, $\tau_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varrho, \varsigma\}\}$, $\mu = \{\varsigma, \varphi\}$.

$\mu_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varsigma\}, \{\varrho, \varsigma\}, \{\varsigma, \varphi\}, \{\varrho, \varsigma, \varphi\}\}$,

$(\mu_R(\mathfrak{X}))^C = \{\mathtt{U}, \emptyset, \{\varpi\}, \{\varpi, \varrho\}, \{\varpi, \varphi\}, \{\varpi, \varrho, \varphi\}\}$,

$\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. $= \Big\{\mathtt{U}, \emptyset, \{\varpi\}, \{\varrho\}, \{\varphi\}, \{\varpi, \varrho\}, \{\varpi, \varphi\}, \{\varrho, \varphi\}, \{\varpi, \varsigma\}, \{\varpi, \varrho, \varphi\}, \{\varpi, \varsigma, \varphi\}, \{\varpi, \varrho, \varsigma\}\Big\}$.

Let $\mathscr{C} = \{\varpi, \varsigma\}$, $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C}$) $= \{\varpi\}$, $\mathcal{M}$-DS($\mathscr{C}$) $= \{\varpi, \varrho, \varphi\}$. Therefore, $\{\varrho, \varphi\}$ is $\mathcal{M}$-$\mathfrak{lmt.pt}$. of $\mathscr{C}$ but not $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{lmt.pt}$. of $\mathscr{C}$.

From the above, we have $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C}$) $\subseteq$ $\mathcal{M}$-DS($\mathscr{C}$).

Theorem 7.7. *The subset $\mathscr{C}$ of the $\mathcal{M}$.T.S., $\mathtt{U}$ is $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Clo. iff it contains all of its $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{lmt.pt}$.*

Proof. **Necessary Part**: Assume that $\mathscr{C}$ is $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Clo. set of $\mathtt{U}$. We claim that $\mathscr{C}$ contains all of its $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{lmt.pt}$. Since $\mathscr{C}$ is $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Clo., $\mathtt{U}-\mathscr{C}$ is $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. in $\mathtt{U}$. Let $\wp$ be a point in $\mathtt{U}-\mathscr{C}$. Since $\mathtt{U}-\mathscr{C}$ is open, by using the Theorem 4.16, we see that $\mathtt{U}-\mathscr{C}$ is the $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{nbhd}$ of the point $\wp$ i.e., $\wp \in \mathtt{U}-\mathscr{C} \subseteq \mathtt{U}-\mathscr{C}$. Since $\wp \notin \mathscr{C}$, the $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{nbhd}$, $\mathtt{U}-\mathscr{C}$, of $\wp$ does not intersect $\mathscr{C}$. Then $\wp$ is not the $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{lmt.pt}$.of $\mathscr{C}$.i.e., no point of $\mathtt{U}-\mathscr{C}$ is the $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{lmt.pt}$. of $\mathscr{C}$. Therefore, $\mathscr{C}$ contains all of its $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{lmt.pt}$.
Sufficient Part: Conversely, assume that $\mathscr{C}$ contains all of its $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{lmt.pt}$.. Suppose that $\wp \notin \mathscr{C}$. i.e., $\wp$ is not the $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{lmt.pt}$.of $\mathscr{C}$. Then $\wp \in \mathtt{U}-\mathscr{C}$. Also from $\wp \notin \mathscr{C}$, we obtain $\wp \notin \mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C}$). Then By the Definition7.1, we see that the intersection of $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{nbhd}$ of $\wp$ and $\mathscr{C}$ is empty. i.e No point of $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{nbhd}$ of $\wp$ contains the point of $\mathscr{C}$. Therefore $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{nbhd}$ of $\wp$ contains the all the points of $\mathtt{U}-\mathscr{C}$. Therefore by the Theorem 4.16, we obtain that $\mathtt{U}-\mathscr{C}$ is $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Op. Hence, the subset $\mathscr{C}$ is $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Clo. □

Theorem 7.8. *Every $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS is $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Clo.*

Proof. Let $\mathscr{C}$ be the subset of the $\mathcal{M}$.T.S., $\mathtt{U}$ and let $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C}$) denotes $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS of $\mathscr{C}$. We now claim that $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C}$) is $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Clo. By using the Theorem 7.7, it is enough to show that $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C}$) contains all of its $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{lmt.pt}$.. Let $\wp$ be the $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{lmt.pt}$. of $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C}$). Then $\wp \in \mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C}$)).

By Definition7.1 $\exists$ a $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{nbhd}$, $\mathscr{D}$ of $\wp$ such that its intersection with $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C}$) contains at least one point other than $\wp$. i.e., $\mathscr{D} \cap (\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C}$) $- \{\wp\}) \neq \emptyset$. Then it follows that $\mathscr{D} \cap ((\mathscr{C}) - \{\wp\}) \neq \emptyset$ Since every $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{nbhd}$ of $\mathscr{D}$ contains at least one point of $\mathscr{C}$. Therefore, $\mathscr{D} \cap (\mathscr{C} - \{\wp\}) \neq \emptyset$ results that $\wp$ is also the $\mathfrak{lmt.pt}$. of $\mathscr{C}$. i.e., $\wp \in \mathscr{D} \subseteq \mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C}$). Hence, $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C}$)) $\subseteq$ $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C}$)). Hence, $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C}$) contains all of its $\mathcal{M}$-$g^{\star}\mathfrak{S}$.$\mathfrak{lmt.pt}$. Therefore, By Theorem 7.7, $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C}$) is $\mathcal{M}$-$g^{\star}\mathfrak{S}$.Clo. □

Theorem 7.9. *Let $\mathtt{U}$ be a $\mathcal{M}$.T.S.. For any two subsets $\mathscr{C}$ and $\mathscr{D}$ of $\mathtt{U}$, the following condition holds.*

(i) $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\emptyset$) $= \emptyset$.
(ii) *If $\mathscr{C} \subseteq \mathscr{D}$, then* $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C}$) $\subseteq$ $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{D}$).
(iii) *If $\wp \in \mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C}$), then $\wp \in \mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C} - \{\wp\}$).*
(iv) $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C}$) $\cup$ $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{D}$) $\subseteq$ $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C} \cup \mathscr{D}$).
(v) $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C}$) $\cap$ $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{D}$) $\supseteq$ $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS($\mathscr{C} \cap \mathscr{D}$).

Proof. (i) It is true that $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\emptyset) = \emptyset$ since $\exists$ no $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$ intersecting $\emptyset$.

(ii) Let $\wp \in \mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{C})$. Then by the Definition 7.1 $\exists$ a $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$, $\mathtt{V}$ of $\wp$ intersects $\mathscr{C}$ other than the point $\wp$. i.e., $\mathtt{V} \cap (\mathscr{C} - \{\wp\}) \neq \emptyset$. Given that $\mathscr{C} \subseteq \mathscr{D}$, Therefore the above inequality changes to $\mathtt{V} \cap (\mathscr{D} - \{\wp\}) \neq \emptyset$, then $\wp$ is the $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{lmt.pt}$. of $\mathscr{D}$. Hence, $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{C}) \subseteq \mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{D})$.

(iii) Let $\wp \in \mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{C})$ implies that $\wp$ is the $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{lmt.pt}$. of $\mathscr{C}$. Therefore by the Definition7.1, $\exists$ a $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{nbhd}$ of $\wp$ such that its intersection with $\wp$ is non-empty. Then $\wp$ is a $\mathcal{M}$-$g^{\star}\mathfrak{S}.\mathfrak{lmt.pt}$. of $(\mathscr{C} - \{\wp\})$. Hence, $\wp \in \mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{C} - \{\wp\})$.

(iv) By the property of set theory, we have $\mathscr{C} \subseteq \mathscr{C} \cup \mathscr{D}$. Therefore using Theorem 7.9 (ii), we have $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{C}) \subseteq \mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{C} \cup \mathscr{D})$. Similarly, from $\mathscr{D} \subseteq \mathscr{C} \cup \mathscr{D}$, we obtain $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{D}) \subseteq \mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{C} \cup \mathscr{D})$. From the above $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{C}) \cup \mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{D}) \subseteq \mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{C} \cup \mathscr{D})$ which concludes the proof.

(v) By the property of set theory, we have $\mathscr{C} \cap \mathscr{D} \subseteq \mathscr{C}$. By using Theorem7.9 (ii), we have $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{C} \cap \mathscr{D}) \subseteq \mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{C})$. Also, $\mathscr{C} \cap \mathscr{D} \subseteq \mathscr{D}$ implies $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{C} \cap \mathscr{D}) \subseteq \mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{D})$. Hence,

$$\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{DS}(\mathscr{C}) \cap \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{DS}(\mathscr{D}) \supseteq \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{DS}(\mathscr{C} \cap \mathscr{D}). \qquad \square$$

Remark 7.10. The equality $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{C}) = \mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{D})$ does not imply that $\mathscr{C} = \mathscr{D}$.

Example 7.11. Let $\mathtt{U} = \{\varpi, \varrho, \varsigma, \varphi\}$, $\mathtt{U}/R = \{\{\varpi, \varphi\}, \{\varrho, \varsigma\}\}$, $\mathfrak{X} = \{\varrho\}$, $\tau_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varrho, \varsigma\}\}$, $\mu = \{\varsigma\}$.

$$\mu_R(\mathfrak{X}) = \{\mathtt{U}, \emptyset, \{\varsigma\}, \{\varrho, \varsigma\}\},$$
$$(\mu_R(\mathfrak{X}))^C = \{\mathtt{U}, \emptyset, \{\varpi, \varphi\}, \{\varpi, \varrho, \varphi\}\},$$
$$\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{Op.} = \Big\{\mathtt{U}, \emptyset, \{\varrho\}, \{\varsigma\}, \{\varsigma, \varphi\}, \{\varpi, \varsigma\}, \{\varrho, \varsigma\}, \{\varpi, \varrho, \varsigma\}, \{\varpi, \varsigma, \varphi\}, \{\varrho, \varsigma, \varphi\}\Big\},$$
$$\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{Clo.} = \Big\{\mathtt{U}, \emptyset, \{\varpi\}, \{\varrho\}, \{\varphi\}, \{\varpi, \varrho\}, \{\varpi, \varphi\}, \{\varrho, \varphi\}, \{\varpi, \varrho, \varphi\}, \{\varpi, \varsigma, \varphi\}\Big\}.$$

Let $\mathscr{C} = \{\varrho, \varsigma\}$ and $\mathscr{D} = \{\varsigma, \varphi\}$, $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{C}) = \mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{D}) = \{\varpi, \varphi\}$. Hence $\mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{C}) = \mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{D})$ but $\mathscr{C} \neq \mathscr{D}$.

Theorem 7.12. *Let $\mathscr{C}$ be the subset of the $\mathcal{M}$.*T.S. $\mathtt{U}$. *Then $\mathscr{C} \cup \mathcal{M}$-$g^{\star}\mathfrak{S}$.DS$(\mathscr{C})$ is $\mathcal{M}$-$g^{\star}\mathfrak{S}$.*Clo.

Proof. We claim that $\mathscr{C} \cup \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{DS}(\mathscr{C})$ is $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{Clo}$. So, we prove that $(\mathscr{C} \cup \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{DS}(\mathscr{C}))^{C}$ is $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{Op}$. By the property of set theory, we see that $(\mathscr{A} \cup \mathscr{B})^{C} = \mathscr{A}^{C} \cap \mathscr{B}^{C}$. We prove that $(\mathscr{C} \cup \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{DS}(\mathscr{C}))^{C}$ $= \mathscr{C}^{C} \cap (\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{DS}(\mathscr{C}))^{C}$ is $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{Op}$. Let $\wp \in \mathscr{C}^{C} \cap (\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{DS}(\mathscr{C}))^{C}$ which implies that $\wp \in \mathscr{C}^{C}$ and $\wp \in (\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{DS}(\mathscr{C}))^{C}$. Further, from $\wp \in \mathscr{C}^{C}$, we have $\wp \notin \mathscr{C}$ and $\wp \in (\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{DS}(\mathscr{C}))^{C}$ results that $\wp \notin \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{DS}(\mathscr{C})$. Hence, $\wp$ is not a $\mathfrak{lmt.pt}$. of $\mathscr{C}$.

By the Definition7.1, $\exists$ a $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathfrak{nbhd}$, $\mathscr{D}$ of $\wp$ for which $(\mathscr{D} \cap \mathscr{C} - \{\wp\}) = \emptyset$ As a result no point of $\mathscr{D}$ is the $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathfrak{lmt.pt}$. of $\mathscr{C}$. Therefore, $\mathscr{D} \not\subseteq \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{DS}(\mathscr{C})$ which then follows that $\mathscr{D} \subseteq \cup - \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{DS}(\mathscr{C})$ i.e., $\wp \in \mathscr{D} \subseteq \mathfrak{U} - \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{DS}(\mathscr{C})$. Since $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{DS}(\mathscr{C})$ is $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{Clo}.$, $\mathfrak{U} - \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{DS}(\mathscr{C})$ is $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{Op}$. Also, $\mathscr{D} \subseteq \mathscr{C}^{C}$ is $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{Op}$. Therefore, $(\mathscr{C} \cup \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{DS}(\mathscr{C}))^{C} = \mathscr{C}^{C} \cap (\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{DS}(\mathscr{C}))^{C}$ is $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{Op}$. Hence, $\mathscr{C} \cup \mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{DS}(\mathscr{C})$ is $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{Clo}$. □

8. Conclusion

The concepts introduced in this paper contribute to the theoretical development of general topology, particularly in the $\mathcal{M}$.T.S. The notion of a new class of $\mathcal{M}.\mathfrak{nbhd}$ namely a micro generalized star semi neighborhood (briefly $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathfrak{nbhd}$) was introduced and its characteristics were explored. The basic properties of micro generalized star semi interior (briefly, $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{Intr}.$), micro generalized star semi closure (briefly, $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{Clr}.$) and its interrelationships were discussed in both general and particular cases. The notions of micro generalized star semi limit point (briefly, $\mathcal{M}\text{-}g^{\star}\text{-}\mathfrak{S}.\mathfrak{lmt.pt}.$) and micro generalized star semi derived set (briefly, $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{DS}$) are defined and their characteristics and relationship with $\mathcal{M}\text{-}g^{\star}\mathfrak{S}.\mathrm{Clo}.$ sets were discussed briefly. These concepts in turn help in studying additional properties such as separation axioms and their relationships. The results also provide a basis for further research on connectedness, compactness and locally closed sets in $\mathcal{M}$.T.S.

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